Sommerman, H. H.
1934

Teaching techniques applied to plane geometry.

School of Education
July 31, 1934
12322

123225

Ed.
Thesis
Sommerman
1934
Stored

BOSTON UNIVERSITY
SCHOOL OF EDUCATION

Thesis

TEACHING TECHNIQUES APPLIED TO PLANE GEOMETRY

Submitted by
Henry Herman Sommerman
(A.B., Clark University, 1922)

In partial fulfillment of requirements for the
degree of Master of Education.

1934.

First reader: G. M. Wilson, Professor of Education
Second reader: Franklin Roberts, Assistant Professor of Education

Boston University
School of Education
Library

FOREWORD

Many of the references for this thesis were gathered originally in the form of stenographic notes. For this reason it is possible that, in spite of care, errors in quotation may occur. For such errors the author asks leniency from the authors quoted on the ground that they are not made with malice aforethought.

Digitized by the Internet Archive
in 2017 with funding from
Boston Library Consortium Member Libraries

https://archive.org/details/teachingtechniqu00somm

TABLE OF CONTENTS

TEACHING TECHNIQUES APPLIED TO PLANE GEOMETRY.

PART I -- INTRODUCTION Page

A. The Thesis 1

B. Why Motivate, and How Motivate. 7

PART II- ORIENTATION.

A. Introductory. 29

B. Definitions. 37

C. Constructions and Measurements. 58

D. Creating a Felt Need for Geometric Demonstration. 69

E. How to Distinguish the Hypothesis from the Conclusion in Plane Geometry. 89

PART III- PLANE GEOMETRY UNITS.

A. The Teaching of Congruency in Plane Geometry. 97

B. The Base Angles of an Isosceles Triangle Are Equal. 106

C. Proportion and Similarity. 120

D. Loci. 142

PART IV-- GENERAL METHODS.

A. Organization and Continuity. 154

B. Methods of Proof. 163

C. Provision for Individual Differences. 170

D. Classroom Procedure. 182

E. Drill. 197

F. Miscellaneous Devices for Motivation. 207

PART V-- SUMMARY. 230

TABLE OF CONTENTS

TEACHING TECHNIQUES APPLIED TO PLANE GEOMETRY.

PART I -- INTRODUCTION Page

A. The Thesis 1

B. Why Motivate, and How Motivate. 7

PART II- ORIENTATION.

A. Introductory. 29

B. Definitions. 37

C. Constructions and Measurements. 58

D. Creating a Felt Need for Geometric Demonstration. 69

E. How to Distinguish the Hypotheses from the Conclusion in Plane Geometry. 89

PART III- PLANE GEOMETRY UNITS.

A. The Teaching of Congruency in Plane Geometry. 97

B. The Base Angles of an Isosceles Triangle Are Equal. 106

C. Proportion and Similarity. 120

D. Loci. 142

PART IV-- GENERAL METHODS.

A. Organization and Continuity. 154

B. Methods of Proof. 163

C. Provision for Individual Differences. 170

D. Classroom Procedure. 182

E. Drill. 197

F. Miscellaneous Devices for Motivation. 207

PART V-- SUMMARY. 230

TEACHING TECHNIQUES APPLIED TO PLANE GEOMETRY.

I. INTRODUCTION.

A. The Thesis.

1. Purpose of Thesis.

a). To illustrate how devices for motivation may be applied to classroom teaching of plane geometry.

The primary purpose of this thesis is to bring together for possible use of teachers of mathematics, opportunities for motivation in plane geometry. The procedure has been to gather from literature current practices in motivation in this subject. Reports of how the principles of motivation have been applied to teaching situations have been collected principally from the Mathematics Teacher, 1926-1934, School Science and Mathematics, 1930-1932, the yearbooks of the National Council of Teachers of Mathematics, and various books on teaching secondary school mathematics. The author, who has had twelve years of experience in the teaching of plane geometry, has attempted to show in detail how motivation has entered his classroom, along with the authoritative background for the various procedures used.

2. Delimitation.

a). Not exhaustive in literature study. Although this thesis embraces a bibliography of current literature on the subject, this list is by no means an exhaustive one. Mere collections of material are costly and have but slight value.

TEACHING TECHNIQUES APPLIED TO PLANE GEOMETRY.

I. INTRODUCTION.

A. The Thesis.

1. Purpose of Thesis.

a). To illustrate how devices for motivation may be applied to classroom teaching of plane geometry.

The primary purpose of this thesis is to bring together for possible use of teachers of mathematics, opportunities for motivation in plane geometry. The procedure has been to gather from literature current practices in motivation in this subject. Reports of how the principles of motivation have been applied to teaching situations have been collected principally from the Mathematics Teacher, 1926-1934, School Science and Mathematics, 1930-1932, the yearbooks of the National Council of Teachers of Mathematics, and various books on teaching secondary school mathematics. The author, who has had twelve years of experience in the teaching of plane geometry, has attempted to show in detail how motivation has entered his classroom, along with the authoritative background for the various procedures used.

2. Delimitation.

a). Not exhaustive in literature study. Although this thesis embraces a bibliography of current literature on the subject, this list is by no means an exhaustive one. Here collections of material are costly and have but slight value.

Generalizations are not too valuable, also, in the opinion of Professor Wm. D. Reeve, who writes:[1]

> "The actual trends in teaching of mathematics today can best be discovered by the specific study of the present proofs in books of mathematics. The time and expense to make a study of these practices is questionable and what is more vital, the significance of such a tabulation is in itself unimportant. We do not care in particular what is being done in general, but we are wholeheartedly interested in the indication of future trends as we see them functioning in progressive communities or as recorded in articles by frontier thinkers."

For those interested in exhaustive researches into the literature of this subject, the Committee on Geometry appointed by President Betz of The National Council of Mathematics Teachers gives valuable material. In November, 1932 this committee gave a preliminary report on their work, part of which is as follows:[2]

> "The first item of the committee's plan of action was to assemble the suggestions which have been made since 1900 or thereabouts concerning the teaching of geometry both intuitive and demonstrative.
>
> .....
>
> "The committee is interested to collect ideas of philosophy of geometry and its psychological aspects, to see later how these are reflected in the choice of content.
>
> .....
>
> "With this body of opinion before it, the committee plans to proceed to the second item on its list: Formulation of a few distinct philosphies of teaching geometry."

It will be seen from the above that a collection of material and an evaluation of current practices is being made which is far above the power of any single thesis.

1. "United States". The National Council of Teachers of Mathematics. The Fourth Yearbook. 1929, p. 135.

2. Ralph Beatley, "Preliminary Report of the Committee on Geometry". The Mathematics Teacher. XXV (November, 1932). pp. 427-428.

Generalizations are not too valuable, also, in the opinion of Professor Wm. D. Reeve, who writes:[1]

> "The actual trends in teaching of mathematics today can best be discovered by the specific study of the present proofs in books of mathematics. The time and expense to make a study of these practices is questionable and what is more vital, the significance of such a tabulation is in itself unimportant. We do not care in particular what is being done in general, but we are wholeheartedly interested in the indication of future trends as we see them functioning in progressive communities or as recorded in articles by frontier thinkers."

For those interested in exhaustive researches into the literature of this subject, the Committee on Geometry appointed by President Betz of The National Council of Mathematics Teachers gives valuable material. In November, 1932 this committee gave a preliminary report on their work, part of which is as follows:[2]

> "The first item of the committee's plan of action was to assemble the suggestions which have been made since 1900 or thereabouts concerning the teaching of geometry both intuitive and demonstrative.
>
> .....
>
> "The committee is interested to collect ideas of philosophy of geometry and its psychological aspects, to see later how these are reflected in the choice of content.
>
> .....
>
> "With this body of opinion before it, the committee plans to proceed to the second item on its list: Formulation of a few distinct philosophies of teaching geometry."

It will be seen from the above that a collection of material and an evaluation of current practices is being made which is far above the power of any single thesis.

---

1. "United States". The National Council of Teachers of Mathematics. The Fourth Yearbook. 1929, p. 135.

2. Ralph Beatley, "Preliminary Report of the Committee on Geometry". The Mathematics Teacher. XXV (November, 1932). pp. 427-428.

More recently Ralph Beatley[1] has published a list of the material examined for the above survey and notes that "The committee is proceeding to the formulation of a few distinct philosophies of the teaching of geometry which will later be subjected to experimental trial in cooperation with the schools" He further notes that "The chief purpose (of extending the work of geometry over several years in Europe) seems to be to make the child so familiar with geometric concepts that when he studies demonstrative geometry he may consider it as such with no distractions, devoting much time to originals and methods of proof." This seems to be a key to some of our difficulties in the teaching of geometry. As noted elsewhere in the thesis, difficulties must be isolated. Careful preparation for definitions must be made, and proper introductory work done, before the work on demonstrations is attempted. Probably much of the distaste for geometry in the past has been failure in properly introducing the subject or appreciating the many things which the pupil has to learn. Beatley[2] says: "In our own country we tend to offer a meagre introductory course in the junior high school, followed by a logical presentation in the senior high school modeled after Legendre's ordering of the theorems."

---

1. Second report of the Committee on Geometry, Presented to the National Council of Teachers of Mathematics at its Annual Meeting in Minneapolis on February 24, 1933. The Mathematics Teacher, XXVI (October 1933), pp. 266-271.

2. Ralph Beatley, loc. cit.

More recently Ralph Beatley[1] has published a list of the material examined for the above survey and notes that "The committee is proceeding to the formulation of a few distinct philosophies of the teaching of geometry which will later be subjected to experimental trial in cooperation with the schools." He further notes that "The chief purpose (of extending the work of geometry over several years in Europe) seems to be to make the child so familiar with geometric concepts that when he studies demonstrative geometry he may consider it as such with no distractions, devoting much time to original methods of proof." This seems to be a key to some of our difficulties in the teaching of geometry. As noted elsewhere in the thesis, difficulties must be isolated. Careful preparation for definitions must be made, and proper introductory work done, before the work on demonstrations is attempted. Probably much of the distaste for geometry in the past has been failure in properly introducing the subject or appreciating the many things which the pupil has to learn. Beatley[2] says: "In our own country we tend to offer a meagre introductory course in the junior high school, followed by a logical presentation in the senior high school modeled after Legendre's ordering of the theorems."

---

1. Second report of the Committee on Geometry, Presented to the National Council of Teachers of Mathematics at its Annual Meeting in Minneapolis on February 24, 1933. The Mathematics Teacher, XXVI (October 1933), pp. 266-271.

2. Ralph Beatley, loc. cit.

Some of the material gathered by the above committee has come to the attention of the author, but it is to be emphasized that the primary function of this thesis is not the collection of literature but suggestions by the teachers themselves and by the author as a teacher of how to motivate in the classroom. Of course it is not implied that the teacher follow necessarily the same suggestions, but rather that these detailed suggestions will serve to act as further suggestions to the teacher as to how to motivate his class in plane geometry. Each teacher's equipment differs from another's; each must choose the course of procedure which will make for his success.

b). Involving actual application of devices to classroom teaching to the exclusion of other related topics.

This thesis does not concern itself with the history of motivation. Neither does it concern itself with the subject of general motivation, nor with the values and aims of the subject, nor with the organization of the course, nor with various debatable points as to the transfer value, the contents, and the combination with solid geometry. Anything general in character is also omitted from the thesis, such as the general subject of interrelationships, or general psychological laws.

Some of the material gathered by the above committee has come to the attention of the author, but it is to be emphasized that the primary function of this thesis is not the collection of literature but suggestions by the teachers themselves and by the author as a teacher of how to motivate in the classroom. Of course it is not implied that the teacher follow necessarily the same suggestions, but rather that these detailed suggestions will serve to act as further suggestions to the teacher as to how to motivate his class in plane geometry. Each teacher's equipment differs from another's; each must choose the course of procedure which will make for his success.

b). Involving actual application of devices to classroom teaching to the exclusion of other related topics.

This thesis does not concern itself with the history of motivation. Neither does it concern itself with the subject of general motivation, nor with the values and aims of the subject, nor with the organization of the course, nor with various debatable points as to the transfer value, the contents, and the combination with solid geometry. Anything general in character is also omitted from the thesis, such as the general subject of interrelationships, or general psychological laws.

The bulk of the work is confined strictly to the application of specific devices in particular situations with the characteristic aim of being suggestive for further application.

3. Books and Theses.

a). Number of reprints of book by Arthur Schultze. It is recognized that when one comes to particular applications, one is treading on dangerous ground. General suggestions are so widespread in their application that they are usually quite safe, but when the attempt is made to help by applying in a definite manner, the writer has to be exceedingly careful that the material which is so live and vital in his particular hands will not become so much dead wood when placed on paper. It is hoped that this thesis is sufficiently vivid to point the ways to motivation of plane geometry to the various readers. That the need of efficient helps in classroom teaching is greatly desired is evident from the reception accorded various theses and texts which have been successful in giving real helps to the teachers.

It is interesting to note in this connection that the book by Arthur Schultze, "The Teaching of Mathematics in Secondary Schools", first printed in 1912, went through a number of reprints up to 1928, so eager were teachers for real helps in the teaching of plane geometry.

b). Theses bearing on the subject. The author has been unable to discover, however, any thesis on the application of motivation techniques to classroom teaching in plane geometry. The National Education Association of the United States reports that

The bulk of the work is confined strictly to the application of specific devices in particular situations with the characteristic aim of being suggestive for further application.

3. Books and Theses.

a). Number of reprints of book by Arthur Schultze. It is recognized that when one comes to particular applications, one is treading on dangerous ground. General suggestions are so widespread in their application that they are usually quite safe, but when the attempt is made to help by applying in a definite manner, the writer has to be exceedingly careful that the material which is so live and vital in his particular hands will not become so much dead wood when placed on paper. It is hoped that this thesis is sufficiently vivid to point the ways to motivation of plane geometry to the various readers. That the need of efficient helps in classroom teaching is greatly desired is evident from the reception accorded various theses and texts which have been successful in giving real helps to the teachers.

It is interesting to note in this connection that the book by Arthur Schultze, "The Teaching of Mathematics in Secondary Schools", first printed in 1912, went through a number of reprints up to 1928, so eager were teachers for real helps in the teaching of plane geometry.

b). Theses bearing on the subject. The author has been unable to discover, however, any thesis on the application of motivation techniques to classroom teaching in plane geometry. The National Education Association of the United States reports that

it is unable to locate in its files any questionnaires on the teaching of geometry.

(1). Among valuable writings on the subject is the thesis by Louise Barbour Eddy, University of Chicago, 1930, "Motivating the Study of Geometry". This thesis involved the collection of a considerable body of material from mathematical literature. The study includes tracing the historical development of ways of motivating the study of plane geometry, the motivating devices used in text books, and the change in tendencies in the last forty-five years. No attempt at evaluation and comparison of motivation material is made by that author, nor does it touch on suggestions for motivated units of instruction. The material does suggest motivation on short exercises and shows that teachers realize the importance of interest in emphasizing geometry as an appreciation unit.

(2). A second thesis is that of Alma Wolf.[1] This contains an evaluation of researches in secondary mathematics for the years 1926-1931. Miss Wolf reviews the Eddy thesis.

(3). A third thesis is that written by Agnes Wallace Templeton.[2] This thesis consists of a summary of the 417 articles appearing in "The Mathematics Teacher" over a period of twenty-two years. It is valuable as a source of quick contact with the literature in this magazine.

1. The Teaching of Secondary Mathematics, 1932, Boston University Unpublished Master of Arts Thesis.
2. A survey of the Magazine, "The Mathematics Teacher", 1932. Unpublished Master of Arts Thesis.

it is unable to locate in its files any questionnaires on the teaching of geometry.

(1). Among valuable writings on the subject is the thesis by Louise Barbour Eddy, University of Chicago, 1930, "Motivating the Study of Geometry". This thesis involved the collection of a considerable body of material from mathematical literature. The study includes tracing the historical development of ways of motivating the study of plane geometry, the motivating devices used in text books, and the change in tendencies in the last forty-five years. No attempt at evaluation and comparison of motivation material is made by that author, nor does it touch on suggestions for motivated units of instruction. The material does suggest motivation on short exercises and shows that teachers realize the importance of interest in emphasizing geometry as an appreciation unit.

(2). A second thesis is that of Alma Wolf.[1] This contains an evaluation of researches in secondary mathematics for the years 1926-1931. Miss Wolf reviews the Eddy thesis.

(3). A third thesis is that written by Agnes Wallace Tennyson.[2] This thesis consists of a summary of the 417 articles appearing in "The Mathematics Teacher" over a period of twenty-two years. It is valuable as a source of quick contact with the literature in this magazine.

---

1. The Teaching of Secondary Mathematics, 1932, Boston University Unpublished Master of Arts Thesis.
2. A survey of the Magazine, "The Mathematics Teacher", 1932. Unpublished Master of Arts Thesis.

Although the theses mentioned above all have valuable material to offer for those interested in an acquaintance with the literature available, they leave the field open for suggestions on the actual motivation of geometry. Their purpose is largely historical in nature. The writer of this thesis has endeavored to consider the dynamic aspects of plane geometry as the subject is in his classroom.

Assuming that the view that school subjects should be motivated is accepted, the next consideration is the special difficulties of geometry toward which motivation is to be directed.

B. Why Motivate and How Motivate.

1. Geometry has special difficulties.

a). General Discussion. A frank statement to the geometry class by the teacher of the task to be accomplished is of use in making the work definite and in showing the pupil the reasons for the difficulties of this particular subject. Next comes, then, a consideration of the difficulties peculiar to plane geometry, followed by suggestions as to how to master these situations.

An additional difficulty of plane geometry beyond its abstract nature is that it is a flat surface subject. This characteristic accentuates the difficulty of the abstract nature. The solution of these two difficulties is closely linked and will be considered shortly. While solid geometry has the advantage of being three-dimensional in nature, plane geometry has to be motivated into a solid figure. The discussion of from symbol to object, p.11, suggests how this may be done.

Although the theses mentioned above all have valuable material to offer for those interested in an acquaintance with the literature available, they leave the field open for suggestions on the actual motivation of geometry. Their purpose is largely historical in nature. The writer of this thesis has endeavored to consider the dynamic aspects of plane geometry as the subject is in his classroom.

Assuming that the view that school subjects should be motivated is accepted, the next consideration is the special difficulties of geometry toward which motivation is to be directed.

B. Why Motivate and How Motivate.

1. Geometry has special difficulties.

a). General Discussion. A frank statement to the geometry class by the teacher of the task to be accomplished is of use in making the work definite and in showing the pupil the reasons for the difficulties of this particular subject. Next comes, then, a consideration of the difficulties peculiar to plane geometry, followed by suggestions as to how to master these situations.

An additional difficulty of plane geometry beyond its abstract nature is that it is a flat surface subject. This characteristic accentuates the difficulty of the abstract nature. The solution of these two difficulties is closely linked and will be considered shortly. While solid geometry has the advantage of being three-dimensional in nature, plane geometry has to be motivated into a solid figure. The discussion of from symbol to object, p.11, suggests how this may be done.

Other difficulties peculiar to plane geometry are the cumulative nature of the subject, the new procedure in the demonstration, and the fact that it is a reflective subject. Of these three difficulties, the first and third are found in some of the other school subjects, but the second is largely peculiar to geometry. In this unit plans are suggested for the overcoming of these difficulties. The suggestions for the difficulty from the cumulative nature is the isolation of the difficulty and proper testing through adequate reviews. Another remedy taken up in the unit on Continuity and Organization is the grouping of theorems for better understanding and retention. The discussion of the solution to the new procedure and the reflective subject difficulty is also considered shortly.

Following the discussion of the difficulties and remedies, may come consideration of ideas aiding. Among these are the margin of safety idea, the universality of plane geometry, and lastly the power of analogies. The first two are discussed in this section; the last is taken up in detail in the unit on miscellaneous devices on attitude, adjacent to the interrelationships with other subjects.

b). Flatland and its remedies. First is the idea which William D. Reevel presents in his title of plane geometry as "Geometry of Flatland" which is studied, as he says while living in a world of three dimensions. There is much discussion today in regard to a combined course of plane and solid geometry.

1. "United States". The National Council of Teachers of Mathematics, The Fourth Yearbook, 1929, p. 162.

Other difficulties peculiar to plane geometry are the cumulative nature of the subject, the new procedure in the demonstration, and the fact that it is a reflective subject. Of these three difficulties, the first and third are found in some of the other school subjects, but the second is largely peculiar to geometry. In this unit plans are suggested for the overcoming of these difficulties. The suggestions for the difficulty from the cumulative nature is the isolation of the difficulty and proper testing through adequate reviews.

Another remedy taken up in the unit on Continuity and Organization is the grouping of theorems for better understanding and retention. The discussion of the solution to the new procedure and the reflective subject difficulty is also considered shortly.

Following the discussion of the difficulties and remedies, may come consideration of ideas aiding. Among these are the margin of safety idea, the universality of plane geometry, and lastly the power of analogies. The first two are discussed in this section; the last is taken up in detail in the unit on miscellaneous devices on attitude, adjacent to the interrelationships with other subjects.

b). Flatland and its remedies. First is the idea which William D. Reeves presents in his title of plane geometry as "Geometry of Flatland" which is studied, as he says while living in a world of three dimensions. There is much discussion today in regard to a combined course of plane and solid geometry.

1. "United States". The National Council of Teachers of Mathematics. The Fourth Yearbook. 1929. p. 128.

It is not the purpose here to take sides, nor to present the history or status of this question, but one can say that solid geometry attached to plane geometry would give to it an added impetus and take it out of the "Geometry of Flatland". It is well to give a parting tribute to plane geometry in connection with an introduction to solid geometry and tell the pupils that plane geometry is comparable to the moving pictures when seen from near the edge of the screen, while solid geometry is more like the actual play on the stage.

c). Abstract nature and its remedies. Next consider the peculiar difficulty of the subject because of its abstract nature. While it is not symbolic to the extent that the traditional algebra has been, yet it is far removed from the sciences which have for their exhibits actual materials, whether they be biological, chemical, physical, or astronomical. From Marie Montessori came vividly to us the necessity of the object; this was a step beyond Comenius with his picture book. Maeterlinck realized the necessity of adequately presenting general principles and abstract relationships when he wrote his play "The Blue Bird". Here one has motivation through the object attached to human beings. "Bread" slices a piece from his stomach; "Sugar" breaks off his fingers of sticks of candy; "Fire" and "Water" have their interplay represented in appropriate colors and lines by human forms, and finally "Happiness" itself is in the little home bluebird. Color, music, and human symbolization have vitalized the abstration of the moral lesson from a faint vague impression to a beautiful

It is not the purpose here to take sides, nor to present the history or status of this question, but one can say that solid geometry attached to plane geometry would give to it an added impetus and take it out of the "Geometry of Flatland". It is well to give a parting tribute to plane geometry in connection with an introduction to solid geometry and tell the pupils that plane geometry is comparable to the moving pictures when seen from near the edge of the screen, while solid geometry is more like the actual play on the stage.

c). Abstract nature and its remedies. Next consider the peculiar difficulty of the subject because of its abstract nature. While it is not symbolic to the extent that the traditional algebra has been, yet it is far removed from the sciences which have for their exhibits actual materials, whether they be biological, chemical, physical, or astronomical. From Maria Montessori came vividly to us the necessity of the object; this was a step beyond Comenius with his picture book. Masterfinck realized the necessity of adequately presenting general principles and abstract relationships when he wrote his play "The Blue Bird". Here one has motivation through the object attached to human beings. "Bread" allows a piece from his stomach; "Sugar" breaks off his fingers of sticks of candy; "Fire" and "Water" have their interplay represented in appropriate colors and lines by human forms, and finally "Happiness" itself is in the little home bluebird. Color, music, and human symbolization have vitalized the abstraction of the moral lesson from a faint vague impression to a beautiful

vivid imprint on the human mind. And so one must go with Betz[1] back into the early dawn of civilization, and forward with the modern astronomers of the universe to find our subject of plane geometry controlling these concrete and beautiful things of the earth and of the heaven. At every opportunity one must lead the pupil to attach these shapes to life. He must be led backward from the abstract symbolism through the language, which is symbolic except that by long acquaintance it has become less strange, through the picture, and finally to the object. Learning must go through these steps. Only then will the channel between the subject of plane geometry and the pupil be opened. It is the teacher's task to arrange for the flow of activity from pupil to subject; it is his work and privilege to assist in the continuance of the flow and his success will be measured by the further continuance when his is no longer there.

(1). Symbol to object. Below is a parallel grouping of examples to show how the pupil may slowly get a comprehension of the need of symbolism in geometry. At one end of the line is the vividness of the object and at the other end is the shorthand symbol for the object. The former is valuable for its ease of comprehension; the latter valuable for its ease and speed of notation, but the former must be understood before the latter is significant. Like all new things, learning the symbol requires effort but pays big dividends in greater power and speed.

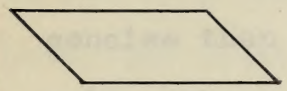
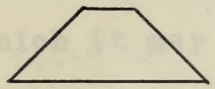
1. "The Teaching of Intuitive Geometry". The National Council of Teachers of Mathematics, The Eighth Yearbook. 1933. pp.84-95.

vivid imprint on the human mind. And so one must go with Betzi back into the early dawn of civilization, and forward with the modern astronomers of the universe to find our subject of plane geometry controlling these concrete and beautiful things of the earth and of the heaven. At every opportunity one must lead the pupil to attach these shapes to life. He must be led backward from the abstract symbolism through the language, which is symbolic except that by long acquaintance it has become less strange, through the picture, and finally to the object. Learning must go through these steps. Only then will the channel between the subject of plane geometry and the pupil be opened. It is the teacher's task to arrange for the flow of activity from pupil to subject; it is his work and privilege to assist in the continuance of the flow and his success will be measured by the further continuance when his is no longer there.

(1). <u>Symbol to object.</u> Below is a parallel grouping of examples to show how the pupil may slowly get a comprehension of the need of symbolism in geometry. At one end of the line is the vividness of the object and at the other end is the short-hand symbol for the object. The former is valuable for its ease of comprehension; the latter valuable for its ease and speed of notation, but the former must be understood before the latter is significant. Like all new things, learning the symbol requires effort but pays big dividends in greater power and speed.

1. "The Teaching of Intuitive Geometry". The National Council of Teachers of Mathematics, <u>The Eighth Yearbook</u>. 1933. pp.84-93.

These examples are illustrative. Each geometric concept may find its way from symbol to object through these successive steps. The pupil will find his familiar approach by first seeing the object and then being taken gradually to the picture and then to the language and finally to the symbol.

Symbol	Verbal	Picture	Object.
$\cong$	Congruent. Same size and shape.	Two triangles	Triangles of wood, apple-pie, etc.
$\sim$	Similar--Same shape	Two polygons or almost any picture or photograph.	Models of apparatus--Toy autos, etc.
	Parallelogram. Opposite sides parallel.	Figures.	Tennis courts, baseball diamond boxes.
	Trapezoid--one-pair opposite sides parallel	Figure--Picture of dam.	Actual dam reviewed.

As noted above where parallel columns of illustrations were made of the symbol to the object, this idea is important to emphasize. In the process of learning, the four degrees of difficulty might arrange themselves in the following order;

(a) The object, the material thing, can be handled, seen, weighed and sometimes smelled, involving at all times the tactile, visual, and barometric senses, and sometimes the olfactory.

(b) The picture uses the visual approach, the eye. It is an illuminated symbol of the object; it is a plane object which still permits the use of the tactile sense to some extent, as in the case of wooden triangles for example,

These examples are illustrative. Each geometric concept may find its way from symbol to object through these successive steps. The pupil will find his familiar approach by first seeing the object and then being taken gradually to the picture and then to the language and finally to the symbol.

Symbol	Verbal	Picture	Object.
	Congruent. Same size and shape.	Two triangles	Triangles of wood, apple-pie, etc.
	Similar--Same shape	Two polygons or almost any picture or photograph.	Models of apparatus--Toy autos, etc.
	Parallelogram. Opposite sides parallel.	Figures.	Tennis courts, baseball diamond boxes.
	Trapezoid--one-pair opposite sides parallel	Figure--picture of dam.	Actual dam reviewed.

As noted above where parallel columns of illustrations were made of the symbol to the object, this idea is important to emphasize. In the process of learning, the four degrees of difficulty might arrange themselves in the following order:

(a) The object, the material thing, can be handled, seen, weighed and sometimes smelled, involving at all times the tactile, visual, and barometric senses, and sometimes the olfactory.

(b) The picture uses the visual approach, the eye. It is an illuminated symbol of the object; it is a plane object which still permits the use of the tactile sense to some extent, as in the case of wooden triangles for example.

but the barometric and olfactory avenues of approach are not open.

(c) The language, which gains its power from early association and constant use, may be visual or auditory in approach. It awakens the other senses only when the reader or listener becomes emotionally identified with the situation. One lives through tales of adventure written in books; one becomes thirsty as the lecturer tells of the lost people in the desert; one grows hungry and cold with the explorer in the frozen north.

(d) The shorthand expression of symbolism, which is more concise than the symbolic word, is the last and most difficult. There are fewer of the various sense organs through which it may enter the mind; it is a language, but an unfamiliar language; it lacks the early associates which the written word has. It has not the power of bringing emotional identification as with the word. Its only sensory approach is through the eye. Whatever thoughts it arouses in the consciousness must come through association.

Because of the richness of sensory approaches of the object and the poverty of the approaches through the symbol, in teaching it is important to lead up gradually to the symbol, beginning with the object. It is because we have made pupils go directly to the learning of the symbol that much of the distaste for school subjects has come. In concluding, however, it would be fair to say that the approach through the symbol has one advantage. Many young people like mystery and like to use a language of symbols.

but the barometric and olfactory avenues of approach are not open.

(c) The language, which gains its power from early association and constant use, may be visual or auditory in approach. It awakens the other senses only when the reader or listener becomes emotionally identified with the situation. One lives through tales of adventure written in books; one becomes thirsty as the lecturer tells of the lost people in the desert; one grows hungry and cold with the explorer in the frozen north.

(d) The shorthand expression of symbolism, which is more concise than the symbolic word, is the last and most difficult. There are fewer of the various sense organs through which it may enter the mind; it is a language, but an unfamiliar language; it lacks the early associates which the written word has. It has not the power of bringing emotional identification as with the word. Its only sensory approach is through the eye. Whatever thoughts it arouses in the consciousness must come through association.

Because of the richness of sensory approaches of the object and the poverty of the approaches through the symbol, in teaching it is important to lead up gradually to the symbol, beginning with the object. It is because we have made pupils go directly to the learning of the symbol that much of the distaste for school subjects has come. In concluding, however, it would be fair to say that the approach through the symbol has one advantage. Many young people like mystery and like to use a language of symbols.

One can recall his clubs with their favorite symbolic emblems, his secret codes between friends, and his delight at writing "hog-latin".

(2). Presenting the familiar ideas. Continuing the progress to overcome the difficulty in plane geometry arising from its abstract nature, the object may be made interesting to the pupil by linking it with that which is familiar and pleasant to him. It is this idea which is behind all persuasion both in the commercial and the educational world. "Have you ever seen anything like that?" is the question which brings back the memories. It is for this reason that geometry reaches out into the room, the home, the local environment, the school buildings, and then to the reading experience of the pupil, and into their actual experiences with life itself. Eagerly the child grasps his mother's hand in the strange locality; whereas he lightly romps away from her at home. So it is through life. Songs have capitalized this feeling of familiarity. Reunions have exemplified it, and the tenderness of early home scenes are examples of it. No teacher can do too much in encouraging the flow of activity by means of the oar of familiarity. This is why the teacher who knows the whole pupil has much material to make the work successful. The humaneness of human beings makes possible the power through interest in their families, in their moods, in their interests, and in their desires. It is here where the individual assignments play their part. The teacher's opportunity is to take the concept and mount the magic carpet and soar off into space.

One can recall his clubs with their favorite symbolic emblems, his secret codes between friends, and his delight at writing "hog-latin".

(2). Presenting the familiar ideas. Continuing the progress to overcome the difficulty in plane geometry arising from its abstract nature, the object may be made interesting to the pupil by linking it with that which is familiar and pleasant to him. It is this idea which is behind all persuasion both in the commercial and the educational world. "Have you ever seen anything like that?" is the question which brings back the memories. It is for this reason that geometry reaches out into the room, the home, the local environment, the school buildings, and then to the reading experience of the pupil, and into their actual experiences with life itself. Eagerly the child grasps his mother's hand in the strange locality; whereas he lightly romps away from her at home. So it is through life. Songs have capitalized this feeling of familiarity. Reunions have exemplified it, and the tenderness of early home scenes are examples of it. No teacher can go too much in encouraging the flow of activity by means of the ear of familiarity. This is why the teacher who knows the whole pupil has much material to make the work successful. The humanness of human beings makes possible the power through interest in their families, in their moods, in their interests, and in their desires. It is here where the individual assignments play their part. The teacher's opportunity is to take the concept and mount the magic carpet and soar off into space.

For the boys, many sports are concerned with geometric figures, such as the baseball field which is a parallelogram, or double-triangle.

Pleasantness may also come from relating geometric figures to candy, pie or ice cream.

Expecially important is this vitalizing by instilling life into the form when the pupil has reached some absurdity in his reasoning. An example of this occurs when the pupil states that since vertical angles are equal, the arcs subtended by them are also equal, and therefore the opposite areas of the sectors of the circle are equal. A rapid drawing on the board, complying exactly with his directions, shows the pupil the falsity of his reasoning. Humor may be brought into the situation for the benefit of the class by calling the areas "ice-cream" areas, and then gravely accepting the larger of the two sectors which he has declared are equal. The pupil sits limply nursing his false assumption and his appreciably smaller "ice-cream" circle area, amid the amused smiles of his classmates. When given the opportunity he quickly corrects the figures, making the angles central angles.

Anything which will magnify an error so that it will not be made again is increased surety for an early correction of the starting image, so necessary to successful procedure.

In the consideration of overcoming the abstract nature of plane geometry, then, we can by means of leading back to the interesting object, motivate an attitude toward geometry with emphasis on the appreciation viewpoint.

For the boys, many sports are concerned with geometric figures, such as the baseball field which is a parallelogram, or double-triangle.

Pleasantness may also come from relating geometric figures to candy, pie or ice cream.

Especially important is this vitalizing by instilling life into the form when the pupil has reached some absurdity in his reasoning. An example of this occurs when the pupil states that since vertical angles are equal, the arcs subtended by them are also equal, and therefore the opposite areas of the sectors of the circle are equal. A rapid drawing on the board, complying exactly with his directions, shows the pupil the falsity of his reasoning. Humor may be brought into the situation for the benefit of the class by calling the areas "ice-cream" areas, and then gravely accepting the larger of the two sectors which he has declared are equal. The pupil sits limply nursing his false assumption and his appreciably smaller "ice-cream" circle area, amid the amused smiles of his classmates. When given the opportunity he quickly corrects the figures, making the angles central angles.

Anything which will magnify an error so that it will not be made again is increased surely for an early correction of the starting image, as necessary to successful procedure.

In the consideration of overcoming the abstract nature of plane geometry, then, we can by means of leading back to the interesting object, motivate an attitude toward geometry with emphasis on the appreciation viewpoint.

d). Cumulative nature. Another difficulty not found in some subjects is the cumulative nature. This is more true of social studies than in mathematics. The success of the day's work depends quite largely on the success of much of the past work. The pupil should be told that the best method of gaining success is to do the work carefully each day. He cannot neglect one portion and then suddenly do successful work in the part following.

Thus if triangles are carefully studied, when pupils study the circle, part of the difficulty of the latter is already overcome, since triangles are in the circles. Again, propositions are built on previous propositions. Solving originals depends largely on a thorough knowledge of prior theorems. The pupil must be friends with them in order to have them friendly to him. He must smile at them that they may smile at him.

(1). The difficulty isolated.

(a). General. Much of the success in teaching and learning depends on how sharply the light of attention may be focussed on the particular matter at hand. Another way of putting it would be the learning of one thing at a time. The new difficulty can be mastered best if all surrounding difficulties are removed. If the light of attention has to be spread over a variety of difficulties at once, learning is apt not to be successful. The journey of the difficulty in plane geometry, then, as in other school subjects must be first in a simple situation, then with other elements, then in complex situations, and finally with original exercises.

d). Cumulative nature. Another difficulty not found in some subjects is the cumulative nature. This is more true of social studies than in mathematics. The success of the day's work depends quite largely on the success of much of the past work. The pupil should be told that the best method of gaining success is to do the work carefully each day. He cannot neglect one portion and then suddenly do successful work in the part following.

Thus if triangles are carefully studied, when pupils study the circle, part of the difficulty of the latter is already overcome, since triangles are in the circles. Again, propositions are built on previous propositions. Solving originals depends largely on a thorough knowledge of prior theorems. The pupil must be friends with them in order to have them friendly to him. He must smile at them that they may smile at him.

(1). The difficulty isolated.

(a). General. Much of the success in teaching and learning depends on how sharply the light of attention may be focussed on the particular matter at hand. Another way of putting it would be the learning of one thing at a time. The new difficulty can be mastered best if all surrounding difficulties are removed. If the light of attention has to be spread over a variety of difficulties at once, learning is apt not to be successful. The journey of the difficulty in plane geometry, then, as in other school subjects must be first in a simple situation, then with other elements, then in complex situations, and finally with original exercises.

As an example of the journey of the difficulty, let us consider the construction of the perpendicular bisector of a line. First would come the teaching and learning of the perpendicular bisector of a line in the customary horizontal position on the blackboard. Then would follow the bisection of the line drawn in any position on the blackboard, slant or vertical. The next degree of difficulty would be the construction of the bisector when the line was the side of a polygon. Finally, an original problem could be presented which would involve the drawings of a bisector of the line as an auxiliary line. The newer texts contain the application of this idea in their exercises which work up in complexity as they go along.

Just as the difficulty must be isolated when considering the degree of complexity, so may we isolate the mechanical aspects of the proposition. For instance, in considering the geometry figure, the part which is being considered in connection with a polygon may be brought out in bold relief by means of dotted lines, colored chalk, heavy lines, or simply by the erasing of all surrounding parts which returns it to its original simplicity. In the moving picture, this task is accomplished by enlarging the part of the picture to which attention is to be directed and thus having it take the entire screen. For instance, if attention is to be directed to the movement of a hand in a mystery story, then the hand is enlarged to take up the entire screen and to show its movements in detail.

As an example of the journey of the difficulty, let us consider the construction of the perpendicular bisector of a line. First would come the teaching and learning of the perpendicular bisector of a line in the customary horizontal position on the blackboard. Then would follow the bisection of the line drawn in any position on the blackboard, slant or vertical. The next degree of difficulty would be the construction of the bisector when the line was the side of a polygon. Finally, an original problem could be presented which would involve the drawing of a bisector of the line as an auxiliary line. The newer texts contain the application of this idea in their exercises which work up in complexity as they go along.

Just as the difficulty must be isolated when considering the degree of complexity, so may we isolate the mechanical aspects of the proposition. For instance, in considering the geometry figure, the part which is being considered in connection with a polygon may be brought out in bold relief by means of dotted lines, colored chalk, heavy lines, or simply by the erasing of all surrounding parts which returns it to its original simplicity. In the moving picture, this task is accomplished by enlarging the part of the picture to which attention is to be directed and thus having it take the entire screen. For instance, if attention is to be directed to the movement of a hand in a mystery story, then the hand is enlarged to take up the entire screen and to show its movements in detail.

The following comparative table shows that the various steps in the developing of a skill are closely the same regardless of what particular field the skill lies. It would seem, then, that any method of procedure which ignores this scale of steps, is not following out the law of gradualness in the learning process, and is weakening the structure by means of gaps.

(b). Parallel steps.

General	Athletics	Music	Geometry.
Known to unknown			Known to unknown. Geometric designs in life to technical text book work.
1. Simple parts	1. Simple exercises.	1. Simple finger exercises	1. Simple separate parts, lines or angles.
2. Complex parts	2. Complex exercises	2. Complex finger exercises	2. Complex separate parts. Lines and angles.
3. Practice in wholes.	3. Practicing for events.	3. Scales, etc.	3. Fundamental concepts with new part. Ways to prove lines parallel.
4. Trial performance.	4. Class meets.	4. Practice pieces	4. Propositions with new part. Theorems involving parallelograms.
5. Public performance	5. Inter-school meets	5. Public performance	5. Originals involving parallelograms.

The following comparative table shows that the various steps in the developing of a skill are closely the same regardless of what particular field the skill lies. It would seem, then, that any method of procedure which ignores this scale of steps, is not following out the law of gradualness in the learning process and is weakening the structure by means of gaps.

(b). Parallel steps.

General	Athletics	Music	Geometry.
Known to unknown			Known to unknown. Geometric designs in life to technical text book work.
1. Simple parts	1. Simple exercises.	1. Simple finger exercises	1. Simple separate parts. lines or angles.
2. Complex parts	2. Complex exercises	2. Complex finger exercises	2. Complex separate parts. Lines and angles.
3. Practice in wholes.	3. Practicing for events.	3. Scales, etc.	3. Fundamental concepts with new part. Ways to prove lines parallel.
4. Trial performance.	4. Class meets.	4. Practice pieces	4. Propositions with new part. Theorems involving parallelograms.
5. Public performance	5. Inter-school meets	5. Public performance	5. Originals involving parallelograms.

Any pupil who has been through the various steps in the development of one skill becomes accustomed to the idea that there is a sequence of steps, and therefore is able to grasp the importance of conquering each step as it is presented.

Whether in mathematics, music, or athletics, a failure to get a correct start or to conquer the fundamentals means a failure in the ultimate results.

It is sometimes a tedious job to put the puzzle together, or to practise for plays, or musical performances, or to to through the drudgery of learning a foreign language, but if one can keep one's eye on the ultimate goal and see the joy of the finished performance, it will encourage one to keep on toward this ultimate success.

(c). <u>See-it-grow method.</u> Much has been said about the smallness which there must be between the gaps in the learning process. Of late, text books have begun to print more than one figure for a construction example. Each small step may be represented by a different figure. The process may be likened

Any pupil who has been through the various steps in the development of one skill becomes accustomed to the idea that there is a sequence of steps, and therefore is able to grasp the importance of conquering each step as it is presented.

Whether in mathematics, music, or athletics, a failure to get a correct start or to conquer the fundamentals means a failure in the ultimate results.

It is sometimes a tedious job to put the puzzle together, or to practice for plays, or musical performances, or to go through the drudgery of learning a foreign language, but if one can keep one's eye on the ultimate goal and see the joy of the finished performance, it will encourage one to keep on toward this ultimate success.

(c). <u>See-it-grow method.</u> Much has been said about the smallness which there must be between the gaps in the learning process. Of late, text books have begun to print more than one figure for a construction example. Each small step may be represented by a different figure. The process may be likened

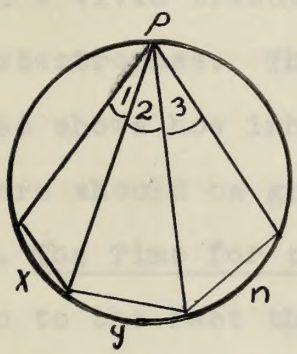
to the little booklets of dancers which used to be printed. Each picture showed a slightly different position. When the pages were thumbed rapidly, the pictures appeared to be in motion. The slow motion picture in the movies present an analogous situation to slow motion in construction work. The printing in a text book of the various figures is the nearest approach to the class-built exercise that a book can get. Here again, the principle at work is the isolation of the difficulty. The complexities are not built into a single figure, but learned and observed separately. Verbally, the principle takes the form of ample time for thought by the pupil after the observance is made by the teacher, and slow enough speech by the teacher or fellow pupil to have the ear receive the initial stimulus clearly.

(d). Restoration to the simple situation. It has already been noted that erasing surrounding parts may restore a figure to original simplicity. When the simple drawing is mastered, the lines may again be sketched in and the figure restored to

to the little booklets of dancers which used to be printed. Each picture showed a slightly different position. When the pages were thumbed rapidly, the pictures appeared to be in motion. The slow motion picture in the movies presents an analogous situation to slow motion in construction work. The printing in a text book of the various figures is the nearest approach to the class-built exercise that a book can get. Here again, the principle at work is the isolation of the difficulty. The complexities are not built into a single figure, but learned and observed separately. Verbally, the principle takes the form of ample time for thought by the pupil after the observance is made by the teacher, and enough speech by the teacher or fellow pupil to have the ear receive the initial stimulus clearly.

(5). Restoration to the simple situation. It has already been noted that erasing surrounding parts may restore a figure to original simplicity. When the simple drawing is mastered, the lines may again be sketched in and the figure restored to

its original form. The following drawing is an example. The figure at the left represents the original figure; the one at the right represents the difficulty isolated with the surrounding lines erased. After the difficulty has been conquered, the surrounding lines may be restored so that the figure at the left again appears. It is noted that the angles of the polygon may be thought of as inscribed angles in a circle as shown by the two figures. In fact this idea of double identity occurs repeatedly in plane geometry and is a powerful tool in the proofs of propositions.



In the above figures, to show that angle P is measured by 1/2 arc xn, one may first show that angle P is made up of three inscribed angles and is itself an inscribed angle. The right hand figure shows the inscribed angle, the difficulty in isolation. Many other examples, of course, could be cited. This method is quite important in the overlapping of triangles, or the choosing of the proper triangles as similar triangles.

its original form. The following drawing is an example. The figure at the left represents the original figure; the one at the right represents the difficulty isolated with the surrounding lines erased. After the difficulty has been conquered, the surrounding lines may be restored so that the figure at the left again appears. It is noted that the angles of the polygon may be thought of as inscribed angles in a circle as shown by the two figures. In fact this idea of double identity occurs repeatedly in plane geometry and is a powerful tool in the proofs of propositions.

In the above figures, to show that angle P is measured by 1/2 arc xn, one may first show that angle P is made up of three inscribed angles and is itself an inscribed angle. The right hand figure shows the inscribed angle, the difficulty in isolation. Many other examples, of course, could be cited. This method is quite important in the overlapping of triangles, or the choosing of the proper triangles as similar triangles.

e). New Procedure. Still another difficulty peculiar to plane geometry is the new type of procedure in the demonstration. This is considered under the topic of geometry as a reflective subject following this, and motivation is provided for in the units on illusions and the selection of hypothesis and conclusion.

f). The reflective nature and its cures.

(1). General discussion. In general, success comes when sudden shocks are eliminated. Many experiences are adequately met if proper time is given. Innoculations in the medical world are a vivid tribute to this necessity for avoidance of sudden catastrophes. The baby with its primary fear of sudden loud tones shows how inherent is this law. In geometry at every stage there should be gradual procedure.

(2). The Time for the answer. Many writers have called attention to the fact that geometry is a reflective subject. It requires time for pupils to comprehend. Questions must be followed by a reasonable elapse of time if the answer is to be a thought answer. The voice, too, must be distinct and the speaking must be slow enough so that the pupils may have ample time to grasp the question. The solution, or the appreciation of geometry tasks, cannot be forced. To attempt to speed up the answer of the pupil is to force him into dishonesty by saying that he understands and sees, when he is fabricating to escape an embarassing situation before the class. The pupil should not be hemmed in by speed demands in comprehending a geometry situation involving thought work.

e). New Procedure. Still another difficulty peculiar to plane geometry is the new type of procedure in the demonstration. This is considered under the topic of geometry as a reflective subject following this, and motivation is provided for in the units on illusions and the selection of hypothesis and conclusion.

f). The reflective nature and its cures.

(1). General discussion. In general, success comes when sudden shocks are eliminated. Many experiences are adequately met if proper time is given. Innoculations in the medical world are a vivid tribute to this necessity for avoidance of sudden catastrophes. The baby with its primary fear of sudden loud tones shows how inherent is this law. In geometry at every stage there should be gradual procedure.

(2). The Time for the answer. Many writers have called attention to the fact that geometry is a reflective subject. It requires time for pupils to comprehend. Questions must be followed by a reasonable elapse of time if the answer is to be a thought answer. The voice, too, must be distinct and the speaking must be slow enough so that the pupils may have ample time to grasp the question. The solution, or the appreciation of geometry tasks, cannot be forced. To attempt to speed up the answer of the pupil is to force him into dishonesty by saying that he understands and sees, when he is fabricating to escape an embarrassing situation before the class. The pupil should not be hemmed in by speed demands in comprehending a geometry situation involving thought work.

Most of the work is of this reflective nature. That is why quick drills play so small a part in this study and why there are so few drill units. It is primarily a slower process study. Failure to recognize this by the teacher has resulted in much disappointment to both teacher and pupil.

(3). The demonstration. As already noted, the demonstration is a new procedure and because there are no past experiences to which to refer, careful attention must be given to the approach that it may be made gradual.

The demonstration, itself, also, should be a gradual process. The following is a suggested gradation.

1. Demonstration by class with all group helping, including the teacher.
2. Demonstration by class with help from group chiefly.
3. Demonstration by pupil with group helping.
4. Demonstration by pupil with no help.

The speed of mastery of the demonstration will, of course, depend largely upon the individual pupil. The slower pupils follow the brighter pupils in the demonstration, and the slower have more repetitions of the explanation before they attempt the work.

(4). Exercises. As an example of showing how there can be a slow process of arriving at the difficulty in its complexity, Schultze[1] may be cited in the following six classes of triangles.

1. With numerical exercises.
2. With no overlapping.
3. With overlapping.
4. With choice between special pairs of equal triangles.
5. With triangles to be constructed.
6. With several pairs of equal triangles necessary to proof.

1. The Teaching of Mathematics in Secondary Schools. pp.115-118.

Most of the work is of this reflective nature. That is why quick drills play so small a part in this study and why there are so few drill units. It is primarily a slower process study. Failure to recognize this by the teacher has resulted in much disappointment to both teacher and pupil.

(3). The demonstration. As already noted, the demonstration is a new procedure and because there are no past experiences to which to refer, careful attention must be given to the approach that it may be made gradual.

The demonstration, itself, also should be a gradual process. The following is a suggested gradation.

1. Demonstration by class with all group helping, including the teacher.
2. Demonstration by class with help from group chiefly.
3. Demonstration by pupil with group helping.
4. Demonstration by pupil with no help.

The speed of mastery of the demonstration will, of course, depend largely upon the individual pupil. The slower pupils follow the brighter pupils in the demonstration, and the slower have more repetitions of the explanation before they attempt the work.

(4). Exercises. As an example of showing how there can be a slow process of arriving at the difficulty in its complexity, Schultze[1] may be cited in the following six classes of triangles.

1. With numerical exercises.
2. With no overlapping.
3. With overlapping.
4. With choice between special pairs of equal triangles.
5. With triangles to be constructed.
6. With several pairs of equal triangles necessary to proof.

1. The Teaching of Mathematics in Secondary Schools. pp.115-116.

(5). Testing. Also one may note that slow motion may be evident in tests through many repetitions, focussing on the important parts, and keeping an index of the progress by the pupil. The author has found that tests should be slowly brought up to the peak of completion. This is largely because the method of plane geometry is new.

Thus it is seen that since geometry is a reflective subject there must be time allowed for the pupil to properly approach and properly absorb it. In this thesis are introductory units to make this procedure gradual. Of such a nature are the units on Orientation, choosing the Hypothesis and Conclusion, and Illusions. In geometry, if anywhere, the old saying "Haste makes waste" is vitally true. Texts are recognizing the need of gradual and small-gap treatment as exampled by one text[1] which has adopted what might be called the shadow proposition. The subject happens to be solid rather than plane geometry but the method might be the same in either division. The method used is to have on the page preceding the theorem, parts of it which are printed as exercises. These exercises are the shadows of the theorem to come. They prove to be exceedingly helpful to the proof of the theorem itself.

2. General suggestions.

a). Safety margin. Still another important truth for the pupil to know is that there must be a margin of safety. The pupil must not only learn but have the matter in mind so that it can be given at the time called for. Just as the actor on

1. Stone-Mallory Modern Solid Geometry. p. 520.

(5). Testing. Also one may note that slow motion may be evident in tests through many repetitions, focussing on the important parts, and keeping an index of the progress by the pupil. The author has found that tests should be slowly brought up to the peak of completion. This is largely because the method of plane geometry is new.

Thus it is seen that since geometry is a reflective subject there must be time allowed for the pupil to properly approach and properly absorb it. In this thesis are introductory units to make this procedure gradual. Of such a nature are the units on Orientation, choosing the Hypothesis and Conclusion, and Illustrations. In geometry, if anywhere, the old saying "Haste makes waste" is vitally true. Texts are recognizing the need of gradual and small-gap treatment as exemplied by one text which has adopted what might be called the shadow proposition. The subject happens to be solid rather than plane geometry but the method might be the same in either division. The method used is to have on the page preceding the theorem, parts of it which are printed as exercises. These exercises are the shadows of the theorem to come. They prove to be exceedingly helpful to the proof of the theorem itself.

2. General suggestions.

a). Safety margin. Still another important truth for the pupil to know is that there must be a margin of safety. The pupil must not only learn but have the matter in mind so that it can be given at the time called for. Just as the author on

1. Stone-Mallory Modern Solid Geometry. p. 320.

the stage must learn his part so that he may act even in the face of extra difficulties attendant on audience situations, or his own physical health, so a pupil, to pass an examination, must have the material so well in mind that it can be recalled when he is not feeling at his best, or when he is rather rushed for time, or when he has not had opportunity just previously to review. A firm grasp of t he subject will prevent nervousness as confidence will be greater.

b). Universality. Sometimes the calling of the attention of the pupil to the broadness of application of a geometric truth aids in creating a favorable attitude. For instance, proportion is a subject which may be discussed in general life situations. Illusions in geometry are but a particular phase of what happens in the world of thought generally. Human individuals so twist things, and use isolated statements which make the thought different than it originally was. Politics have been especially imbedded with these devices to deceive. Rudyard Kipling expresses the ideas in his poem "If".

> "If you can bear to hear the truth you've spoken
> Twisted by knaves to make a trap for fools."

3. Personality of teacher. Of prime importance is the equipment of the teacher. She it is who provides the enriched materials to which she attracts her pupils. Then, by per personal equalities of enthusiasm, humor, tactfulness, and resourcefulness, she makes the channel of communication pleasant. With all the attention given to mental hygiene to-day, and with the primary reward of self-activity, pleasantness, and satisfaction

the stage must learn his part so that he may act even in the face of extra difficulties attendant on audience situations, or his own physical health, as a pupil, to pass an examination, must have the material so well in mind that it can be recalled when he is not feeling at his best, or when he is rather rushed for time, or when he has not had opportunity just previously to review. A firm grasp of the subject will prevent nervousness as confidence will be greater.

b). Universality. Sometimes the calling of the attention of the pupil to the broadness of application of a geometric truth aids in creating a favorable attitude. For instance, proportion is a subject which may be discussed in general life situations. Illustrations in geometry are but a particular phase of what happens in the world of thought generally. Human individuals so twist things, and use isolated statements which make the thought different than it originally was. Politics have been especially imbedded with these devices to deceive. Rudyard Kipling expresses the idea in his poem "If".

"If you can bear to hear the truth you've spoken
Twisted by knaves to make a trap for fools."

3. Personality of teacher. Of prime importance is the equipment of the teacher. She is who provides the enriched material to which she attracts her pupils. Then, by her personal qualities of enthusiasm, humor, teachfulness, and resourcefulness, she makes the channel of communication pleasant. With all the attention given to mental hygiene to-day, and with the primary reward of self-activity, pleasantness, and satisfaction

too much cannot be said in regard to the importance of the teacher. She makes the task attractive by making it joyous, clear, interesting, sensible, suited to the capacities of the pupils. She makes it definite, individual, social, and alive.

The central core of all human activity is the achieving of the ego. This means to have the ability to take one's place in the world and to be important because of something one does. So dominant in lives is this that the severest punishment is the puncturing of the ego, a favorite amusement of school comrades. In school, also, this achieving of the ego makes the audience situation of prime importance; there must be a purpose for the creation of the work at hand.

4. Immediate and ultimate results from success.

a). Fear. Not only must the channel be kept clear by the teacher for the proper flow of power development between pupil and goal, but mental hygiene must urge the waters against any sluggish motion so that they may flow swiftly and clearly onward to success. It is one's task not only to put all the positive elements for motivation along the channel to insure flow of activity, but to remove all negative elements. Chief of the latter is fear. Mental Hygiene has given one a rather vivid picture of this force in lives; it has shown how it inhibits, how it dwarfs, how it drives to desperation, how it demoralizes the human being, and how it defames character. William H. Burnham,[1,2] writes of this in his two books. We see it on every hand in the twisted button on Johnny's coat, or the twisted ribbon on Mary's dress as each on the platform

1. The Normal Mind, vii * 702 pp.
2. The Wholesome Personality, vii*713 pp.

too much cannot be said in regard to the importance of the teacher. She makes the task attractive by making it joyous, clear, interesting, sensible, suited to the capacities of the pupils. She makes it definite, individual, social, and alive.

The central core of all human activity is the achieving of the ego. This means to have the ability to take one's place in the world and to be important because of something one does. So dominant in lives is this that the severest punishment is the puncturing of the ego, a favorite amusement of school comrades. In school, also, this achieving of the ego makes the audience situation of prime importance; there must be a purpose for the creation of the work at hand.

4. Immediate and ultimate results from success.

a). Fear. Not only must the channel be kept clear by the teacher for the proper flow of power development between pupil and goal, but mental hygiene must urge the waters against any sluggish motion so that they may flow swiftly and clearly onward to success. It is one's task not only to put all the positive elements for motivation along the channel to insure flow of activity, but to remove all negative elements. Chief of the latter is fear. Mental Hygiene has given one a rather vivid picture of this force in lives; it has shown how it inhibits, how it dwarfs, how it drives to desperation, how it demoralizes the human being, and how it defames character. William H. Burnham,[1,2] writes of this in his two books. We see it on everyhand in the twisted button on Johnny's coat, or the twisted ribbon on Mary's dress as each on the platform

1. The Normal Mind, vii * 202 pp.
2. The Wholesome Personality, vii*415 pp.

forgets his or her piece. One sees the cribbing, the bluffing, the leading which pupils in the classroom practise to steer away from their weak point because of fear. To remove fear of a subject, then, is one of the most vital tasks of the teacher. How it is done will be considered somewhat in detail in the next section on general principles of motivation. It is important to pause here simply to note that the teacher has the supreme hand in this control of the pupil. A pupil may be made or broken through the instilling or through the removal of fear.

b). Attitude from past. Closely allied to the part which teachers play toward the removal or instilling of fear is the general attitude toward the subject built up by the pupils by past antipathies of other pupils, or parents, and of the public in general. The unhappy experience of these people creates in the mind of the pupil approaching plane geometry an attitude of fear and unpleasantness. The teacher may remove this attitude by his enthusiasm and his skill.

If plane geometry is properly presented, the inhibitions of fear and antagonism built up from recitals of other pupils of past failures will be vanquished. The task will bring to the pupil the sensibleness of his activity. It will be within his capacity; and because it is his task, it will be more interesting, and he will seek to accomplish it. It will be social and alive to him because of the audience in the classroom waiting for his contribution. It will link to the familiar, will involve the activity and self-discovery of the pupil, and will show the significance of the relationship.

forgets his or her place. One sees the cribbing, the bluffing, the leading which pupils in the classroom practise to steer away from their weak point because of fear. To remove fear of a subject, then, is one of the most vital tasks of the teacher. How it is done will be considered somewhat in detail in the next section on general principles of motivation. It is important to pause here simply to note that the teacher has the supreme hand in this control of the pupil. A pupil may be made or broken through the instilling or through the removal of fear.

b). Attitude from past. Closely allied to the part which teachers play toward the removal or instilling of fear is the general attitude toward the subject built up by the pupils by past antipathies of other pupils, or parents, and of the public in general. The unhappy experience of these people creates in the mind of the pupil approaching plane geometry an attitude of fear and unpleasantness. The teacher may remove this attitude by his enthusiasm and his skill.

If plane geometry is properly presented, the inhibitions of fear and antagonism built up from recitals of other pupils of past failures will be vanquished. The task will bring to the pupil the sensibleness of his activity. It will be within his capacity; and because it is his task, it will be more interesting, and he will seek to accomplish it. It will be social and alive to him because of the audience in the classroom waiting for his contribution. It will link to the familiar, will involve the activity and self-discovery of the pupil, and will show the significance of the relationship.

Not only for the immediate results of having the pupil feel satisfied but even more valuable for ultimate results is the necessity of setting the stage for the success by the pupil. He becomes an enthusiastic supporter of geometry. He tells of the good times he had and the success he met. Years later, as a parent, his attitude toward school and school tasks is helpful, and as a citizen he tends to become an ardent supporter of educational activities. Today school would have more friends if more of the former pupils had found happiness and success in their tasks.

Thus one sees the difficulties of motivation in plane geometry. It has been shown that because it is a geometry of "Flatland", is abstract and symbolic in character, is taken by a wide range of pupils today, and is approached by those pupils with fears and antagonisms built up from past experiences of other pupils, and embraces a different method from what has ever been encountered by the pupil in his school world,-- because of these various factors, difficulties in its motivation must be overcome.

5. Organizing the techniques. Having now considered some general channels through which motivation may flow, an attempt will now be made to organize definite techniques in the various types of situations in which teachers of geometry find themselves. Taking a lesson plan, it is found that there are three situations such as the creating of a favorable mind set toward the lesson, the continuing of that set throughout the actual teaching and finally the attitude when the lesson is

Not only for the immediate results of having the pupil feel satisfied but even more valuable for ultimate results is the necessity of setting the stage for the success by the pupil. He becomes an enthusiastic supporter of geometry. He tells of the good times he had and the success he met. Years later, as a parent, his attitude toward school and school tasks is helpful, and as a citizen he tends to become an ardent supporter of educational activities. Today school would have more friends if more of the former pupils had found happiness and success in their tasks.

Thus one sees the difficulties of motivation in plane geometry. It has been shown that because it is a geometry of "Flatland", is abstract and symbolic in character, is taken by a wide range of pupils today, and is approached by those pupils with fears and antagonisms built up from past experiences of other pupils, and embraces a different method from what has ever been encountered by the pupil in his school world,--because of these various factors, difficulties in its motivation must be overcome.

5. <u>Organizing the techniques</u>. Having now considered some general channels through which motivation may flow, an attempt will now be made to organize definite techniques in the various types of situations in which teachers of geometry find themselves. Taking a lesson plan, it is found that there are three situations such as the creating of a favorable mind set toward the lesson, the continuing of that set throughout the actual teaching and finally the attitude when the lesson is

finished. So it may be with plane geometry. There are three tasks: to create a proper attitude; to move the devices about that set stage in a proper way that the pupil will make a success; and to leave him with a feeling of satisfaction. It is like a play; it must have adequate preparation; the actual play must take advantage of the setting; and finally there must be the feeling of satisfaction and interest. Now the difficulty in organizing the techniques in this way lies in the fact that these three objectives are constantly operating within a very small situation in addition to the larger. For instance, the whole study of geometry has these three situations, yet almost any given lesson or even part of a lesson depends on the three fields of activity, namely the making of a favorable mind set so that activity may start, the motives to continue the activity, and finally a satisfying result so as to have a potential value to the individual in terms of courage in attacking a new situation and facing life in general.

finished. So it may be with plane geometry. There are three tasks: to create a proper attitude; to move the devices about that set stage in a proper way that the pupil will make a success; and to leave him with a feeling of satisfaction. It is like a play: it must have adequate preparation; the actual play must take advantage of the setting; and finally there must be the feeling of satisfaction and interest. Now the difficulty in organizing the technique in this way lies in the fact that these three objectives are constantly operating within a very small situation in addition to the larger. For instance, the whole study of geometry has these three situations, yet almost any given lesson or even part of a lesson depends on the three fields of activity, namely the making of a favorable mind set so that activity may start, the motives to continue the activity, and finally a satisfying result so as to have a potential value to the individual in terms of courage in attacking a new situation and facing life in general.

PART II.

ORIENTATION.

A. Introductory.

It was noted in the introduction that there were three situations for motivation; namely, at the start, during the task, and at the completion of the task. It is now the purpose to consider these three situations in detail. Therefore, this part of the thesis is concerned with the situation at the start and involves the introductory units, first of which is the orientation of the pupil in geometry.

Proceeding to the method of orientating the pupil by means of a general talk which leads to an assignment and which by means of the individual-interest list gives to the teacher information on the special tasks involving problem, verbal, art, and manual training contributions, the purpose of this lesson is to create a favorable attitude toward geometry. The device at work is the relation to familiar objects. One notes that geometry is not simply a book subject, but concerns objects all around us. The next step is the general talk.

PART II.

ORIENTATION.

A. Introductory.

It was noted in the introduction that there were three situations for motivation; namely, at the start, during the task, and at the completion of the task. It is now the purpose to consider these three situations in detail. Therefore, this part of the thesis is concerned with the situation at the start and involves the introductory units, first of which is the orientation of the pupil in geometry.

Proceeding to the method of orientating the pupil by means of a general talk which leads to an assignment and which by means of the individual-interest list gives to the teacher information on the special tasks involving problem, verbal, art, and manual training contributions, the purpose of this lesson is to create a favorable attitude toward geometry. The device at work is the relation to familiar objects. One notes that geometry is not simply a book subject, but concerns objects all around us. The next step is the general talk.

1. General Talk to Pupils.

a). Purpose. We are living in such an in such an interesting world to-day with so many fine things that we all like to know when we take up a school subject what that subject is and why we should study it. What it is and of what use it is are two things we will try to answer to-day.

(1). Meaning of word "Geometry". First, geometry comes from two Greek words which mean to measure land. The first part of the word makes us think of land, geo-logy, a study of rocks on the land, geo-graphy, a study of the form of land such as mountains, valleys, etc., while we have geo-metry which means land measurement.

(2). Scope of the study. Geometry is a study of the shape of objects like the circle or the square. It does not matter what the material is. It may be a circle of paper or a pie, or a wheel; it is a circle if it has a certain shape. So with the square; it may be a baseball diamond, or a puzzle board, or a black board; if it has a certain shape, it is a square.

(3). Relation to insects, minerals, buildings, gardens, and games. Anything that has a definite shape has a geometric shape. The cell in the honeycomb has a definite shape. How many sides has that figure? Minerals are sometimes classified in museums with reference to their definite shapes. Sometimes it takes the microscope to prove that a speck of mineral has a definite shape.

I. General Talk to Pupils.

a). Purpose. We are living in such an interesting world to-day with so many fine things that we all like to know when we take up a school subject what that subject is and why we should study it. What it is and of what use it is are two things we will try to answer to-day.

(1). Meaning of word "Geometry". First, geometry comes from two Greek words which mean to measure land. The first part of the word makes us think of land, geo-logy, a study of rocks on the land, geo-graphy, a study of the form of land such as mountains, valleys, etc., while we have geo-metry which means land measurement.

(2). Scope of the study. Geometry is a study of the shape of objects like the circle or the square. It does not matter what the material is. It may be a circle of paper or a pie, or a wheel; it is a circle if it has a certain shape. So with the square; it may be a baseball diamond, or a puzzle board, or a black board; if it has a certain shape, it is a square.

(3). Relation to insects, minerals, buildings, gardens, and games. Anything that has a definite shape has a geometric shape. The cell in the honeycomb has a definite shape. How many sides has that figure? Minerals are sometimes classified in museums with reference to their definite shapes. Sometimes it takes the microscope to prove that a speck of mineral has a definite shape.

A speck of salt or alum has a particular form. Perhaps we can borrow a microscope from the laboratory this week and find out what shape salt has. Snow flakes have very beautiful shapes. Look in your book and see how beautiful they are. Did you ever read the story of "Kay" written by Hans Anderson? He was taken to the palace of the snow queen which was very beautiful with geometric forms.

In the life around us we may see many examples of geometric design. We can look from some tall building over a city and see beautiful geometric designs around public buildings, in parks, and public gardens. The flower beds are laid out in pretty shapes. There are beautiful bridges, also. "The National Geographic Magazine" in the library contains pictures which have many geometrical designs in them. Most games have geometric forms to them. The parchesi board, the checker board, the cribbage board, cards, the baseball field, the football field, and the hockey rink all have a definite shape.

(4) Geometry of the classroom. Are there any decorations or other things in the room here which have a definite shape? The circle is illustrated by the clock face, the bell, or a cylinder; the square or rectangle by the panes of glass. Sometimes the clock has eight sides--an octagon - oct - meaning eight. Octave in music means the eight notes of the scale.

(5) Happiness from geometry. If we can recognize these forms, it will make us happier because it will make life more interesting to us. Every time we meet them, they will seem like old friends, and we can say, "How do you do, Mr. Circle, Mr. Square, or

A speck of salt or alum has a particular form. Perhaps we can borrow a microscope from the laboratory this week and find out what shape salt has. Snow flakes have very beautiful shapes. Look in your book and see how beautiful they are. Did you ever read the story of "Kay" written by Hans Anderson? He was taken to the palace of the snow queen which was very beautiful with geometric forms.

In the life around us we may see many examples of geometric design. We can look from some tall building over a city and see beautiful geometric designs around public buildings, in parks, and public gardens. The flower beds are laid out in pretty shapes. There are beautiful bridges, also. "The National Geographic Magazine" in the library contains pictures which have many geometrical designs in them. Most games have geometric forms to them. The parchesi board, the checker board, the cribbage board, cards, the baseball field, the football field, and the hockey rink all have a definite shape.

(4) Geometry of the classroom. Are there any decorations or other things in the room here which have a definite shape? The circle is illustrated by the clock face, the bell, or a cylinder; the square or rectangle by the panes of glass. Sometimes the clock has eight sides--an octagon - oct - meaning eight. Octave in music means the eight notes of the scale.

(5) Happiness from geometry. If we can recognize these forms, it will make us happier because it will make life more interesting to us. Every time we meet them, they will seem like old friends, and we can say, "How do you do, Mr. Circle, Mr. Square, or

Mr. Rectangle.

The reason the baby gets tired so quickly of looking at one thing is that he does not have many thoughts regarding it. He sees a ball, but it is not round to him, not made up of many circles. It is simply bright to him and he puts it in his mouth. He thinks perhaps it is something to eat. When we learn more about each object, we do not get tired of it so quickly.

Geometry, then, will be useful to us in making us happy,--in knowing something about the beauty of the work,--how so many things have a definite form like the flowers in the field and beautiful architecture. It will be fun meeting something you have known before. When you visit your friends or relatives whom you have visited before, you know how nice it is to see the same things and the same people again. So with geometry when you meet these beautiful forms again and again.

2. The Assignment.

a). Bringing to class descriptions and designs of geometric shapes. I shall expect some of the boys tomorrow to tell me what shapes athletic fields have. You can find the names at the beginning of the text book. The girls can also look these up, or they may bring in some pictures of linoleum designs or embroidery. One daily paper, "The Telegram", has a design for embroidery in it each day. See if you can find it and bring it in. Sometimes in the advertisements in these magazines you will find forms. Mount these geometric forms on paper and attach to them the names of the forms.

Mr. Rectangle.

The reason the baby gets tired so quickly of looking at one thing is that he does not have many thoughts regarding it. He sees a ball, but it is not round to him, not made up of many circles. It is simply bright to him and he puts it in his mouth. He thinks perhaps it is something to eat. When we learn more about each object, we do not get tired of it so quickly.

Geometry, then, will be useful to us in making us happy,--in showing something about the beauty of the work,--how so many things have a definite form like the flowers in the field and beautiful architecture. It will be fun meeting something you have known before. When you visit your friends or relatives whom you have visited before, you know how nice it is to see the same things and the same people again. So with geometry when you meet these beautiful forms again and again.

2. The Assignment.

a). Bringing to class descriptions and designs of geometric shapes. I shall expect some of the boys tomorrow to tell me what shapes athletic fields have. You can find the names at the beginning of the text book. The girls can also look these up, or they may bring in some pictures of linoleum designs or embroidery. One daily paper, "The Telegram", has a design for embroidery in it each day. See if you can find it and bring it in. Sometimes in the advertisements in these magazines you will find forms. Mount these geometric forms on paper and attach to them the names of the forms.

3. The Questionnaire.

a) Filling out sheet of suggested special activities in order of preference. On the sheet I am now passing out are lists of activities in which you are interested. Will you place after the activity you are most interested in the figure 1, and the thing you are next interested in the figure 2? If there are some names on the list in which you are not interested, leave the space after these names blank. Mark only those you like and in the order you like them as well as you can. If you are not sure which you like best, write down the order you think. Note: This assignment leads to a list of geometrical terms which may be placed on the blackboard, each member contributing to the growing lists.

3. The Questionnaire.

a) Filling out sheet of suggested special activities in order of preference. On the sheet I am now passing out are lists of activities in which you are interested. Will you place after the activity you are most interested in the figure 1, and the thing you are next interested in the figure 2? If there are some names on the list in which you are not interested, leave the space after these names blank. Mark only those you like and in the order you like them as well as you can. If you are not sure which you like best, write down the order you think.

Note: This assignment leads to a list of geometrical terms which may be placed on the blackboard, each member contributing to the growing lists.

Manual Training.

Astronomy

Photography

Writing

Plays

Games

Reading

Talking

Drawing

Painting

Coloring

Mapping

Puzzles

Designing

Lettering

Mechanical Drawing

Look at this list and find the name of the thing you think you like best and put (1) after it. Then go over the list again and find the thing you like second best and put (2) after it, etc.

Do not mark any number after those things you are not interested in.

From this list can be integrated the class strength toward their respective contributions during the year such as are outlined in the unit on individual differences. In these early lessons, history of various geometricians and objects to illustrate symmetry may be brought in from time to time. Flowers illustrate polygons; berries and grapes, spheres; starfish, sand dollars, shells, and leaves all illustrate symmetry.[1]

---

[1] Vesta A. Richmond, A Number of Things for Beginners in Geometry, The Mathematics Teacher, Vol. XX, (March, 1927) pp. 142-9.

Manual Training.

Astronomy

Photography

Writing

Plays

Games

Reading

Talking

Drawing

Painting

Coloring

Mapping

Puzzles

Designing

Lettering

Mechanical Drawing

Look at this list and find the name of the thing you think you like best and put (1) after it. Then go over the list again and find the thing you like second best and put (2) after it, etc.

Do not mark any number after those things you are not interested in.

From this list can be integrated the class strength toward their respective contributions during the year such as are outlined in the unit on individual differences. In these early lessons history of various geometricians and objects to illustrate symmetry may be brought in from time to time. Flowers illustrate polygons; beetles and grapes, spheres; starfish, sand dollars, shells, and leaves all illustrate symmetry.[1]

---

[1] Vesta A. Richmond, A Number of Things for Beginners in Geometry, The Mathematics Teacher, Vol. XX, (March, 1927) pp. 142-9.

4. Symmetry. Yesterday we ended our lesson by noting that many objects were symmetrical, such as flowers, berries, starfish, and leaves. Today we shall tell how to know when an object is symmetrical. Concluding the sample talk to the pupils, let us now turn to the ideas as presented by various authors.

a) Definition of symmetry. If a figure consists of two halves so that if it is folded over, the left-hand part will exactly fit over the right or vice-versa, it is said to be symmetrical. Let the pupil draw a line on a sheet of paper and mark an ink dot on the paper somewhere outside of the line. He then folds the paper over the line and another dot is obtained. The paper is unfolded and the two dots are joined by a line segment. The crease is the line of symmetry of the line segment.

The line of symmetry of the angle is the bisector. An equilateral triangle has three lines of symmetry, a square has four lines, a circle has any number.

b) Illustrations of symmetry. Connected with the human body are examples of symmetry in eyes, ears, hands, Clothing, gloves and shoes afford the other examples. In the home illustrations include furniture, pictures, and vases. In nature may be found butterflies, trees, leaves, apples, orange sections, and blossoms. Games mentioned are the teeter board, basketball floor, checkerboard, and parchesi board. Other objects are the balanced stage, kite, and isosceles triangle.

We live in a world of symmetrical objects. We like to have balance. We have the salt and pepper shaker made alike except for the difference in size of the holes at the top. Will you bring in tomorrow five pictures of objects which are symmetrical

4. Symmetry. Yesterday we ended our lesson by noting that many objects were symmetrical, such as flowers, berries, starfish, and leaves. Today we shall tell how to know when an object is symmetrical. Concluding the sample talk to the pupils, let us now turn to the ideas as presented by various authors.

a) Definition of symmetry. If a figure consists of two halves so that if it is folded over, the left-hand part will exactly fit over the right or vice-versa, it is said to be symmetrical. Let the pupil draw a line on a sheet of paper and mark an ink dot on the paper somewhere outside of the line. He then folds the paper over the line and another dot is obtained. The paper is unfolded and the two dots are joined by a line segment. The crease is the line of symmetry of the line segment.

The line of symmetry of the angle is the bisector. An equilateral triangle has three lines of symmetry, a square has four lines, a circle has any number.

b) Illustrations of symmetry. Connected with the human body are examples of symmetry in eyes, ears, hands. Clothing, gloves and shoes afford the other examples. In the home illustrations include furniture, pictures, and vases. In nature may be found butterflies, trees, leaves, apples, orange sections, and blossoms. Games mentioned are the teeter board, basketball floor, checker-board, and parchesi board. Other objects are the balanced stage, kite, and isosceles triangle.

We live in a world of symmetrical objects. We like to have balance. We have the salt and pepper shaker made alike except for the difference in size of the holes at the top. Will you bring in tomorrow five pictures of objects which are symmetrical

and fold them to show that they are symmetrical?[1] For a more scientific consideration of symmetry the teacher may present special work to members of the class interested in the following ideas: These pupils may like design work. This work in design has the advantage of offering definite use for the congruency of triangles and for construction work, such as the perpendicular bisector of a line.

c) Symmetry and construction of concrete objects. John W. Young[2] links construction with symmetry and with concrete objects, such as the vase. He concludes his discussion with:

> "Such consideration as the above, stripped perhaps of some of their formality, may therefore be presented to a class very near the outset of its study of geometry. They would indeed seem to have their chief pedagogical value during the introductory parts of the subject."

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics. p. 302.

2. Symmetry, The National Council of Teachers of Mathematics, The Fifth Yearbook, 1930. pp. 145-148.

and fold them to show that they are symmetrical.[1] For a more scientific consideration of symmetry the teacher may present special work to members of the class interested in the following ideas. These pupils may like design work. This work in design has the advantage of offering definite use for the congruency of triangles and for construction work, such as the perpendicular bisector of a line.

c). Symmetry and construction of concrete objects. John W. Young[2] links construction with symmetry and with concrete objects, such as the vase. He concludes his discussion with:

> "Such consideration as the above, stripped perhaps of some of their formality, may therefore be presented to a class very near the outset of its study of geometry. They would indeed seem to have their chief pedagogical value during the introductory parts of the subject."

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics. p. 302.

2. Symmetry, The National Council of Teachers of Mathematics, The Fifth Yearbook, 1930. pp. 145-148.

B. Definitions.

1. General Rules.

After the general orientation talk, a second introductory unit may be that concerning definitions. This topic will embrace a wider area of the geometry text and the various ideas are grouped together here simply for convenience, and not with the thought that this part should be taught as a unit.

As in the approach to any new difficulty, it is well for the teacher to remember some of the general rules. These might be cited as:

1. Begin with the interesting facts and experiments.
2. Lead to necessary definitions and laws.
3. Unify through association with practical applications.
4. Have pupils self-active both physically and mentally.
5. Relate the various parts together into an organized whole.
6. Repeat the principles involved in order to thoroughly instill them in the minds of the pupils.

The order of procedure is somewhat in the nature of analysis. That is, one goes to the conclusion and works back to the beginning. For instance, in dealing with an abstract definition, first consider the concrete object and then go back to the picture and through language to the symbolic expression.

As an illustration, one thinks of a triangle perhaps as a piece of pie or a cake of chocolate or anything else which would relate the triangle to some interesting object. It is next presented as made of wood which can be handled. After that pictures of triangles attached to practical applications are shown. Finally the group talks about the triangle and its properties in verbal language and then apply to it and to its parts the geometric symbols which represent the shorthand of geometry.

B. Definitions.

1. General Rules.

After the general orientation talk, a second introductory unit may be that concerning definitions. This topic will embrace a wider area of the geometry text and the various ideas are grouped together here simply for convenience, and not with the thought that this part should be taught as a unit.

As in the approach to any new difficulty, it is well for the teacher to remember some of the general rules. These might be cited as:

1. Begin with the interesting facts and experiments.
2. Lead to necessary definitions and laws.
3. Unify through association with practical applications.
4. Have pupils self-active both physically and mentally.
5. Relate the various parts together into an organized whole.
6. Repeat the principles involved in order to thoroughly instill them in the minds of the pupils.

The order of procedure is somewhat in the nature of analysis. That is, one goes to the conclusion and works back to the beginning. For instance, in dealing with an abstract definition, first consider the concrete object and then go back to the picture and through language to the symbolic expression.

As an illustration, one thinks of a triangle perhaps as a piece of pie or a cake of chocolate or anything else which would relate the triangle to some interesting object. It is next presented as made of wood which can be handled. After that pictures of triangles attached to practical applications are shown. Finally the group talks about the triangle and its properties in verbal language and then apply to it and to its parts the geometric symbols which represent the abstract of geometry.

2. Necessity.

To emphasize the need of understanding the language of geometry, that is, the need for definitions,one can refer to a local daily paper which has a cartoon each day of "They don't speak our language." This is useful to the pupils in showing them that every activity must have a special language. Geometry is no different from any other activity. That this is vital was brought to the writer's attention recently when he assigned a bright student to copy some geometric material on the blackboard. The mistakes made brought out in relief the unfamiliarity of the pupil with the language of geometry.

The teacher sometimes is impatient with the slow progress of the pupil in the early stages because of being unaware of the many difficulties to be overcome simultaneously by the pupil. Chief of these early difficulties is that of lack of clarity in understanding geometric definitions.

2. Necessity.

To emphasize the need of understanding the language of geometry, that is, the need for definitions, one can refer to a local daily paper which has a cartoon each day of "They don't speak our language." This is useful to the pupils in showing them that every activity must have a special language. Geometry is no different from any other activity. That this is vital was brought to the writer's attention recently when he assigned a bright student to copy some geometric material on the blackboard. The mistakes made brought out in relief the unfamiliarity of the pupil with the language of geometry.

The teacher sometimes is impatient with the slow progress of the pupil in the early stages because of being unaware of the many difficulties to be overcome simultaneously by the pupil. Chief of these early difficulties is that of lack of clarity in understanding geometric definitions.

DAILY TELEGRAM, THURSDAY, AUGUST 31, 1933.

THEY DON'T SPEAK OUR LANGUAGE

3. First assignment.

a) Bringing geometrical designs from a world familiar to the pupil. After the pupil has been shown the need in general for definitions the next step is the assigning to the pupil of tasks which will further emphasize this need.

If the teacher refers to that which is familiar within the realm of experience of the pupil, the learner obtains a comforting feeling of security and the feeling of fear of the new and strange is avoided. Thus when he is asked to bring in material from current literature or to observe the classroom or buildings in his community he is dealing with objects which are familiar to him. An early assignment may consist of the bringing in of a colored picture from a magazine. The pupil is then asked to describe the various forms he sees on the walls, the linoleum or wallpaper in this picture. This difficulty of finding names is suggestive of help. If possible some of this help should come from the class. On the blackboard there is gradually gathered a list of names and opposite them the figures to which they belong. This list grows at the suggestion of the pupils and at the speed the pupils need. Discussion is held in regard to the classroom and the objects in it. Descriptions are asked which again involve definitions. Geometric figures are drawn on the board, and the pupils are asked to talk about them, comparing, contrasting, etc. This again shows the necessity of knowing the names of the various parts. All these drawings at first should be made as concrete as possible by colors, and by representing them as actual

3. First assignment.

a) Bringing geometrical designs from a world familiar to the pupil. After the pupil has been shown the need in general for definitions the next step is the assigning to the pupil of tasks which will further emphasize this need.

If the teacher refers to that which is familiar within the realm of experience of the pupil, the learner obtains a comforting feeling of security and the feeling of fear of the new and strange is avoided. Thus when he is asked to bring in material from current literature or to observe the classroom or buildings in his community he is dealing with objects which are familiar to him. An early assignment may consist of the bringing in of a colored picture from a magazine. The pupil is then asked to describe the various forms he sees on the walls, the linoleum or wallpaper in this picture. This difficulty of finding names is suggestive of help. If possible some of this help should come from the class. On the blackboard there is gradually gathered a list of names and opposite them the figures to which they belong. This list grows at the suggestion of the pupils and at the speed the pupils need. Discussion is held in regard to the classroom and the objects in it.

Descriptions are asked which again involve definitions. Geometric figures are drawn on the board, and the pupils are asked to talk about them, comparing, contrasting, etc. This again shows the necessity of knowing the names of the various parts. All these drawings at first should be made as concrete as possible by colors, and by representing them as actual

objects. For instance, the problem is not to have two lines cross, but to have two fences cross, not to describe a circle but an uncut apple pie.

b) Finding center of a line with compass and straight edge. The lesson to be assigned is in the nature of a challenge. It consists of the use of the compass and the straight edge, and the pupil is asked to find the middle of the line without measuring it. He may use a text if he wishes, and he may find the information in any way he will. This not only serves as an introduction to construction work but presents another situation where he must name certain parts when he is asked to describe what he did. In short, he is again faced with a situation which demands definitions.

One writer[1] uses work sheets which have the new words on them and, in order to perform the activity, the words must have meaning. Thus a felt need is created and the teacher's help is solicited.

4. Applied problem. The intersection of lines should never be isolated from concrete situations at the start. The writer uses the familiar and practical in his illustrations.

a) Locating a point.

(1) A Forest fire. Most of the children have been to a local lookout fire station. The lookout man has observed a fire through his telescope but, because of the distance away, cannot locate on his direction line the exact spot. He telephones to the nearest lookout station and asks the direction as found

1. W. R. Ramon, Geometry Measures Land, The Mathematics Teacher, XXIII (April, 1930), p. 246.

objects. For instance, the problem is not to have two lines cross, but to have two fences cross, not to describe a circle but an uncut apple pie.

b) Finding center of a line with compass and straight edge. The lesson to be assigned is in the nature of a challenge. It consists of the use of the compass and the straight edge, and the pupil is asked to find the middle of the line without measuring it. He may use a text if he wishes, and he may find the information in any way he will. This not only serves as an introduction to construction work but presents another situation where he must name certain parts when he is asked to describe what he did. In short, he is again faced with a situation which demands definitions.

One writer[1] uses work sheets which have the new words on them and, in order to perform the activity, the words must have meaning. Thus a felt need is created and the teacher's help is solicited.

4. Applied problem. The intersection of lines should never be isolated from concrete situations at the start. The writer uses the familiar and practical in his illustrations.

a) Locating a point.

(1) A Forest fire. Most of the children have been to a local lookout fire station. The lookout man has observed a fire through his telescope but, because of the distance away, cannot locate on his direction line the exact spot. He telephones to the nearest lookout station and asks the direction as found

1. W. H. Ramon, Geometry measures land, The Mathematics Teacher, XXIII (April, 1930), p. 246.

from that station. This he gets, and from his maps starts his own direction line and also that of the neighboring station, extending them until they cross. This locates a particular point on the map, and he is thus able to telephone the nearest town to direct fire apparatus to fight the fire without loss of time through failure to find the exact location.

(2). A fence corner. The class may be given the problem of completing two ends of fences so that an angle may be formed.

(3). A house in the city. Discussion is then held in regard to location of a house in a city in which it is brought out that there must be an intersection named in order to have definiteness. It is not enough to say Mr. Jones lives on 42nd street, but the description must be either the exact point, number, on 42 nd street, or if at the corner at what intersection. Emphasis should be placed on the need of another line, another direction.

(4). A ship at sea. Discussion is also made of an application to finding location at sea. Both latitude and longitude must be known. These two directions are found by means of plotting the ship's position with the stars in conjunction with tables made up in books. The details need not be taken up here, although a special assignment may be made for a particular pupil to go to the science department and bring back a clear description of the method. In the meantime, the necessity of obtaining two lines is again shown.

Illustrations showing these various situations follow:

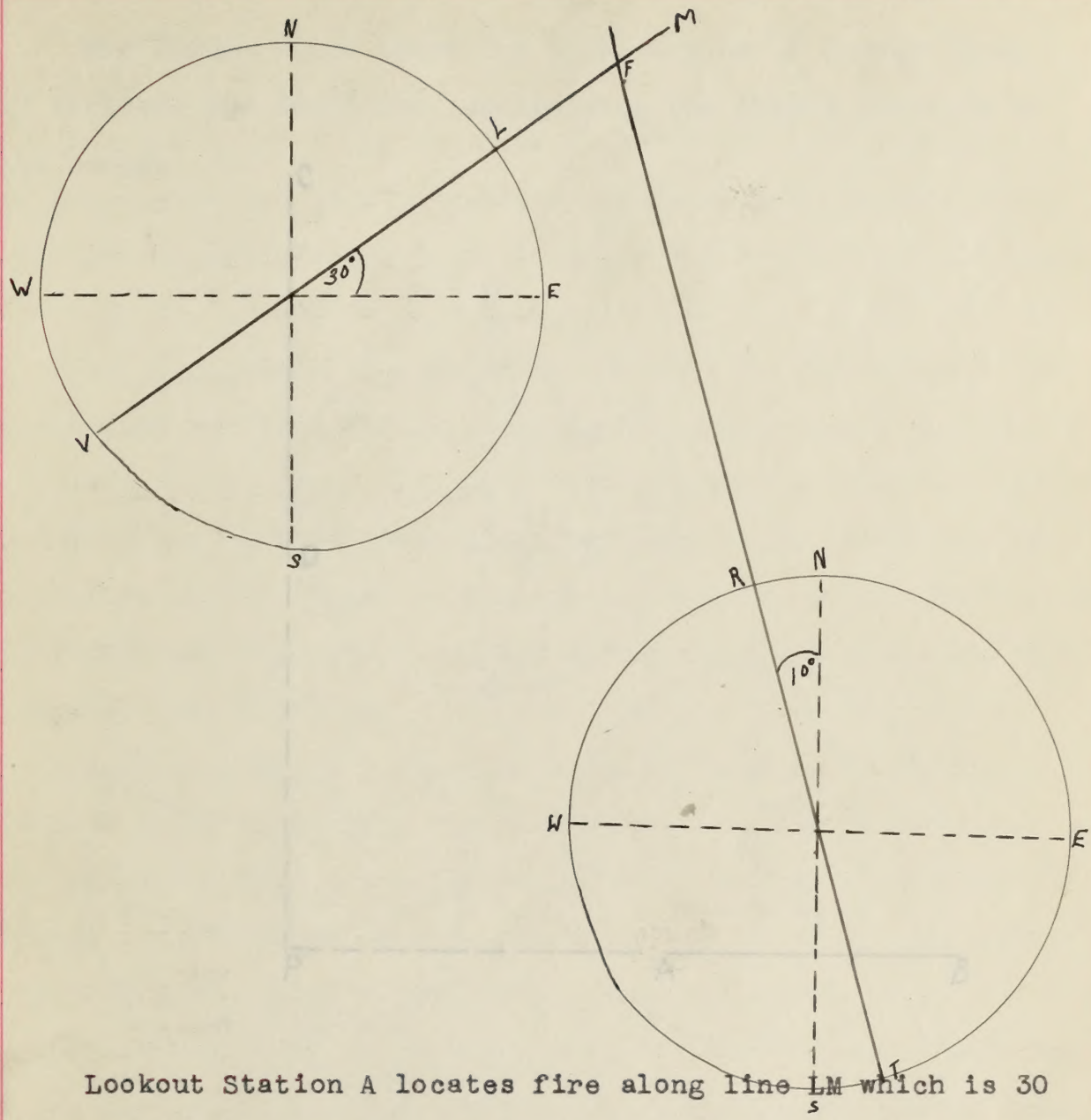
from that station. This he gets, and from his maps starts his own direction line and also that of the neighboring station, extending them until they cross. This locates a particular point on the map, and he is thus able to telephone the nearest town to direct fire apparatus to fight the fire without loss of time through failure to find the exact location.

(2). A fence corner. The class may be given the problem of completing two ends of fences so that an angle may be formed.

(3). A house in the city. Discussion is then held in regard to location of a house in a city in which it is brought out that there must be an intersection named in order to have definiteness. It is not enough to say Mr. Jones lives on 42nd street, but the description must be either the exact point, number, on 42 nd street, or if at the corner at what intersection. Emphasis should be placed on the need of another line, another direction.

(4). A ship at sea. Discussion is also made of an application to finding location at sea. Both latitude and longitude must be known. These two directions are found by means of plotting the ship's position with the stars in conjunction with tables made up in books. The details need not be taken up here, although a special assignment may be made for a particular pupil to go to the science department and bring back a clear description of the method. In the meantime, the necessity of obtaining two lines is again shown.

Illustrations showing these various situations follow:

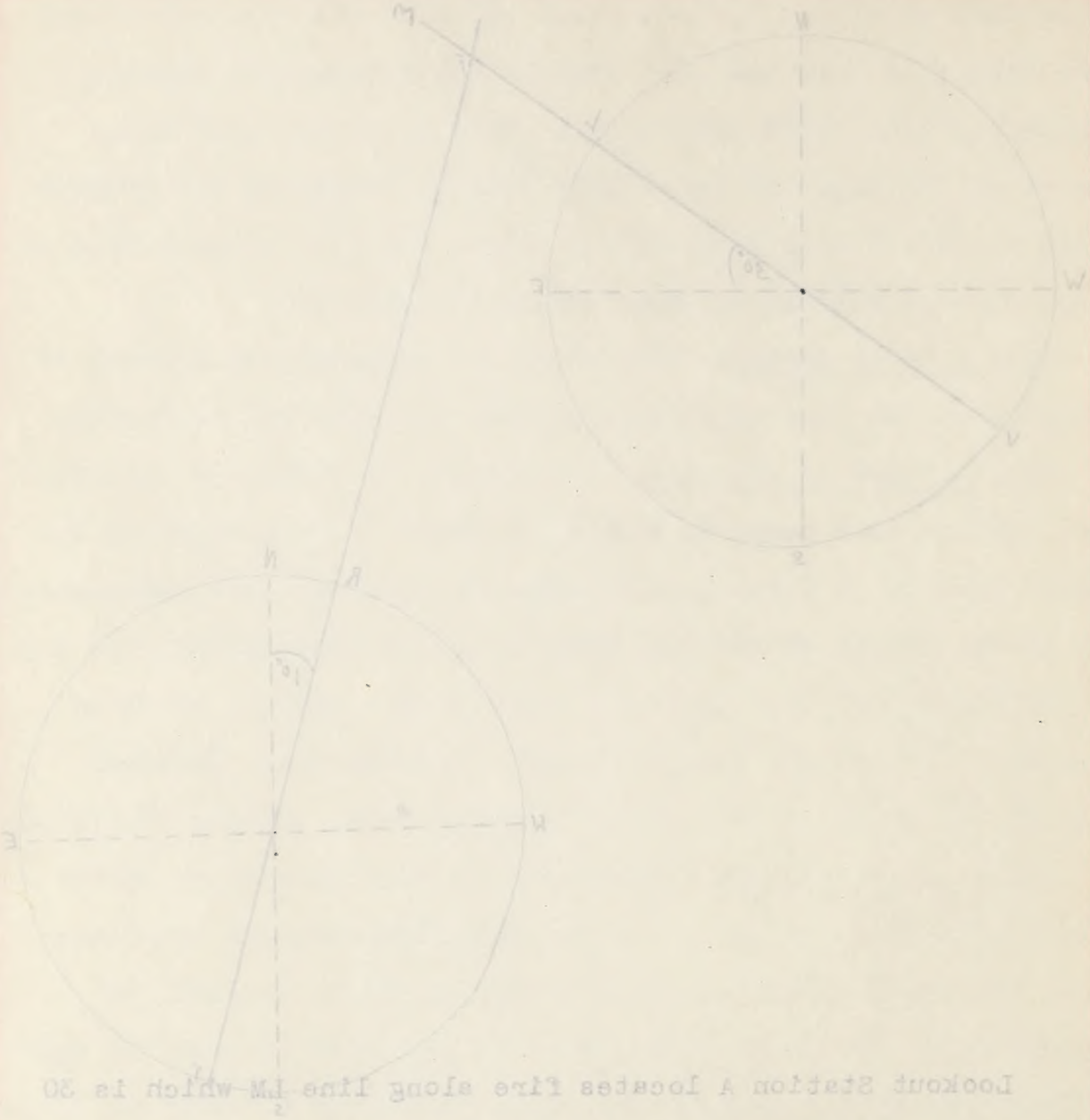


Lookout Station A locates fire along line LM which is 30 degrees north of east.

Lookout Station B locates fire along line RF which is 10 degrees west of north.

The two directions are plotted on a map and the exact point of intersection is found--that is the exact location of the fire.

The purpose of finding this exact location is to telephone the fire department, so that they may reach the fire without unnecessary delay.

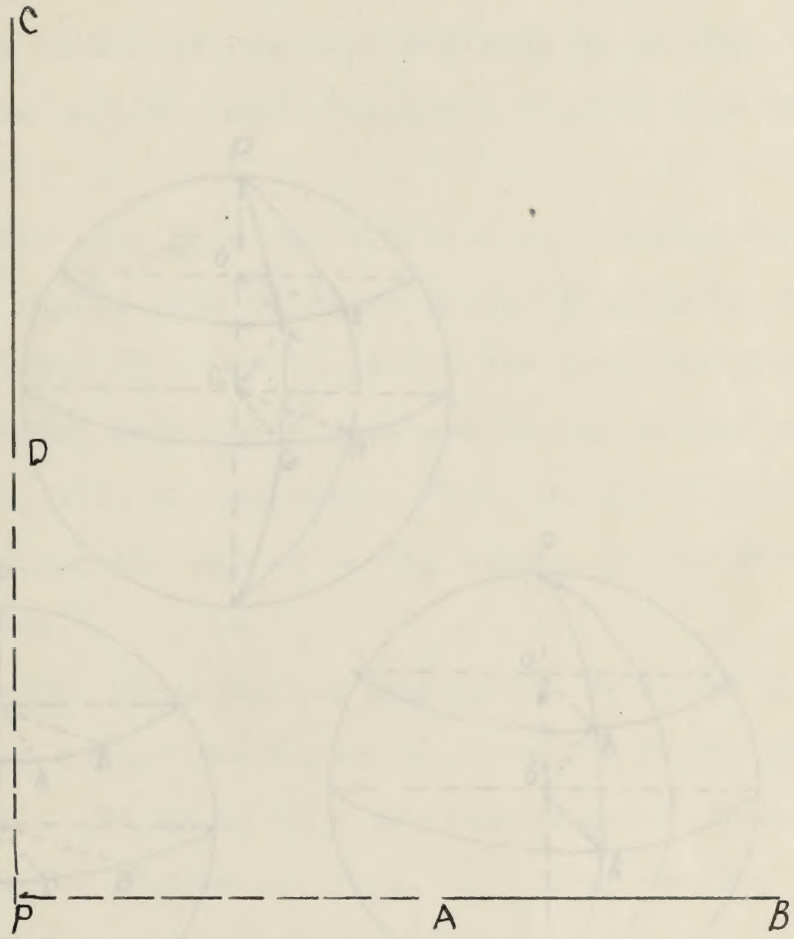


Lookout station A locates fire along line LM which is 30 degrees north of east.

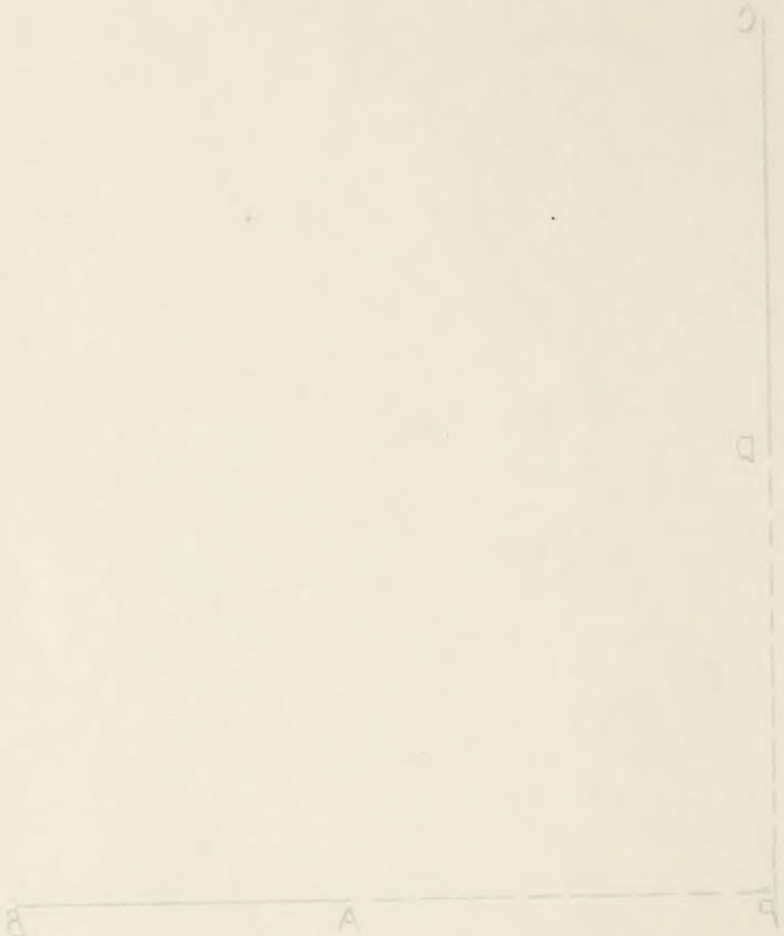
Lookout station B locates fire along line NP which is 10 degrees west of north.

The two directions are plotted on a map and the exact point of intersection is found--that is the exact location of the fire.

The purpose of finding this exact location is to telephone the fire department, so that they may reach the fire without unnecessary delay.

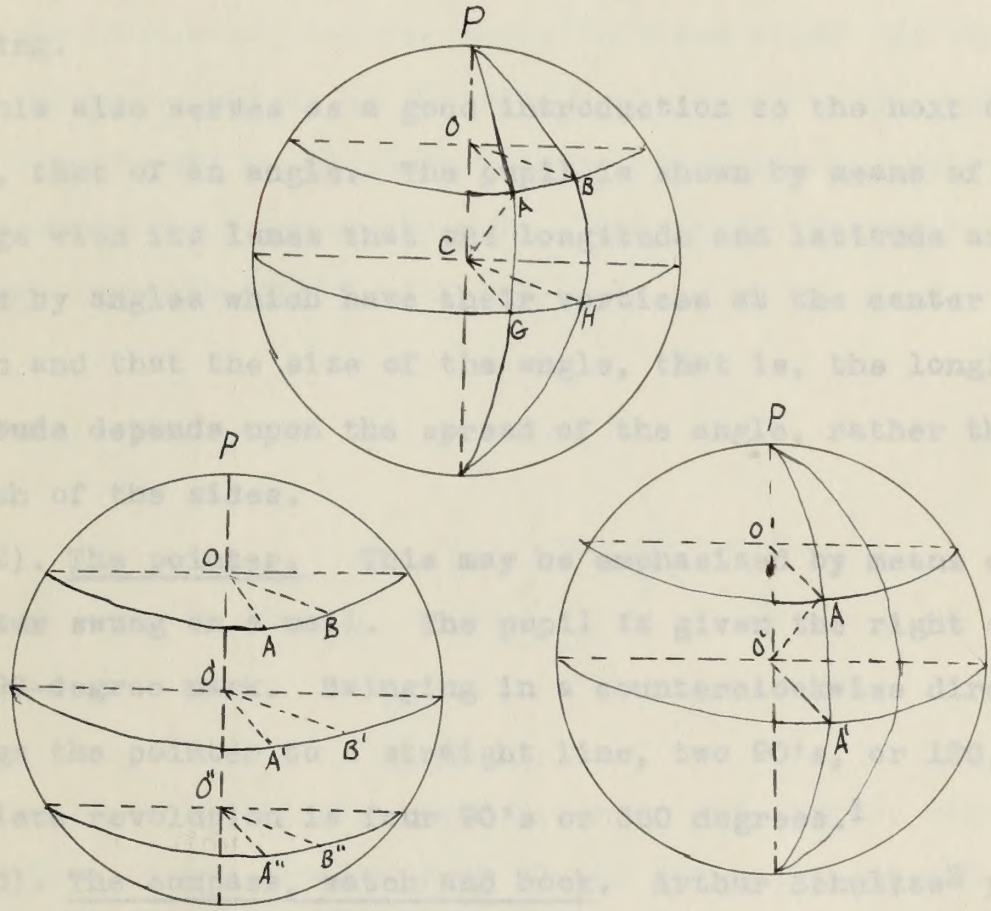


The figure here consists of the two parts of fences which when lengthened meet at P. Two lines, two fences, intersect at a post. Also two posts determine the direction of a fence.



The figure here consists of the two parts of fences which when lengthened meet at P. Two lines, two fences, intersect at a post. Also two posts determine the direction of a fence.

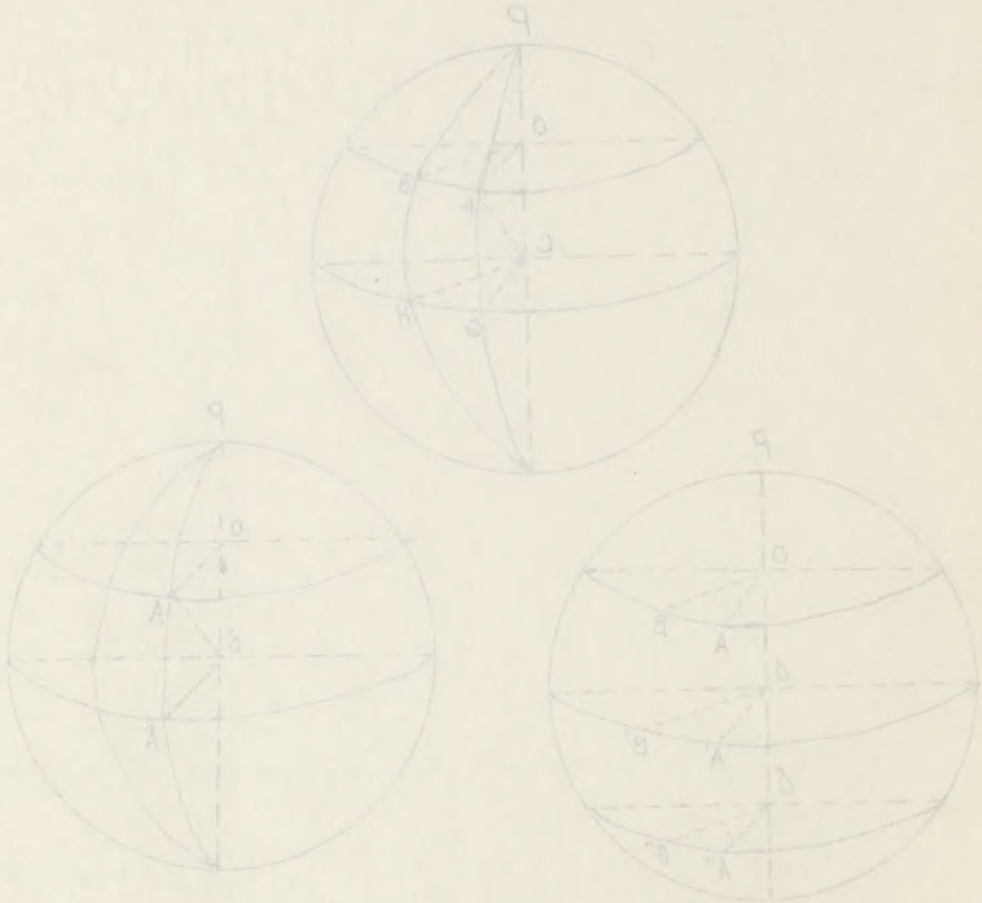
The figure below shows the intersection of curved lines of latitude and longitude, determining the ship's position on the ocean.



This map shows longitude. Lines on the surface of the sphere. This problem may be given to a pupil as individual work. The pupil is to present the idea, including drawings, description, and explanation at some meeting.

This map shows latitude. Angles from the center of the earth. The latitude tells where on the long lines in figure at left--that is, determine the intersection.

The figure below shows the intersection of curved lines of latitude and longitude, determining the ship's position on the ocean.



This map shows longitude. Lines on the surface of the sphere. This problem may be given to a pupil as individual work. The pupil is to present the idea, including drawings, description, and explanation at some meeting.

This map shows latitude. Angles from the center of the earth. The latitude tells where on the long lines in figure at left--that is, determine the intersection.

5. Definition of the angle.

a). Objective material.

(1). The globe. The presence of an actual globe and practise in finding points with latitude and longitude given will be interesting to the pupils, and will give a clearer idea of their meaning.

This also serves as a good introduction to the next definition, that of an angle. The pupil is shown by means of an orange with its lunes that the longitude and latitude are determined by angles which have their vertices at the center of the earth and that the size of the angle, that is, the longitude and latitude depends upon the spread of the angle, rather than the length of the sides.

(2). The pointer. This may be emphasized by means of a pointer swung on a nail. The pupil is given the right angle on the 90-degree mark. Swinging in a counterclockwise direction brings the pointer to a straight line, two 90's, or 180, and a complete revolution is four 90's or 360 degrees.[1]

(3). The compass, watch and book. Arthur Schultze[2] presents a teaching plan for angle definition. He suggests the following:

> "Explain an angle as a rotation, by using a material contrivance that actually shows a rotation of a line. A pair of compasses, or, better, several pairs of different sizes, the hands of a toy watch, or even a book may be used effectively to show that an angle is generated by a rotation, and that the amount of rotation, and hence the angle, does not depend upon the length of the sides. Such illustrations will show clearly what an angle really is, and explain the meaning of straight angle, reflex angle, etc."

1. W. R. Ramon, op. cit. p. 246.
2. The Teaching of Mathematics in Secondary Schools. (1924).p.77.

b. Definition of the angle.

a). Objective material.

(1). The globe. The presence of an actual globe and practice in finding points with latitude and longitude given will be interesting to the pupils, and will give a clearer idea of their meaning.

This also serves as a good introduction to the next definition, that of an angle. The pupil is shown by means of an orange with its lunes that the longitude and latitude are determined by angles which have their vertices at the center of the earth and that the size of the angle, that is, the longitude and latitude depends upon the spread of the angle, rather than the length of the sides.

(2). The pointer. This may be emphasized by means of a pointer swung on a nail. The pupil is given the right angle on the 90-degree mark. Swinging in a counterclockwise direction brings the pointer to a straight line, two 90's, or 180, and a complete revolution is four 90's or 360 degrees.[1]

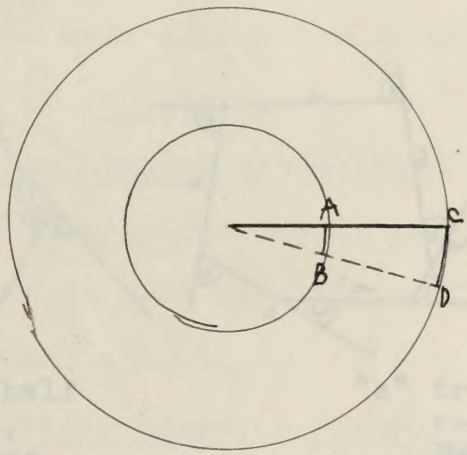
(3). The compass, watch and book. Arthur Schultze[2] presents a teaching plan for angle definition. He suggests the following:

> "Explain an angle as a rotation, by using a material contrivance that actually shows a rotation of a line. A pair of compasses, or, better, several pairs of different sizes, the hands of a toy watch, or even a book may be used effectively to show that an angle is generated by a rotation, and that the amount of rotation, and hence the angle, does not depend upon the length of the sides. Such illustrations will show clearly what an angle really is, and explain the meaning of straight angle, reflex angle, etc."

1. W. H. Hamon, op. cit. p. 246.
2. The Teaching of Mathematics in Secondary Schools. (1924).p.77.

b) Applied problem.

(1). The radio dial. Reference is made to the older radios with their small knobs on which it was difficult to locate radio stations. Later larger knobs were provided and finally gears were constructed to magnify the angle rotation. The figure is drawn as below, and the pupil is asked which the larger angle is. He sees that it is the same angle but a larger length of arc because the second circle is a larger circle, but the degrees of arc remains the same. Length of the angle does not depend upon the length of lines.



The pupil sees that the second circle, the larger, magnifies and makes easier reading even though the angle has not increased. In radio language no change in wave length has been made.

From A to B is a small space and therefore, is harder to read than from C to D.

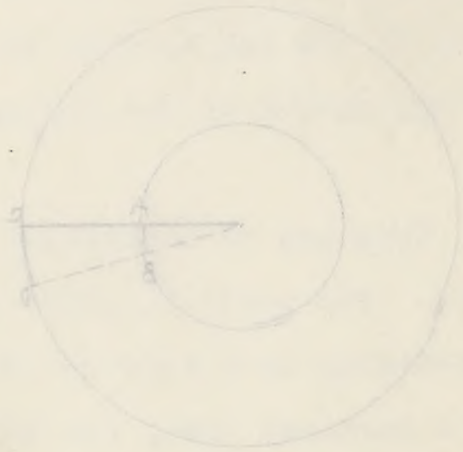
The angle depends on spread, not on length of the sides.

Activity is given the pupil by:

a. Drawing figures on board and naming angles, with increasing complexity of figures.
b. Using numerical exercises.
c. Practising drawing exercises.

b) Applied problem.

(1). The radio dial. Reference is made to the older radios with their small knobs on which it was difficult to locate radio stations. Later larger knobs were provided and finally gears were constructed to magnify the angle rotation. The figure is drawn as below, and the pupil is asked which the larger angle is. He sees that it is the same angle but a larger length of arc because the second circle is a larger circle, but the degrees of arc remains the same. Length of the angle does not depend upon the length of lines.



The pupil sees that the second circle, the larger, magnifies and makes easier reading even though the angle has not increased. In radio language no change in wave length has been made.

From A to B is a small space and therefore, is harder to read than from C to D.

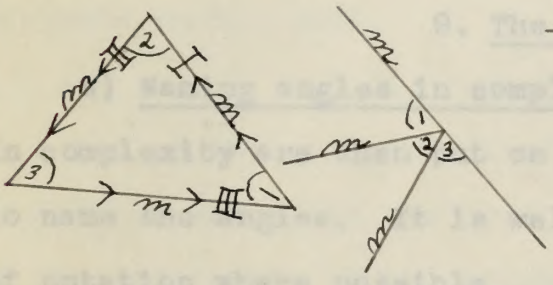
The angle depends on spread, not on length of the sides.

Activity is given the pupil by:

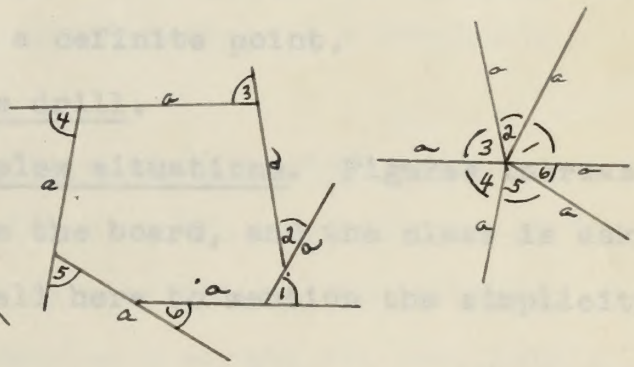
a. Drawing figures on board and naming angles, with increasing complexity of figures.
b. Using numerical exercises.
c. Practicing drawing exercises.

d. Doing laboratory exercises.
e. Finding angles of hands of clock when a given time. (Attaching to concrete and familiar).
f. Finding angles of compass.
g. Making the angle an angle of a pie sector.

The idea of rotation may be applied to the angles of the triangle in showing that the sum of the three angles equals 180 degrees. This must be supplemented by paper cutting also to show this fact. The extension may be made to the exterior angles of a polygon, which leads to the formula for the interior angles. All this further strengthens the idea of rotation as a definition of the term "angle."



"m" travels a half a revolution, 180 degrees.



"a" travels a complete revolution, 360 degrees.

6. The assignment.

a). Find applied problems. The pupils are asked to bring in applications of where the angle is used for measuring, or where two lines cross in a point.

7. The test.

The next period is devoted first to a five-minute test on their understanding of definition of size of an angle, and applications are asked for this and the intersection of two

d. Doing laboratory exercises.
e. Finding angles of hands of clock when a given time. (Attaching to concrete and familiar).
f. Finding angles of compass.
g. Making the angle an angle of a pie sector.

The idea of rotation may be applied to the angles of the triangle in showing that the sum of the three angles equals 180 degrees. This must be supplemented by paper cutting also to show this fact. The extension may be made to the exterior angles of a polygon, which leads to the formula for the interior angles. All this further strengthens the idea of rotation as a definition of the term "angle."

"m" travels a half a revolution, 180 degrees.

"a" travels a complete revolution, 360 degrees.

6. The assignment.

a). Find applied problems. The pupils are asked to bring in applications of where the angle is used for measuring, or where two lines cross in a point.

7. The test.

The next period is devoted first to a five-minute test on their understanding of definition of size of an angle, and applications are asked for this and the intersection of two

lines as a useful part of life.

8. Discussion.

a). Pupils' contributions. Their own contributions are then discussed and at this time, also, individual projects may be provided for magnification without change in angle, such as the radio dial and similar examples, and their various illustrations of two lines intersecting.

b) Pupils' definitions. At this time also formulation of a description of the new law will be natural and the pupils will tell in their own language that it requires the intersection of two lines to make a definite point.

9. The drill.

a) Naming angles in complex situations. Figures increasing in complexity are then put on the board, and the class is asked to name the angles. It is well here to mention the simplicity of notation where possible.

b) Reading angle where there is more than one angle at a vertex. If there is one angle at the vertex, the vertex letter is sufficient, but if there are several angles at the vertex, the three letters must be read, the vertex letter always coming in the center. In the triangle the order of reading is not important but in any polygon the reading must be continuous. Examples of this are given also in order that the pupil may get a clear idea in his mind. In these earlier stages, as mentioned in this unit, all pupils should obtain a mastery since this involves fundamental definitions and ideas. For this reason there should be some drill work and the

lines as a useful part of life.

8. Discussion.

a) Pupils' contributions. Their own contributions are then discussed and at this time, also, individual projects may be provided for magnification without change in angle, such as the radio dial and similar examples, and their various illustrations of two lines intersecting.

b) Pupils' definitions. At this time also formulation of a description of the new law will be natural and the pupils will tell in their own language that it requires the intersection of two lines to make a definite point.

9. The drill.

a) Naming angles in complex situations. Figures increasing in complexity are then put on the board, and the class is asked to name the angles. It is well here to mention the simplicity of notation where possible.

b) Reading angle where there is more than one angle at a vertex. If there is one angle at the vertex, the vertex letter is sufficient, but if there are several angles at the vertex, the three letters must be read, the vertex letter always coming in the center. In the triangle the order of reading is not important but in any polygon the reading must be continuous. Examples of this are given also in order that the pupil may get a clear idea in his mind. In these earlier stages, as mentioned in this unit, all pupils should obtain a mastery since this involves fundamental definitions and ideas. For this reason there should be some drill work and the

frequent calling of the pupil's attention to the fact or idea until mastery is attained by all.

Since the aim is to make certain conceptions automatic, in order that when the pupil is engrossed on a problem which calls for his utmost mental powers, that he may not have to be diverted by attempting to clear up cloudy ideas, every effort must be made that this mastery to an automatic point be accomplished in a very real way. This means that there must be no useless and meaningless repetitions, but rather that the variety of approaches may be such that the pupil will have the definition seep into his system with the continuation of a favorable attitude on his part.

10. Definition of "included" angle.

One of the early difficulties is the significance of the word "included". As has been pointed out by some writers, part of the difficulty of the congruence theorems has come from a failure to thoroughly orientate the pupils in the terms with which these theorems deal. Considerable time should be spent in establishing in the minds of all the pupils what "included" means.

a). Colored chalk. This may be done by means of colored chalk, designating either the angle or side, whichever is being considered.

b). Description. Stress may also be made verbally by explaining that in the included angle, the sides surround the angle and are the sides of the angle, In the case of the included side, that at the extremities of the side are the

frequent calling of the pupil's attention to the fact or idea until mastery is attained by all.

Since the aim is to make certain conceptions automatic, in order that when the pupil is engrossed on a problem which calls for his utmost mental powers, that he may not have to be diverted by attempting to clear up cloudy ideas, every effort must be made that this mastery to an automatic point be accomplished in a very real way. This means that there must be no useless and meaningless repetitions, but rather that the variety of approaches may be such that the pupil will have the definition seep into his system with the continuation of a favorable attitude on his part.

10. Definition of "included" angle.

One of the early difficulties is the significance of the word "included". As has been pointed out by some writers, part of the difficulty of the congruence theorems has come from a failure to thoroughly orientate the pupils in the terms with which these theorems deal. Considerable time should be spent in establishing in the minds of all the pupils what "included" means.

a). Colored chalk. This may be done by means of colored chalk, designating either the angle or side, whichever is being considered.

b). Description. Stress may also be made verbally by explaining that in the included angle, the sides surround the angle and are the sides of the angle. In the case of the included side, that at the extremities of the side are the

angles which include the side.

c.) Board drawings. The best way to practise on these terms is to draw figures on the board and first erase the parts of the triangle which do not apply, and then fill them in. In this way the difficulty is isolated. One reason of the difficulty is that at the tenth year pupils are not over careful in their statements and fail to see the significance of not staying with the definitions.

d). A common error. One way to cure this carelessness is to draw two triangles which have two sides and an angle equal but not the included angle. Purposely make one triangle much larger than the other and then tell the pupil that since he stated these were congruent, that you would be quite satisfied to have the triangle of ice cream on the left, which you have drawn much larger, while he may take the one at the right which is much smaller. Then ask him if that is not fair since he has just stated that that would be how to make the triangles congruent. Amid the smiles of the class he sees the error, and the correction is made. The word "included" now begins to take on meaning, and he will observe it more carefully because of his experience. This may be repeated two or three times in the course of the week before everyone in the class is awake to the word "included", but eventually all get the term firmly in mind.

e) Illustrations. Following are illustrations.

Boston University
School of Education
Library

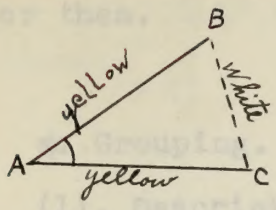
angles which include the side.

c.) Board drawings. The best way to practice on these terms is to draw figures on the board and first erase the parts of the triangle which do not apply, and then fill them in. In this way the difficulty is isolated. One reason of the difficulty is that at the tenth year pupils are not over careful in their statements and fail to see the significance of not staying with the definitions.

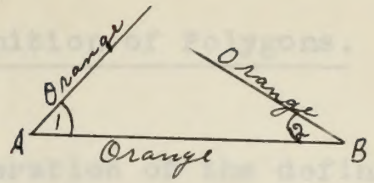
d). A common error. One way to cure this carelessness is to draw two triangles which have two sides and an angle equal but not the included angle. Purposely make one triangle much larger than the other and then tell the pupil that since he stated these were congruent, that you would be quite satisfied to have the triangle of ice cream on the left, which you have drawn much larger, while he may take the one at the right which is much smaller. Then ask him if that is not fair since he has just stated that that would be how to make the triangles congruent. Amid the smiles of the class he sees the error, and the correction is made. The word "included" now begins to take on meaning, and he will observe it more carefully because of his experience. This may be repeated two or three times in the course of the week before everyone in the class is awake to the word "included", but eventually all get the term firmly in mind.

e) Illustrations. Following are illustrations.

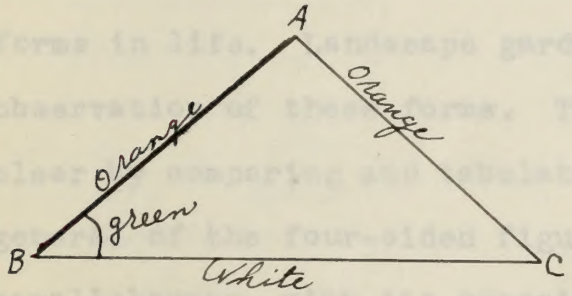
Boston University
School of Education
Library

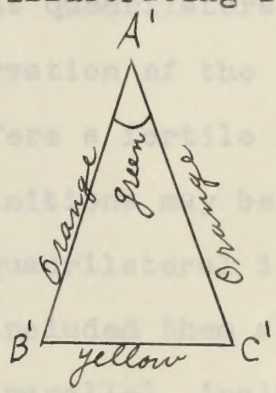


Illustrating included angle.



Illustrating included side





Illustrating two triangles which have two sides and an angle equal but not the included angle. Parts equal may be in the same color chalk. The pupil decides that he wishes he might have the "ice-cream" at the left rather than the right and thus gets an insight into the importance of the word "included".

f). <u>The drill</u>. The practise on definitions may consist of picking out from the figures in the text the parts studied and also in having drawn on the board various figures growing in complexity in which these parts appear. If these figures are drawn on the board ahead of the class period by some geometry pupil, it no only gives him a chance at activity but saves the time in class. At the end of the unit on definitions two sides may be chosen, and either on the blackboard or on large placards various figures may be shown and the pupil asked to describe them in geometric language and also give the geometric symbols.

Illustrating included angle.

Illustrating included side

Illustrating two triangles which have two sides and an angle equal but not the included angle. Parts equal may be in the same color chalk. The pupil decides that he wishes he might have the "ice-cream" at the left rather than the right and thus gets an insight into the importance of the word "included".

f). The drill. The practice on definitions may consist of picking out from the figures in the text the parts studied and also in having drawn on the board various figures growing in complexity in which these parts appear. If these figures are drawn on the board ahead of the class period by some geometry pupil, it no only gives him a chance at activity but saves the time in class. At the end of the unit on definitions two sides may be chosen, and either on the blackboard or on large placards various figures may be shown and the pupil asked to describe them in geometric language and also give the geometric symbols.

for them.

11. Definition of Polygons.

a). Grouping.

(1). Descriptive. A consideration of the definitions of the various figures, such as the various quadrilaterals, may be made interesting first through observation of the various forms in life. Landscape gardening offers a fertile field for observation of these forms. These definitions may be made clear by comparing and tabulating. A quadrilateral is the most general of the four-sided figures; it included them all. The parallelogram, with its opposite sides parallel, includes under it the rectangle, the square, and the rhombus. All of these are special cases, the first two having right angles, and the last named having all sides equal. The trapezoid comes partially under the parallelogram, having two of its sides parallel; and the isosceles trapezoid has two sides equal, similar to the isosceles triangle.

(2). Family relationship. A little humor may be brought in by saying that the quadrilateral is great-grandfather to the square, grandfather to the rectangle and the rhombus, and father to both the parallelogram and the trapezoid. Fun is derived from further consideration of family relationships such as uncles, aunts, nephews, nieces, and cousins. Few in the class know that the children of first cousins are third cousins and not second cousins. A second cousin is the relationship between the first cousin and the child of the second cousin.

for them.

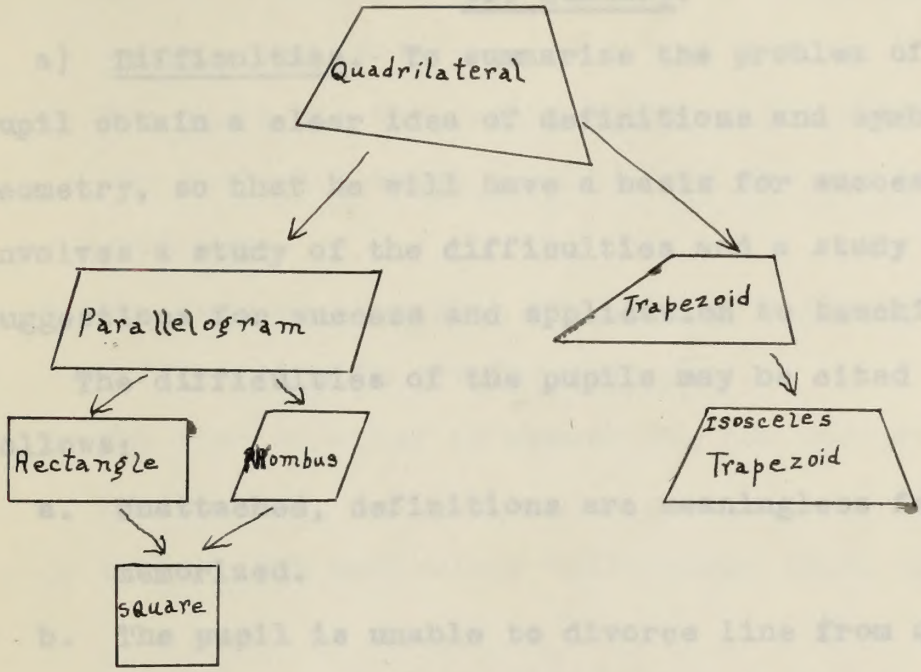
II. Definition of Polygons.

a). Grouping.

(1). Descriptive. A consideration of the definitions of the various figures, such as the various quadrilaterals, may be made interesting first through observation of the various forms in life. Landscape gardening offers a fertile field for observation of these forms. These definitions may be made clear by comparing and tabulating. A quadrilateral is the most general of the four-sided figures; it included them all. The parallelogram, with its opposite sides parallel, includes under it the rectangle, the square, and the rhombus. All of these are special cases, the first two having right angles, and the last named having all sides equal. The trapezoid comes partially under the parallelogram, having two of its sides parallel; and the isosceles trapezoid has two sides equal, similar to the isosceles triangle.

(2). Family relationship. A little humor may be brought in by saying that the quadrilateral is great-grandfather to the square, grandfather to the rectangle and the rhombus, and father to both the parallelogram and the trapezoid. Fun is derived from further consideration of family relationships such as uncles, aunts, nephews, nieces, and cousins. Few in the class know that the children of first cousins are third cousins and not second cousins. A second cousin is the relationship between the first cousin and the child of the second cousin.

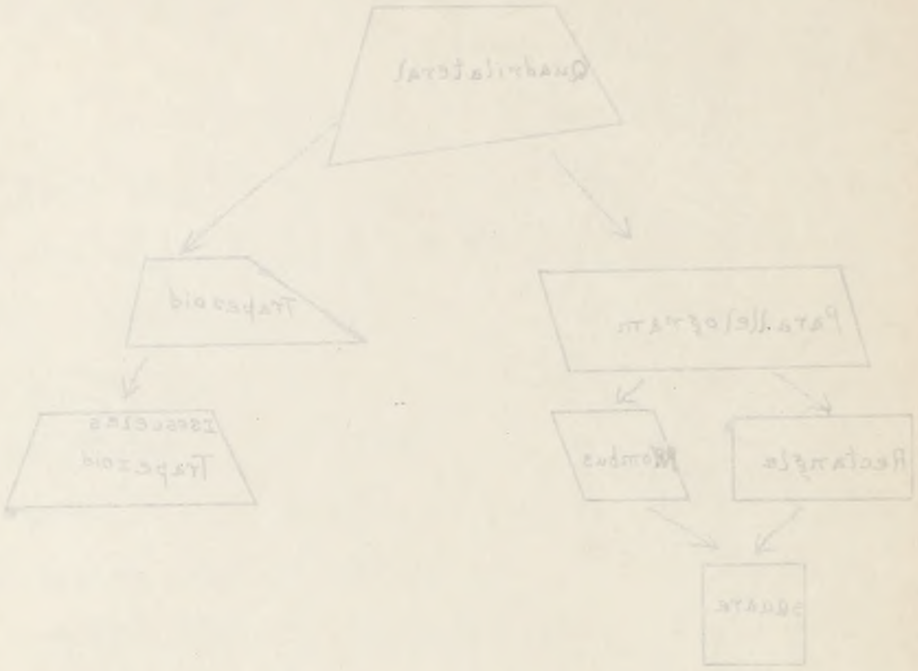
(3). Graphic. Tabulated graphically they might appear as follows:



12. Definition of "distance".

Another troublesome definition is the reading into the word "Distance" "perpendicular distance." This may be made clear by asking the pupils how far you are from the side of the room. This brings out the vagueness of the question until the pupil decides one wishes the shortest distance, which is a line perpendicular to the plane of the side of the room. "As the crow flies" may be brought in to further establish the meaning of the word "distance", and in the adolescent stage one can quote with meaning, "The longest way round is the shortest way home", under some circumstances. A tracing of the mileage for auto trips will reveal that these mileages are not straight

(3). Graphic. Tabulated graphically they might appear as follows:



12. Definition of "distance".

Another troublesome definition is the reading into the word "Distance" "perpendicular distance." This may be made clear by asking the pupils how far you are from the side of the room. This brings out the vagueness of the question until the pupil decides on what is the shortest distance, which is a line perpendicular to the plane of the side of the room. "As the crow flies" may be brought in to further establish the meaning of the word "distance", and in the adolescent stage one can quote with meaning, "The longest way round is the shortest way home", under some circumstances. A tracing of the mileage for auto trips will reveal that these mileages are not straight

distances between the points.

13. Summary.

a) Difficulties. To summarize the problem of having the pupil obtain a clear idea of definitions and symbols in plane geometry, so that he will have a basis for successful work, involves a study of the difficulties and a study of the various suggestions for success and application to teaching situations.

The difficulties of the pupils may be cited somewhat as follows:

a. Unattached, definitions are meaningless facts to be memorized.

b. The pupil is unable to divorce line from qualities of color and width and think only of length in the geometric sense.

c. An angle is thought of as being controlled in size by the length of the sides, since the pupil has considered increased length as an actual increase in other situations. He has a mind set in that direction because of past experiences.

d. Multiplicity of new definitions and symbols confuse the pupil.

e. The pupil fails to extend knowledge from a particular situation to a general; a pupil can recognize a perpendicular in a simple upright figure but has difficulty when the figure is more complex and the perpendicular line is drawn to a line not horizontal.

distances between the points.

13. Summary.

a) Difficulties. To summarize the problem of having the pupil obtain a clear idea of definitions and symbols in plane geometry, so that he will have a basis for successful work, involves a study of the difficulties and a study of the various suggestions for success and application to teaching situations.

The difficulties of the pupils may be cited somewhat as follows:

a. Unattached, definitions are meaningless facts to be memorized.

b. The pupil is unable to divorce line from qualities of color and width and think only of length in the geometric sense.

c. An angle is thought of as being controlled in size by the length of the sides, since the pupil has considered increased length as an actual increase in other situations. He has a mind set in that direction because of past experiences.

d. Multiplicity of new definitions and symbols confuse the pupil.

e. The pupil fails to extend knowledge from a particular situation to a general; a pupil can recognize a perpendicular in a simple upright figure but has difficulty when the figure is more complex and the perpendicular line is drawn to a line not horizontal.

f. He judges from appearance rather than logical proof. He has a tendency to draw special cases such as isosceles triangles.

g. The pupil has the psychological difficulty of fear over the confusion which results from the new language of geometry.

h. The failure to master the idea of definitions leads to difficulties in proving theorems.

i. The time allotted to absorb the new definitions is inadequate.

j. Logical and meticulous definitions drive interest away.

f. He judges from appearance rather than logical proof.
He has a tendency to draw special cases such as isosceles triangles.

g. The pupil has the psychological difficulty of fear over the confusion which results from the new language of geometry.

h. The failure to master the idea of definitions leads to difficulties in proving theorems.

i. The time allotted to absorb the new definitions is inadequate.

j. Logical and meticulous definitions drive interest away.

b). General devices. Devices for aiding in success in teaching definitions.

1. Gradual procedure.
2. Understanding by pupil.
3. Definitions only actually needed presented.
4. Simple terms.
5. Definitions growing out of the pupil's experience.
6. Instruments and constructions.
7. Concrete objects.
8. Concrete illustrations from the classroom and the world outside.
9. Drawing of figures by the pupil.
10. "They don't speak our language". Every subject has its vocabulary to be learned.
11. Motion idea, as in angles.
12. Avoidance of dwelling too long on the definition.
13. The new ideas presented singly and in isolation.
14. Practise on the new definitions in various situations and applications, including construction.
15. A felt need for description created from the pupil's project, the figure on the board, or a figure in life. A necessity for names for the various parts of the figure.
16. Definitions presented when there is need for them.

d). General devices. Devices for aiding in success in teaching definitions.

1. Gradual procedure.
2. Understanding by pupil.
3. Definitions only actually needed presented.
4. Simple terms.
5. Definitions growing out of the pupil's experience.
6. Instruments and constructions.
7. Concrete objects.
8. Concrete illustrations from the classroom and the world outside.
9. Drawing of figures by the pupil.
10. "They don't speak our language". Every subject has its vocabulary to be learned.
11. Motion idea, as in angles.
12. Avoidance of dwelling too long on the definition.
13. The new ideas presented singly and in isolation.
14. Practise on the new definitions in various situations and applications, including construction.
15. A felt need for description created from the pupil's project, the figure on the board, or a figure in life. A necessity for names for the various parts of the figure.
16. Definitions presented when there is need for them.

C. Constructions and Measurements.

So important has seemed the method of building up definitions and concepts by means of constructions and measurements that it has appeared desirable to devote a unit to this topic.

1. When taught.

Among the writers there seems to be general agreement on the wisdom of preceding work in demonstrative geometry by constructions and measurements. Lide[1] calls attention to the work at the English High School in Boston as especially noteworthy and writes that "The preliminary work consists largely of constructions with ruler, compass, and protractor. Throughout these constructions opportunity is taken to build up the mathematical vocabulary of the pupil...."

2. List of constructions.

One may first list the various constructions as noted by the writers, together with the instruments used, and then cite the advantages and disadvantages as found.

a. Bisecting a line.
b. Bisecting an angle.
c. Drawing a perpendicular to a point at a given point.
d. Constructing a perpendicular from a point to a line.
e. Making an angle equal to a given angle.
f. Bisecting an arc of a circle.
g. Trisecting a right angle.
h. Inscribing a regular hexagon in a circle.
i. Constructing a triangle congruent to a given triangle by making certain parts equal.
j. Drawing segments of a line of specified length.
k. Making a circle of given radius.
l. Constructing a triangle, given certain parts: special cases-right, equilateral, isosceles.

1. Edwin S. Lide, Instruction in Mathematics, Bulletin, 1932, no. 17 Monograph 23 U.S. Dept. of the Interior, Office of Education, p. 51.

c. Constructions and Measurements.

So important has seemed the method of building up definitions and concepts by means of constructions and measurements that it has appeared desirable to devote a unit to this topic.

1. When taught.

Among the writers there seems to be general agreement on the wisdom of proceeding work in demonstrative geometry by constructions and measurements. Lidell calls attention to the work at the English High School in Boston as especially noteworthy and writes that "The preliminary work consists largely of constructions with ruler, compass, and protractor. Throughout these constructions opportunity is taken to build up the mathematical vocabulary of the pupil...."

2. List of constructions.

One may first list the various constructions as noted by the writers, together with the instruments used, and then cite the advantages and disadvantages as found.

a. Bisecting a line.
b. Bisecting an angle.
c. Drawing a perpendicular to a point at a given point.
d. Constructing a perpendicular from a point to a line.
e. Making an angle equal to a given angle.
f. Bisecting an arc of a circle.
g. Trisecting a right angle.
h. Inscribing a regular hexagon in a circle.
i. Constructing a triangle congruent to a given triangle by making certain parts equal.
j. Drawing segments of a line of specified length.
k. Making a circle of given radius.
l. Constructing a triangle, given certain parts: special cases-right, equilateral, isosceles.

1. Edwin S. Lide, Instruction in Mathematics, Bulletin, 1922, no. 17 Monograph 23 U.S. Dept. of the Interior, Office of Education, p. 51.

m. Constructing parallel lines.
n. Constructing rectangle, square, octagon.
o. Making scale constructions; congruence and similarity.

Of these constructions, the first five have been noted as the fundamental constructions by Breslich,[1] while Betz[2] names the first four as fundamental.

3. How taught.

How constructions should be taught is suggested by Betz as follows:[3]

> "The fundamental constructions are often taught by a purely imitative procedure. Thus, the pupil is shown how to bisect an angle without understanding the reasons for the various motions. The result is always an undesirable mental uncertainty and confusion. Instead, a thorough understanding of these constructions can be developed organically by studying the symmetric properties of the kite, in which case even very slow seventh-grade pupils quickly grasp the essentials of the construction problems which it suggests.
>
> "Again, the isolated presentation of individual constructions is misleading. They should be used in organic combinations. So far, no better plan has been suggested for this purpose than the use of applied design, or of composite figures, such as Gothic windows, trefoils, quaterfoils, star polygons, and the like. Not only are all the fundamental construction skills introduced in natural combinations when this method is used, but an immediate check is provided for the degree of accuracy that should be attained at any stage of the work."

For those especially interested in design work in geometry, the Bureau of Publications, Teachers College, Columbia University, New York, have published, 1932, Geometry of Repeating Design and Geometry of Design for High Schools. A. Day Bradley.

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics. pp. 247-279.
2. William Betz, "The Teaching of Intuitive Geometry". The National Council of Teachers of Mathematics. The Eighth Yearbook, p. 135. (1930)
3. William Betz, op. cit. The Eighth Yearbook. p. 151.

m. Constructing parallel lines.
n. Constructing rectangle, square, octagon.
o. Making scale constructions; congruence and similarity.

Of these constructions, the first five have been noted as the fundamental constructions by Breslich,[1] while Betz[2] names the first four as fundamental.

3. How taught.

How constructions should be taught is suggested by Betz as follows:[3]

"The fundamental constructions are often taught by a purely imitative procedure. Thus, the pupil is shown how to bisect an angle without understanding the reasons for the various motions. The result is always an undesirable mental uncertainty and confusion. Instead, a thorough understanding of these constructions can be developed organically by studying the symmetric properties of the kite, in which case even very slow seventh-grade pupils quickly grasp the essentials of the construction problems which it suggests.

"Again, the isolated presentation of individual constructions is misleading. They should be used in organic combinations. So far, no better plan has been suggested for this purpose than the use of applied design, or of composite figures, such as Gothic windows, trefoils, quatrefoils, star polygons, and the like. Not only are all the fundamental construction skills introduced in natural combinations when this method is used, but an immediate check is provided for the degree of accuracy that should be attained at any stage of the work."

For those especially interested in design work in geometry, the Bureau of Publications, Teachers College, Columbia University, New York, have published, 1932, Geometry of Repeating Design and Geometry of Design for High Schools, A. Day Bradley.

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics. pp. 247-279.
2. William Betz, "The Teaching of Intuitive Geometry". The National Council of Teachers of Mathematics. The Eighth Yearbook, p. 155. (1930)
3. William Betz, op. cit. The Eighth Yearbook, p. 151.

The instruments used for constructions as noted by various writers are compasses, and the straight edge or ruler, which is according to the law of construction.

4. Discussion of Measurement.

Turning to the measurements suggested, we find that the following are listed from the writings:

William Betz writes:[1]

> "If the new unit related to 'measurement', the preliminary discussion may touch on such items as the following:
> 1. Trades, industries, and professions which depend largely on measurement.
> 2. Occasions for measurements in everyday life.
> 3. The most common measuring instruments.
> 4. The purposes of measurement.
> 5. Interesting measurement projects suggested by the community.
> 6. The meaning of accuracy in measurement.
> 7. The historical development of the common units of measure.
> 8. What it means to 'estimate' lengths."

5. Measurement situations.

Situations in which measurements occur are as follows:

a. Finding sum of angles of a triangle--by
   1. Protractor.
   2. Paper tearing.
   3. Rotation of pencil.
b. Developing ratio inductively by measurement of circumference and diameter, and finding the ratio a constant.
c. Proving area of a parallelogram is equal to area of a rectangle by cutting out the triangles at the side and fitting in on the other side.
d. Cutting out twelve sectors in a circle and fitting in to form a rectangle.
e. Making and using a transit.

1. Intuitive Geometry, New York Syllabus in Junior High School Mathematics. 1928, 1933. The National Council of Teachers of Mathematics. The Eighth Yearbook. p. 135.

The instruments used for constructions as noted by various writers are compasses, and the straight edge or ruler, which is according to the law of construction.

4. Discussion of Measurement.

Turning to the measurements suggested, we find that the following are listed from the writings:

William Betz writes:[1]

"If the new unit related to 'measurement', the preliminary discussion may touch on such items as the following:

1. Trades, industries, and professions which depend largely on measurement.
2. Occasions for measurements in everyday life.
3. The most common measuring instruments.
4. The purposes of measurement.
5. Interesting measurement projects suggested by the community.
6. The meaning of accuracy in measurement.
7. The historical development of the common units of measure.
8. What it means to 'estimate' lengths."

5. Measurement situations.

Situations in which measurements occur are as follows:

a. Finding sum of angles of a triangle--by
   1. Protractor.
   2. Paper tearing.
   3. Rotation of pencil.
b. Developing ratio inductively by measurement of circumference and diameter, and finding the ratio a constant.
c. Proving area of a parallelogram is equal to area of a rectangle by cutting out the triangles at the side and fitting in on the other side.
d. Cutting out twelve sectors in a circle and fitting in to form a rectangle.
e. Making and using a transit.

1. Intuitive Geometry. New York Syllabus in Junior High School Mathematics. 1928, 1935. The National Council of Teachers of Mathematics. The Eighth Yearbook. p. 135.

f. Discovering relationship between area and its diameter through weight comparison. Cutting out from a piece of cardboard a number of circles and compare weights of the cardboard discs with the weight of a unit of area cut from the same material. Making a list of diameters and weights.
g. Computing lengths and distances by numerical trigonometry.
h. Finding volumes.
i. Measuring angles.
j. Drawing angles.
k. Drawing lines.
l. Using scale drawings.
m. Measuring irregular piece of land.
n. Copying by means of pantograph.

William Betz gives the following applications:[1]

1. To show drawings.
2. To interpret blue prints.
3. To make house plans.
4. To design.
5. To apply to pattern making.
6. To survey plots.

6. Instruments used.

Among the instruments for measuring mentioned are:

a. Squared paper--cross section.
b. Protractor.
c. Compass.
d. Ruler.
e. Transit.
f. Drawing instruments.
g. Measuring rods.
h. Calipers.
i. Balances.
j. Sextants.
k. Steel tapes.
l. Thermometers.
m. Planimeters.
n. Level.
o. Hypsometer.
p. Plane Table.
q. Pantograph.

1. Intuitive Geometry, New York Syllabus in Junior High School Mathematics (1928), The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933. p. 135.

f. Discovering relationship between area and its diameter through weight comparison. Cutting out from a piece of cardboard a number of circles and compare weights of the cardboard discs with the weight of a unit of area cut from the same material. Making a list of diameters and weights.
g. Computing lengths and distances by numerical trigonometry.
h. Finding volumes.
i. Measuring angles.
j. Drawing angles.
k. Drawing lines.
l. Using scale drawings.
m. Measuring irregular piece of land.
n. Copying by means of pantograph.

William Bade gives the following applications:[1]

1. To show drawings.
2. To interpret blue prints.
3. To make house plans.
4. To design.
5. To apply to pattern making.
6. To survey plots.

6. Instruments used.

Among the instruments for measuring mentioned are:

a. Squared paper--cross section.
b. Protractor.
c. Compass.
d. Ruler.
e. Transit.
f. Drawing instruments.
g. Measuring rods.
h. Calipers.
i. Balances.
j. Sextants.
k. Steel tapes.
l. Thermometers.
m. Planimeters.
n. Level.
o. Hypsometer.
p. Plane Table.
q. Pantograph.

1. Inductive Geometry. New York Syllabus in Junior High School Mathematics (1928). The National Council of Teachers of Mathematics. The Eighth Yearbook, 1933. p. 136.

Miss Eddy found the following additional instruments named in text books:[1]

a. Carpenter's square.
b. Triangles.
c. Parallel ruler.
d. T-square
e. Diagonal scale.
f. Center disc.
g. Proportional compasses.
h. Carpenter's A level
i. Angle bisector.
j. Plumb line.
k. Angle trisector.
l. Height measurer.
m. Horizontal level.
n. Carpenter's guage.
o. Slide rule.
p. Field protractor.
q. Military compass.
r. Vernier.
s. Parallel bob.

It is evident that the texts lead in the suggestion of use of instruments to the applied problem. The literature in the field features general suggestions following psychological laws which have been stamped with the approval of authoritative educational agencies, but which have fewer definite suggestions or illustrative lessons and thus have less mention of definite instruments of measurements.

7. Advantages.

Writers mention frequently the advantages of constructions and measurements. They may be summed up as follows:

a. To better grasp the theorem.
b. To become thoroughly acquainted with the names and meanings of the fundamental concepts.
c. To acquire a vocabulary of terms with understanding.
d. To acquire accuracy in reading.
e. To develop ability to handle physical tools.

1. Louise Barbour Eddy, M. A. Thesis, "Motivating the Study of Geometry", Unpublished Master's thesis of Chicago University, 1930.

Miss Eddy found the following additional instruments named in text books:[1]

a. Carpenter's square.
b. Triangles.
c. Parallel ruler.
d. T-square
e. Diagonal scale.
f. Center disc.
g. Proportional compasses.
h. Carpenter's A level
i. Angle bisector.
j. Plumb line.
k. Angle trisector.
l. Height measurer.
m. Horizontal level.
n. Carpenter's gauge.
o. Slide rule.
p. Field protractor.
q. Military compass.
r. Vernier.
s. Parallel bob.

It is evident that the texts lead in the suggestion of use of instruments to the applied problem. The literature in the field features general suggestions following psychological laws which have been stamped with the approval of authoritative educational agencies, but which have fewer definite suggestions or illustrative lessons and thus have less mention of definite instruments of measurements.

V. Advantages.

Writers mention frequently the advantages of constructions and measurements. They may be summed up as follows:

a. To better grasp the theorem.
b. To become thoroughly acquainted with the names and meanings of the fundamental concepts.
c. To acquire a vocabulary of terms with understanding.
d. To acquire accuracy in reading.
e. To develop ability to handle physical tools.

1. Louise Barbour Eddy, M. A. Thesis, "Motivating the Study of Geometry", Unpublished Master's thesis of Chicago University, 1930.

f. To have pupils work as individuals.
g. To have pupils work with their own hands at a table, using objective things.
h. To make work experimental in order to supply that which is essential in sense perception and as an aid for abstract thinking.
i. To practise earth measurements thus giving life and reality to the subject.
j. To note that interest centers in the concrete rather than in the abstract.
k. To give the pupil opportunity to become familiar with many of the topics.
l. To convince pupil of his own childish vocabulary and make him willing to acquire the formal language of geometric exposition.
m. To afford an opportunity for the concept of hypothesis and conclusion to be illustrated concretely.
n. To utilize the civil engineering period of boyhood.
o. To break down the prejudice against geometry.
p. To make pupil realize that mathematics is a vital part of life rather than a series of mental gymnastics.
q. To emphasize doing.
r. To be within capacity of child.
s. To make application prominent.
t. To give clear notion of space concepts.
u. To guard against slovely work which often leads to erroneous conclusions.
v. To train pupil to draw neat and accurate figures.
w. To teach use of instruments.
x. To give joy in learning how to make instruments.

8. Disadvantages with excessive use.

Schultz sounds a note of warning on the excessive use of the laboratory method and notes that it is a valuable supplement and aids in the interest in pure mathematics but must not replace it. He cites the following disadvantages:

a. The method is exceedingly slow.
b. It may degenerate into a kind of manual training.
c. It is based on wrong assumption that pupils cannot comprehend and do not enjoy demonstrational mathematics.
d. It does not give training in true mathematical thinking.
e. It makes pupil acquainted with mathematical facts but not mathematical reasoning.

1. Arthur Schultz, The Teaching of Mathematics in Secondary Schools. pp. 48-49.

f. To have pupils work as individuals.
g. To have pupils work with their own hands at a table, using objective things.
h. To make work experimental in order to supply that which is essential in sense perception and as an aid for abstract thinking.
i. To practise earth measurements thus giving life and reality to the subject.
j. To note that interest centers in the concrete rather than in the abstract.
k. To give the pupil opportunity to become familiar with many of the topics.
l. To convince pupil of his own childish vocabulary and make him willing to acquire the formal language of geometric exposition.
m. To afford an opportunity for the concept of hypothesis and conclusion to be illustrated concretely.
n. To utilize the civil engineering period of boyhood.
o. To break down the prejudice against geometry.
p. To make pupil realize that mathematics is a vital part of life rather than a series of mental gymnastics.
q. To emphasize doing.
r. To be within capacity of child.
s. To make application prominent.
t. To give clear notion of space concepts.
u. To guard against slovely work which often leads to erroneous conclusions.
v. To train pupil to draw neat and accurate figures.
w. To teach use of instruments.
x. To give joy in learning how to make instruments.

8. Disadvantages with excessive use.

Schultz sounds a note of warning on the excessive use of the laboratory method and notes that it is a valuable supplement and aids in the interest in pure mathematics but must not replace it. He cites the following disadvantages:

a. The method is exceedingly slow.
b. It may degenerate into a kind of manual training.
c. It is based on wrong assumption that pupils cannot comprehend and do not enjoy demonstrational mathematics.
d. It does not give training in true mathematical thinking.
e. It makes pupil acquainted with mathematical facts but not mathematical reasoning.

1. Arthur Schultz, The Teaching of Mathematics in Secondary Schools, pp. 18-19.

A perusal of the above disadvantages reveals, however, that they are all present only in case of too great an emphasis to the exclusion of the demonstrative geometry. In the proper place, construction and measurement as an introduction to and a supplement to the demonstrational geometry, has no disadvantages, a fact which cannot well be dismissed.

Any particular method of teaching would have disadvantages if overbalanced, just as any technique may be used too much. Like golf, a single club may achieve results in the hands of a good player but not the best results, so any single technique may prove valuable in the hands of a good teacher but not as valuable as using all the available techniques in proper sequence and with proper emphasis.

Hassler and Smith[1] also sound a note of warning when they suggest that "If such work is prolonged or narrowly conceived, the pupil is likely to have a shock when the time for disillusionment comes. When the teacher conducts the work in constructions as a vehicle to carry along the aims that we have set forth in these pages,....Always looking forward to demonstration, and not as an end in itself, he will find it a convenient and interesting approach."

Although one must be careful not to over emphasize the constructive geometry, yet Young[2] points out that it would be a mistake to abandon this side when demonstrative geometry is taken up as a separate subject. Even with the course of formal

1. J. O. Hassler, and R. R. Smith, The Teaching of Secondary Mathematics. p. 324.
2. J. W. Young, The Teaching of Mathematics in the Elementary and Secondary School. p. 263.

A perusal of the above disadvantages reveals, however, that they are all present only in case of too great an emphasis to the exclusion of the demonstrative geometry. In the proper place, construction and measurement as an introduction to and a supplement to the demonstrational geometry, has no disadvantages, a fact which cannot well be dismissed.

Any particular method of teaching would have disadvantages if overbalanced, just as any technique may be used too much. Like golf, a single club may achieve results in the hands of a good player but not the best results, so any single technique may prove valuable in the hands of a good teacher but not as valuable as using all the available techniques in proper sequence and with proper emphasis.

Hassler and Smith[1] also sound a note of warning when they suggest that "If such work is prolonged or narrowly conceived, the pupil is likely to have a shock when the time for disillusionment comes. When the teacher conducts the work in constructions as a vehicle to carry along the aims that we have set forth in these pages,.....always looking forward to demonstration, and not as an end in itself, he will find it a convenient and interesting approach."

Although one must be careful not to over emphasize the constructive geometry, yet Young[2] points out that it would be a mistake to abandon this side when demonstrative geometry is taken up as a separate subject. Even with the course of formal

1. J. O. Hassler, and R. R. Smith, The Teaching of Secondary Mathematics. p. 324.

2. J. W. Young, The Teaching of Mathematics in the Elementary and Secondary School. p. 263.

demonstration in full swing, the concrete instrumentalities may still be used to lead up to propositions, to illustrate them, to study their bearings, properties, and consequences. The use of models is of great value. They should be made by the pupil himself when feasible.

9. Difficulties overcome by construction and measurement.

Various difficulties have been cited as cleared up by the use of constructions and measurements. Breslich[1] notes the tendency of pupils to draw special triangles, such as equilateral, isosceles, or right-angled and then to use these facts in the proofs, although not given in the theorem. He says to grasp better the theorem, draw out the figure and measure. He suggests a method for the teacher to sketch at the board and then check each pupil at each step, and finally to apply immediately to situations.

Hassler and Smith[2] call attention to the benefit of construction in making clear to the pupil the limitation of conditions which a line may fulfill. They state "The requirement that the work should actually be done with ruler and compasses is first aid to the situation."

One error consists of placing too many conditions upon the construction line and assuming that lines, given and constructed, pass through particular points. "A pupil may say, 'Bisect the vertical angle of an isosceles triangle, so that the line is perpendicular to the base'. It will be obvious in the construction that both things are not

1. Ernest R. Breslich, op. cit. pp. 247-279.

2. J. O. Hassler and R. R. Smith, op. cit. pp. 347-348.

demonstration in full swing, the concrete instrumentalities may still be used to lead up to propositions, to illustrate them, to study their bearings, properties, and consequences. The use of models is of great value. They should be made by the pupil himself when feasible.

2. Difficulties overcome by construction and measurement.

Various difficulties have been cited as cleared up by the use of constructions and measurements. Breslich[1] notes the tendency of pupils to draw special triangles, such as equilateral, isosceles, or right-angled and then to use these faces in the proofs, although not given in the theorem. He says to grasp better the theorem, draw out the figure and measure. He suggests a method for the teacher to sketch at the board and then check each pupil at each step, and finally to apply immediately to situations.

Hassler and Smith[2] call attention to the benefit of construction in making clear to the pupil the limitation of conditions which a line may fulfill. They state "The requirement that the work should actually be done with ruler and compasses is first aid to the situation."

One error consists of placing too many conditions upon the construction line and assuming that lines, given and constructed, pass through particular points. "A pupil may say, 'Bisect the vertical angle of an isosceles triangle, so that the line is perpendicular to the base'. It will be obvious in the construction that both things are not

1. Ernest R. Breslich, op. cit. pp. 247-279.

2. J. O. Hassler and R. R. Smith, op. cit. pp. 347-348.

"done. Either angle is bisected or a perpendicular is constructed from the vertex to the base. Either relation is the result of the other. A more abstract discussion may follow. A line is determined by two points or a point and a direction. If a line connects two points or passes through a point in a particular direction (as in the case when an angle is bisected or a perpendicular is erected to a line through a given point), no other condition may be placed on the line."[1].

a) Placing too many conditions upon the construction line.

"In considering the theorem, 'Any two parallel chords drawn to the extremities of a diameter are equal', most pupils find it very easy to 'prove' this exercise by drawing a line AD, assuming that it passes through the center. When asked, 'How do you know that AD will pass through O?' they cannot answer. When asked, 'How is AD determined?' and they answer 'by A and D,' they realize that they have no right to assume that it passes through O. Then they will wish to compromise by drawing AO and OD, but will innocently fall into the error of assuming that AOD is a straight line. Errors of this type can be eliminated for the time being by requiring the solution of several exercises in a series where each exercise contains the possibility of one of these errors. A discussion of the possible errors will cause the pupils to be on the watch for them. The mistake is likely to manifest itself at some later date, however, if the matter is not frequently touched upon."

b) Constructing a line in a complex situation. Another type of error on the part of the teacher is exposed by construction according to Hassler and Smith[2]. They state:

"The teacher will learn from this concrete work in constructions how small must be the gaps over which the average intelligence of the tenth-year pupil can jump. A pupil may be able to bisect a line or an angle, but be at a loss when asked to bisect the sides of angles of a triangle. Intermediate exercises have to be prepared to help bridge the gap. If this is so in simple work as construction exercises, the teacher must be doubly on his guard when the more abstract work is begun."

1. J.O.Hassler and R.R.Smith, op. cit. pp. 347-348.

2. J.O.Hassler and R.R.Smith, op. cit. pp. 323-324.

"done. Either angle is bisected or a perpendicular is constructed from the vertex to the base. Either relation is the result of the other. A more abstract discussion may follow. A line is determined by two points or a point and a direction. If a line connects two points or passes through a point in a particular direction (as in the case when an angle is bisected or a perpendicular is erected to a line through a given point), no other condition may be placed on the line."[1].

a) Placing too many conditions upon the construction line.

"In considering the theorem, 'Any two parallel chords drawn to the extremities of a diameter are equal', most pupils find it very easy to 'prove' this exercise by drawing a line AD, assuming that it passes through the center. When asked, 'How do you know that AD will pass through O?' they cannot answer. When asked, 'How is AD determined?' and they answer 'by A and D,' they realize that they have no right to assume that it passes through O. Then they will wish to compromise by drawing AO and OD, but will innocently fall into the error of assuming that AOD is a straight line. Errors of this type can be eliminated for the time being by requiring the solution of several exercises in a series where each exercise contains the possibility of one of these errors. A discussion of the possible errors will cause the pupils to be on the watch for them. The mistake is likely to manifest itself at some later date, however, if the matter is not frequently touched upon."

b) Constructing a line in a complex situation. Another type of error on the part of the teacher is exposed by construction according to Hassler and Smith[2]. They state:

"The teacher will learn from this concrete work in constructions how small must be the gaps over which the average intelligence of the tenth-year pupil can jump. A pupil may be able to bisect a line or an angle, but be at a loss when asked to bisect the sides of angles of a triangle. Intermediate exercises have to be prepared to help bridge the gap. If this is so in simple work as construction exercises, the teacher must be doubly on his guard when the more abstract work is begun."

1. J.O.Hassler and R.R.Smith, op. cit. pp. 347-348.

2. J.O.Hassler and R.R.Smith, op. cit. pp. 323-324.

10. Construction of a transit.

T. L. Engle[1] gives in detail directions for the making of a transit and calls attention to the stage or development which most boys pass through which might be called a civil engineering period.

Smith-Reeve-Morse[2] gives the idea of not compressing the drawings into a single diagram. This is in line with the isolation of the difficulty for which the author has striven in his classes. Some of the modern texts are beginning to feature this idea. Instead of one finished construction appearing, there are several drawings as the construction progresses.

In the unit on proportion description of a pantograph is given and the direction for construction.

11. The class out-of-doors.

Since the subject of this unit is general in character, it would not be particularly valuable to tie it to a particular illustrative lesson. Anyone who has witnessed the enthusiasm of a class on a field trip does not need to be convinced of its efficiency. The writer has taken his classes outside for measuring heights of the flagpole in the school yard, the height of the school building, and the height of various public buildings in the community. The water tanks may also be measured and the distance around found and the volume computed. In measuring heights, pupils have drawn a protractor scale on oak tag and attached it to a frame constructed in the manual training

1. Constructing a Transit as a Project in Geometry. The Mathematics Teacher, XXIV (November, 1931) pp. 444-447.
2. Text and Tests in Plane Geometry. The Mathematics Teacher. XXVI (February, 1933) pp. 119-121.

10. Construction of a transit.

T. I. Engle[1] gives in detail directions for the making of a transit and calls attention to the stage or development which most boys pass through which might be called a civil engineering period.

Smith-Reeve-Morse[2] gives the idea of not compressing the drawings into a single diagram. This is in line with the isolation of the difficulty for which the author has striven in his classes. Some of the modern texts are beginning to feature this idea. Instead of one flatness construction appearing, there are several drawings as the construction progresses.

In the unit on proportion description of a pantograph is given and the direction for construction.

11. The class out-of-doors.

Since the subject of this unit is general in character, it would not be particularly valuable to tie it to a particular illustrative lesson. Anyone who has witnessed the enthusiasm of a class on a field trip does not need to be convinced of its efficiency. The writer has taken his classes outside for measuring heights of the flagpole in the school yard, the height of the school building, and the height of various public buildings in the community. The water tanks may also be measured and the distance around found and the volume computed. In measuring heights, pupils have drawn suproteactor scale on oak tag and attached it to a frame constructed in the manual training

1. Constructing a Transit as a Project in Geometry. The Mathematics Teacher. XIX (November, 1931) pp. 444-449.
2. Text and Tests in Plane Geometry. The Mathematics Teacher. XXVI (February, 1933) pp. 112-121.

department. This enlarged protractor with its wooden pointer is used to sight tops of buildings, and the base line is measured with steel tape. Allowance is made when the tip of the tower is over a point inside the building which makes the problem of estimating the distance from the outside of the building to the point beneath the peak. The class is required to find the center of the rectangle or a square on which the tower is built, a problem which involves finding the mid-point of a diagonal, Various members of the class sight at different base distances, and the computations are made and presented to the class to demonstrate the accuracy of the method.

department. This enlarged protractor with its wooden pointer is used to sight tops of buildings, and the base line is measured with steel tape. Allowance is made when the tip of the tower is over a point inside the building which makes the problem of estimating the distance from the outside of the building to the point beneath the peak. The class is required to find the center of the rectangle or a square on which the tower is built, a problem which involves finding the mid-point of a diagonal. Various members of the class sight at different base distances, and the computations are made and presented to the class to demonstrate the accuracy of the method.

D. Creating a felt need for geometric demonstration.

1. Illusions.

a). Introduction. In one's experience as a teacher one finds that books of psychology are of great use in their illustrations to show optical illusions. Many of these are brought to class and charts are drawn of them. Newspaper items of these illusions are presented. Recitals of mistakes people have made in what they thought they saw or heard are enumerated. Pupils are encouraged to tell of their experiences in the dark when going along a road or passing a deserted house, or when ill. A written assignment will bring interesting anecdotes. It was Stevenson's imagination when he was ill which led to many of his exciting adventures. The bear turns out to be a rock in the distant pasture; the calling of one's name with no one around, and the feeling of the hand on one's shoulder which is really not there, are illusions. The way one's mind fills in on certain situations according to the mental set, or the way one makes the situation or the word take on meaning, are all witnesses to the mistakes which the sensory organs and even intellects make. From instances similar to these many superstitions have been born. The after image of the eye until understood was mystical. All these show the necessity of proof. A second way of showing need of the demonstration comes from the life situations.

D. Creating a felt need for remedial demonstration.

1. Illusions.

a). Introduction. In one's experience as a teacher one finds that books of psychology are of great use in their illustrations to show optical illusions. Many of these are brought to class and charts are drawn of them. Newspaper items of these illusions are presented. Realists of mistakes people have made in what they thought they saw or heard are enumerated. Pupils are encouraged to tell of their experiences in the dark when going along a road or passing a deserted house, or when ill. A written assignment will bring interesting anecdotes. It was Stevenson's imagination when he was ill which led to many of his exciting adventures. The bear turns out to be a rock in the distant pasture; the calling of one's name with no one around, and the feeling of the hand on one's shoulder which is really not there, are illusions. The way one's mind fills in on certain situations according to the mental set, or the way one makes the situation or the word take on meaning, are all witnesses to the mistakes which the memory organs and even intellects make. From instances similar to these many superstitions have been born. The after image of the eye until understood was misleadingly. All these show the necessity of proof. A second way of showing need of the demonstration comes from the life situations.

Would one believe this individual? The salesmen as the prestidigitator who tries to keep one's mind on the movements he wishes one to notice and not to make a complete inventory of the situation is like the magician who makes one's eye follow the right hand while the "magic" is being performed by the left hand. The necessity of having authoritative information on the subject, of unbiased information, and not just accepting the thing because some one says it, or some book states it, should be shown. This reveals the need of complete proof. And this method of analysis and evaluation is surely of use in all situations in life involving choice, and decisions, among a collection of influences and facts.

Having thus oriented the beginning student into the need of plane geometry through the methods above described in the attempt to open the channel by favorable attitudes, one may next consider other guide posts along the footpath to success.

Would one believe this individual? The salesman as the prestidigitator who tries to keep one's mind on the movements he wishes one to notice and not to make a complete inventory of the situation is like the magician who makes one's eye follow the right hand while the "magic" is being performed by the left hand. The necessity of having authoritative information on the subject, of unbiased information, and not just accepting the thing because some one says it, or some book states it, should be shown. This reveals the need of complete proof. And this method of analysis and evaluation is surely of use in all situations in life involving choice, and decisions,among a collection of influences and facts.

Having thus oriented the beginning student into the need of plane geometry through the methods above described in the attempt to open the channel by favorable attitudes, one may next consider other guide posts along the footpath to success.

b). Influence of the mind set. Immediately preceding the beginning of demonstrational geometry there will need to come a consideration of the necessity for proof. Local material will be of greatest interest. The writer has introduced the subject somewhat as follows.

For several days in the daily newspapers there had been attention called to a coming high spot in a murder trial in which a nine-year old child was to testify that on a certain morning four months previously when she called to her father, he answered her. To too many of the public this is good evidence, but to one who considers the age of the child, the time elapsed since the act, and the fact that this was a repeated process each morning, this type of evidence will be less substantial, no matter how honest the child may be within herself. Nurses who have cared for sick people over a long period of time continue to hear them call after they are gone. So strong is the mind set under similar conditions that actual absence of the stimulus will often make no difference in the mind of the individual. Examples of this are in typewriting where when one word or a part of a word is started the mind automatically puts down what is customary. For instance, in this paragraph in the second line is the word "high". The writer immediately typed the word "school" after it even though "spot" was the word to be typed. This was because so many times previously the word "high" had been followed by the word "school".

b). Influence of the mind set. Immediately preceding the beginning of demonstrational geometry there will need to come a consideration of the necessity for proof. Local material will be of greatest interest. The writer has introduced the subject somewhat as follows.

For several days in the daily newspapers there had been attention called to a coming high spot in a murder trial in which a nine-year old child was to testify that on a certain morning four months previously when she called to her father, he answered her. To too many of the public this is good evidence, but to one who considers the age of the child, the time elapsed since the act, and the fact that this was a repeated process each morning, this type of evidence will be less substantial, no matter how honest the child may be within herself. Nurses who have cared for sick people over a long period of time continue to hear them call after they are gone. So strong is the mind set under similar conditions that actual absence of the stimulus will often make no difference in the mind of the individual. Examples of this are in typewriting where when one word or a part of a word is started the mind automatically puts down what is customary. For instance, in this paragraph in the second line is the word "high". The writer immediately typed the word "school" after it even though "spot" was the word to be typed. This was because so many times previously the word "high" had been followed by the word "school".

Our sensory impressions must receive careful checking.

This error of the senses has been a great factor in trials. Felix Frankfurter brought out in "The Atlantic Monthly" that at the distance away from the auto which the witness said he was and at the speed the auto was going, it was practically impossible for any eye to identify the occupants of the automobile, but the witness in spite of these obvious facts did identify the occupants.

McDill[1] calls attention to this mind-set indirectly when he says: "The average text book in assuming that principles should be reasoned out rather than taken by intuition, gives one or two optical illusions, showing that mistakes are made in intuition, but never hints that any one could make a mistake in formal reasoning."

How the mind reads into the situation is well exemplified by the terrors of the small child lost at night. Every shadow is a fearful shape which in daylight fades away. The writer has had many experiences of mistaking various objects at dusk. A rabbit became an inanimate clump of moss, while a gnarled tree trunk became a distant bear on his hind legs, At this stage, the pupil can be encouraged to give, in writing usually, his own experiences in this direction and thus the feeling of familiarity and contact with the subject may be made. In fact, this may well be the original point of contact of the discussion, and then the enlarging may be in the direction of the court cases etc.

1. R.M.McDill, "Laboratory Work in Geometry for the Ninth Year" The Mathematics Teacher, XXIV, (January, 1931). p. 16.

Our sensory impressions must receive careful checking.

This error of the senses has been a great factor in trials. Felix Frankfurter brought out in "The Atlantic Monthly" that at the distance away from the auto which the witness said he was and at the speed the auto was going, it was practically impossible for any eye to identify the occupants of the automobile, but the witness in spite of these obvious facts did identify the occupants.

McJill calls attention to this mind-set indirectly when he says: "The average text book in assuming that principles should be reasoned out rather than taken by intuition, gives one or two optical illusions, showing that mistakes are made in intuition, but never hints that any one could make a mistake in formal reasoning."

How the mind reads into the situation is well exemplified by the terrors of the small child lost at night. Every shadow is a fearful shape which in daylight fades away. The writer has had many experiences of mistaking various objects at dusk. A rabbit became an inanimate clump of moss, while a gnarled tree trunk became a distant bear on his hind legs. At this stage, the pupil can be encouraged to give, in writing usually, his own experiences in this direction and thus the feeling of familiarity and contact with the subject may be made. In fact, this may well be the original point of contact of the discussion, and then the enlarging may be in the direction of the court cases etc.

1. R.M.McJill, "Laboratory Work in Geometry for the Ninth Year" The Mathematics Teacher, XXIV, (January, 1931). p. 16.

The above examples are to illustrate mind-set errors and the errors from surrounding circumstances. Other errors creep in because of the structure of the eye which like other organs of the body is subject to fatigue and to muscle development.

Another interesting procedure is to look at a partial picture of an individual. If you gaze steadily at the picture below, you will finally see the clear picture of a noted young man.

MYSTERY PICTURE

Something New and Novel

"LINDBERGH"

Instructions—Take card in both hands, look steadily at small diamond shaped speck on nose try not to blink and count to 50 slow, then follow any of the instructions listed.

Look up at the sky day or night or on a light wall and photograph will appear greatly enlarged. Keep looking at one spot for 15 seconds.

Result—The actual photograph will appear and disappear several times.

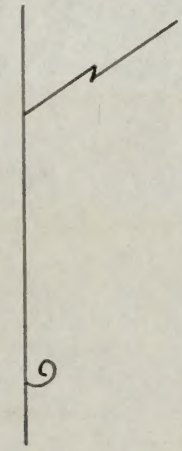
There's no mystery to good picture taking for the best in photography patronize

GEER'S STAR PHOTO SHOP

Block Island, R. I.

(over)

c). Influence of surrounding circumstances. Reading into the situation may be illustrated by the following drawing. When attention is first called the pupils see nothing. When it is explained that it is a bayonet end and that a soldier has just entered with his dog, the situation is clear. They see that the dog has a curly tail.

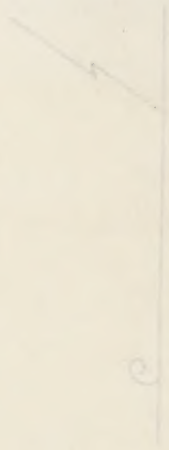


Prominent men are continually having the thing they say taken out of its setting and put in another setting. Perhaps Kipling in his poem "If" best expresses this when he says:

"If you can bear to hear the truth you've spoken
Twisted by knaves to make a trap for fools."

Similarly an experiment was made with a college class which showed the errors possible in sensory recordings. This happened as follows: Without warning one day the door of the classroom burst open and certain people entered, did certain things, and hastily departed. The class were asked to record

c). <u>Influence of surrounding circumstances.</u> Reading into the situation may be illustrated by the following drawing. When attention is first called the pupils see nothing. When it is explained that it is a bayonet and that a soldier has just entered with his dog, the situation is clear. They see that the dog has a curly tail.



Prominent men are continually having the thing they say taken out of its setting and put in another setting. Perhaps Kipling in his poem "If" best expresses this when he says:

"If you can bear to hear the truth you've spoken
Twisted by knaves to make a trap for fools."

Similarly an experiment was made with a college class which showed the errors possible in sensory recordings. This happened as follows: Without warning one day the door of the classroom burst open and certain people entered, did certain things, and hastily departed. The class were asked to record

what happened. Many and varied were the reports. Practically no one agreed on any one item. Even the number of people who entered the room was in doubt. The general impression was that someone had fired a revolver because the flash was seen and the report was heard. This was later verified as consisting only of a paper bag which had been blown up and suddenly smashed which accounted for the "report". A yellow banana waved in the air was the "red flare". And so the story went! Briefly, even a large class of intelligent individuals with average sensory capacities found it impossible to record accurately their sensations, partly because of the stimuli and partly because of the clever leadings away from what really happened as shown by the explosion of the paper bag.

This shows the necessity of thinking carefully and slowly about a situation and not rushing to conclusions from appearances. One remembers the college professor who said that he never bought a book until he had thought about it over night. Life is full of situations where the other individual presses you to make an immediate decision on the surface facts which look good from the sensory approach.

what happened. Many and varied were the reports. Practically no one agreed on any one item. Even the number of people who entered the room was in doubt. The general impression was that someone had fired a revolver because the flash was seen and the report was heard. This was later verified as consisting only of a paper bag which had been blown up and suddenly smashed which accounted for the "report". A yellow banana waved in the air was the "red flare". And so the story went! Briefly, even a large class of intelligent individuals with average sensory capacities found it impossible to record accurately their sensations, partly because of the stimuli and partly because of the clever leadings away from what really happened as shown by the explosion of the paper bag.

This shows the necessity of thinking carefully and slowly about a situation and not rushing to conclusions from appearances. One remembers the college professor who said that he never bought a book until he had thought about it over night. Life is full of situations where the other individual presses you to make an immediate decision on the surface facts which look good from the sensory approach.

Figure 2 is another variation of figure [1]. Surrounding lines make an optical illusion.

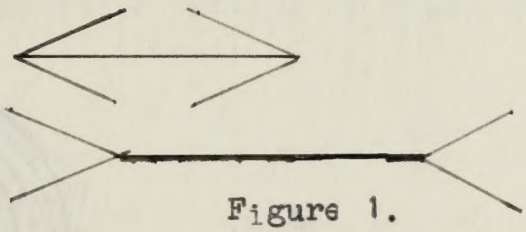


Figure 1.

The above diagrams[2] on both sides of the page again relate to the influence of surrounding lines to produce mistakes in measurements with the eyes.

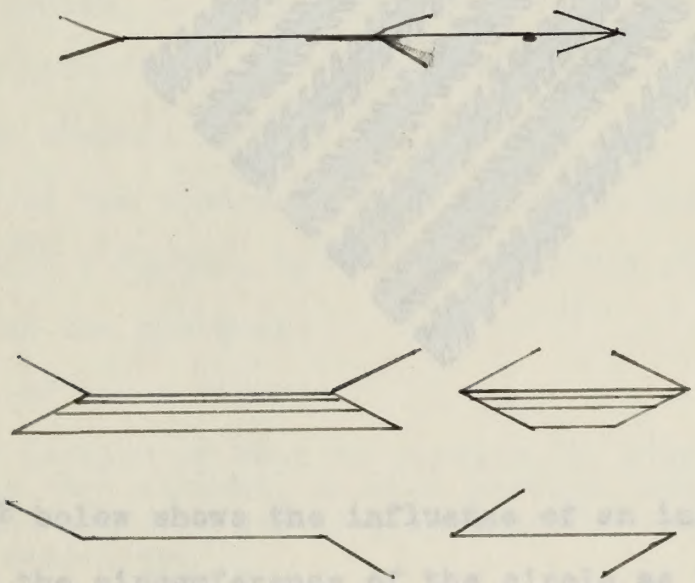


Figure 2.

---

1. W. B. Pillsbury, The Essentials of Psychology, p. 198.

2. Charles S. Myers, A Text Book of Experimental Psychology, 1911. pp. 287-288.

Figure 2 is another variation of Figure 1. Surrounding lines make an optical illusion.

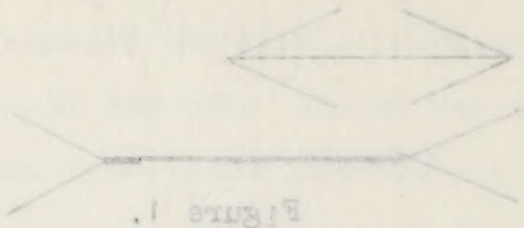


Figure 1.

The above diagrams[2] on both sides of the page again relate to the influence of surrounding lines to produce mistakes in measurements with the eye.

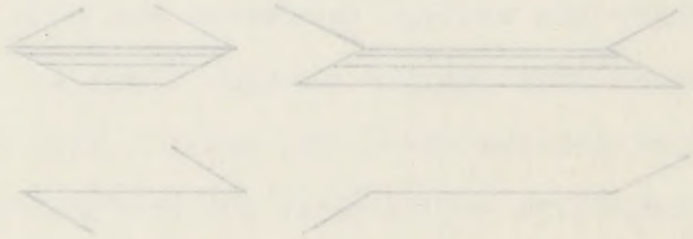


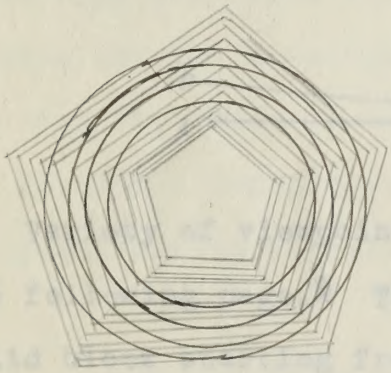
Figure 2.

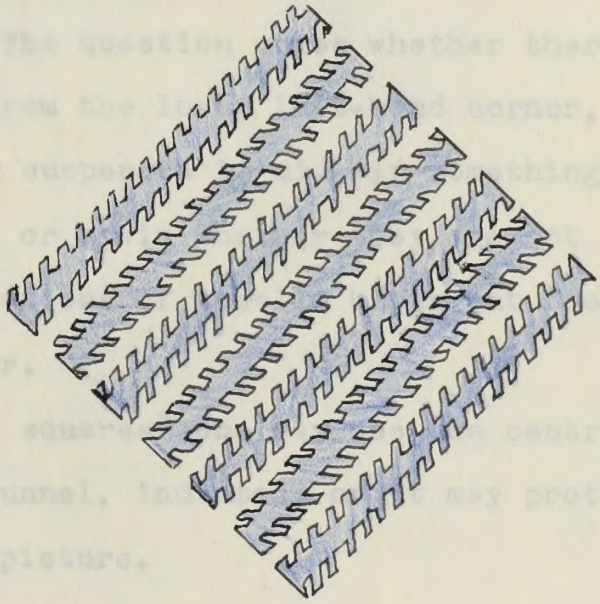
---

1. W. B. Pillsbury, The Essentials of Psychology, p. 198.

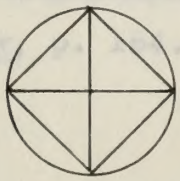
2. Charles S. Myers, A Text Book of Experimental Psychology, 1911. pp. 287-288.

The diagrams attached on this page again show the influence of surrounding lines.[1]





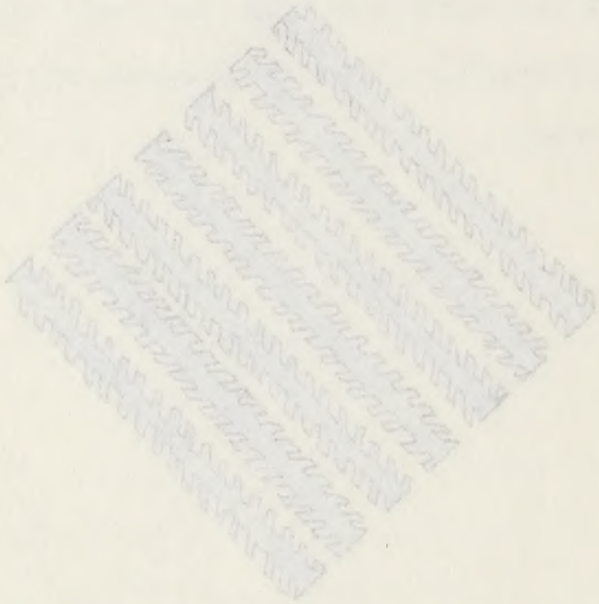
The figure[2] below shows the influence of an inscribed quadrilateral on the circumference of the circle as it appears to the eye.



1. Charles M. Myers, op. cit., p. 290.

2. W. B. Pillsbury, op. cit. p. 196.

The diagrams attached on this page again show the influence of surrounding lines.[1]

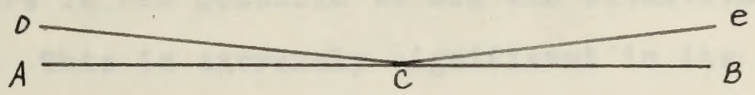


The figure[2] below shows the influence of an inscribed quadrilateral on the circumference of the circle as it appears to the eye.

1. Charles M. Myers, op. cit., p. 290.

2. W. B. Pillsbury, op. cit. p. 192.

Still another illusion attendant upon surrounding lines is given in the illustration below.[1] Here the horizontal lines seem bent in each instance because of these surrounding lines.



Variety of viewpoints are illustrated by stairs shown on the following page.[2] The question comes whether there is a solid block starting from the lower left-hand corner, or whether the stairs are suspended in mid-air something like the folds in an accordian, or again whether they are not stairs at all, but simply indentations of a solid block cut from the upper right-hand corner.

In the case of the squares[3] one may see the central square as the rear end of a funnel, indented, or it may protrude to the front part of the picture.

Angell[4] says of these figures:

> "While a portion of what we perceive is always supplied from without, another portion, and often the dominant portion, is supplied from within ourselves."

1. Mary W. Calkins, A First Book in Psychology, p. 72.
2. J. R. Angell, Psychology, p. 164.
3. Ibid. p. 164.
4. Ibid. p. 163.

Still another illusion attendant upon surrounding lines is given in the illustration below.[1] Here the horizontal lines seem bent in each instance because of these surrounding lines.

Variety of viewpoints are illustrated by stairs shown on the following page.[2] The question comes whether there is a solid block starting from the lower left-hand corner, or whether the stairs are suspended in mid-air something like the folds in an accordion, or again whether they are not stairs at all, but simply indentations of a solid block cut from the upper right-hand corner.

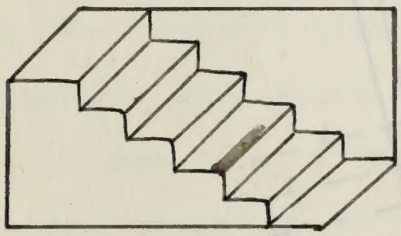
In the case of the squares[3] one may see the central square as the rear end of a tunnel, indented, or it may protrude to the front part of the picture.

Angell[4] says of these figures:

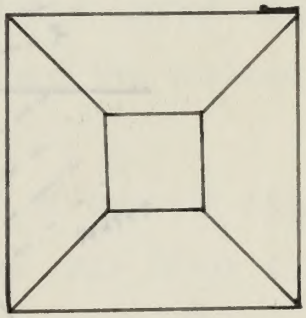
> "While a portion of what we perceive is always supplied from without, another portion, and often the dominant portion, is supplied from within ourselves."

1. Mary W. Calkins, A First Book in Psychology, p. 72.
2. J. R. Angell, Psychology, p. 164.
3. Ibid. p. 164.
4. Ibid. p. 163.

This is what has been spoken of as the mind set. The writer had an experience only recently when his wife saw seven squares in the diagram while he could see only six. Neither one could show the other what had been seen. It was some few minutes before it was possible to see the situation as the other had seen it. This is extremely significant in its relation to not forcing a child to see something which he cannot. Time must be given, and force will not do.



Three ways to see the above figure: a solid block in the lower left-hand corner; a solid block in the upper right; and the steps suspended in mid-air.



The smaller square protrudes or is in the background.

Figure 2 shows again the effect of surrounding lines. In the upper figure the line in the middle looks longer.

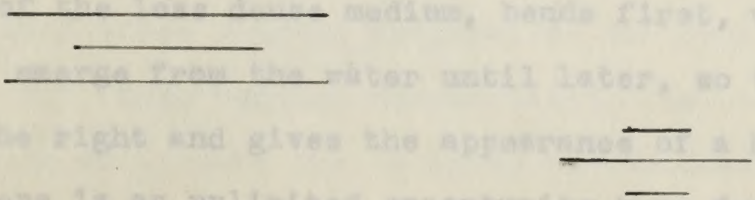
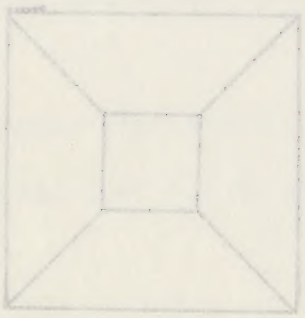


Figure 2.

1. J. R. Angell, Psychology, p. 187, 164.

This is what I has been spoken of as the mind set. The writer had an experience only recently when his wife saw seven squares in the diagram while he could see only six. Neither one could show the other what had been seen. It was some few minutes before it was possible to see the situation as the other had seen it. This is extremely significant in its relation to not forcing a child to see something which he cannot. Time must be given, and force will not do.

Three ways to see the above figure: a solid block in the lower left-hand corner; a solid block in the upper right; and the steps suspended in mid-air.



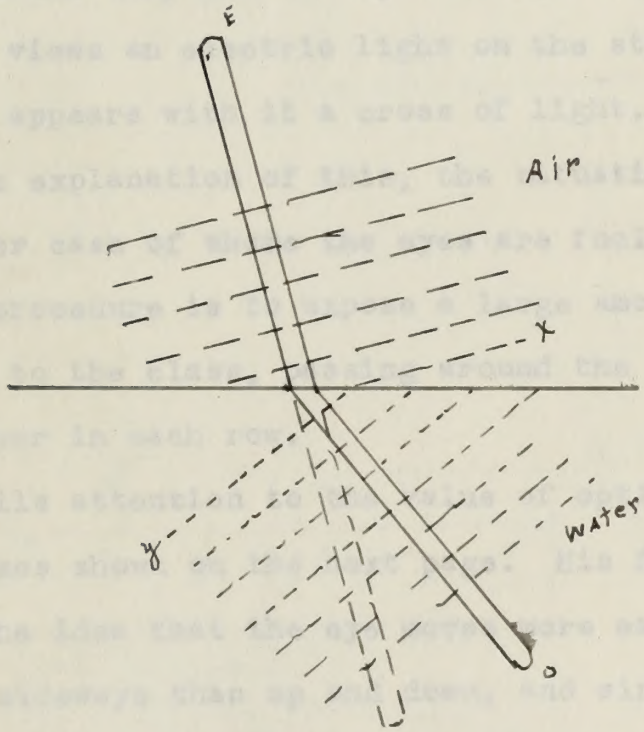
The smaller square protrudes or is in the background.

Figure 3 shows again the effect of surrounding lines. In the upper figure the line in the middle looks longer.

Figure 3.

I. J. R. Angell, Psychology, p. 187, 1904.

Everyone is familiar with the optical illusion of the broken oar in water. This may be illustrated before the class with something smaller, and then the simple explanation may be given which calls in the interrelationship of physics. The ray of light travelling from the oar end to the eye is bent as it leaves the water because of the difference of speed of light when entering a different density medium, such as water and air. The picture may be made somewhat as follows to illustrate this.



It will be noted that as the wave leaves the water at the surface that the right hand end "x" strikes the air first, and because of the less dense medium, bends first, while the "y" end does not emerge from the water until later, so that it bends toward the right and gives the appearance of a broken oar.

There is an unlimited opportunity here for pupils to bring in experiences they have had, or that their families may have had,

Everyone is familiar with the optical illusion of the broken oar in water. This may be illustrated before the class with something similar, and then the simple explanation may be given which calls in the interrelationship of physics. The ray of light travelling from the oar and to the eye is bent as it leaves the water because of the difference of speed of light when entering a different density medium, such as water and air. The picture may be made somewhat as follows to illustrate this.

It will be noted that as the wave leaves the water at the surface that the right hand end "x" strikes the air first, and because of the less dense medium, bends first, while the "y" end does not emerge from the water until later, so that it bends toward the right and gives the appearance of a broken oar.

There is an unlimited opportunity here for pupils to bring in experiences they have had, or that their families may have had.

or readings they have made, in regard to their topics. Mirages would come under this topic. The advantage of all this is that there is a linking with the familiar, sometimes with the actual families, a clearing away of superstitions, a realization of the necessity for proof. When this ground work is properly laid, there will be less chance that later in geometry pupils will be looking at a figure and telling the teacher that it does not need proof because they can see by the figure that it is that way.

When one views an electric light on the street through a screen, there appears with it a cross of light. Until one knows the scientific explanation of this, the situation is weird. Here is another case of where the eyes are fooled.

The next procedure is to expose a large amount of illustrative material to the class, passing around the various items, starting a paper in each row.

Reeve[1] calls attention to the value of optical illusions and gives four cases shown on the next page. His first illusion is based on the idea that the eye moves more easily and therefore more quickly sideways than up and down, and since the speed is slower looking up and down, to the eye the lines look higher rather than wider, although the measurements are the same in each case. It is for this reason that tall ladies do not wear dresses with lines running lengthwise; nor do stout ladies wear dresses with lines running horizontally. In each case these lines would accentuate the tallness or stoutness as the case may be.

1. W.D.Reeve. "Demonstrative Geometry for the Ninth Grade". The Mathematics Teacher. (March, 1933). p. 55.

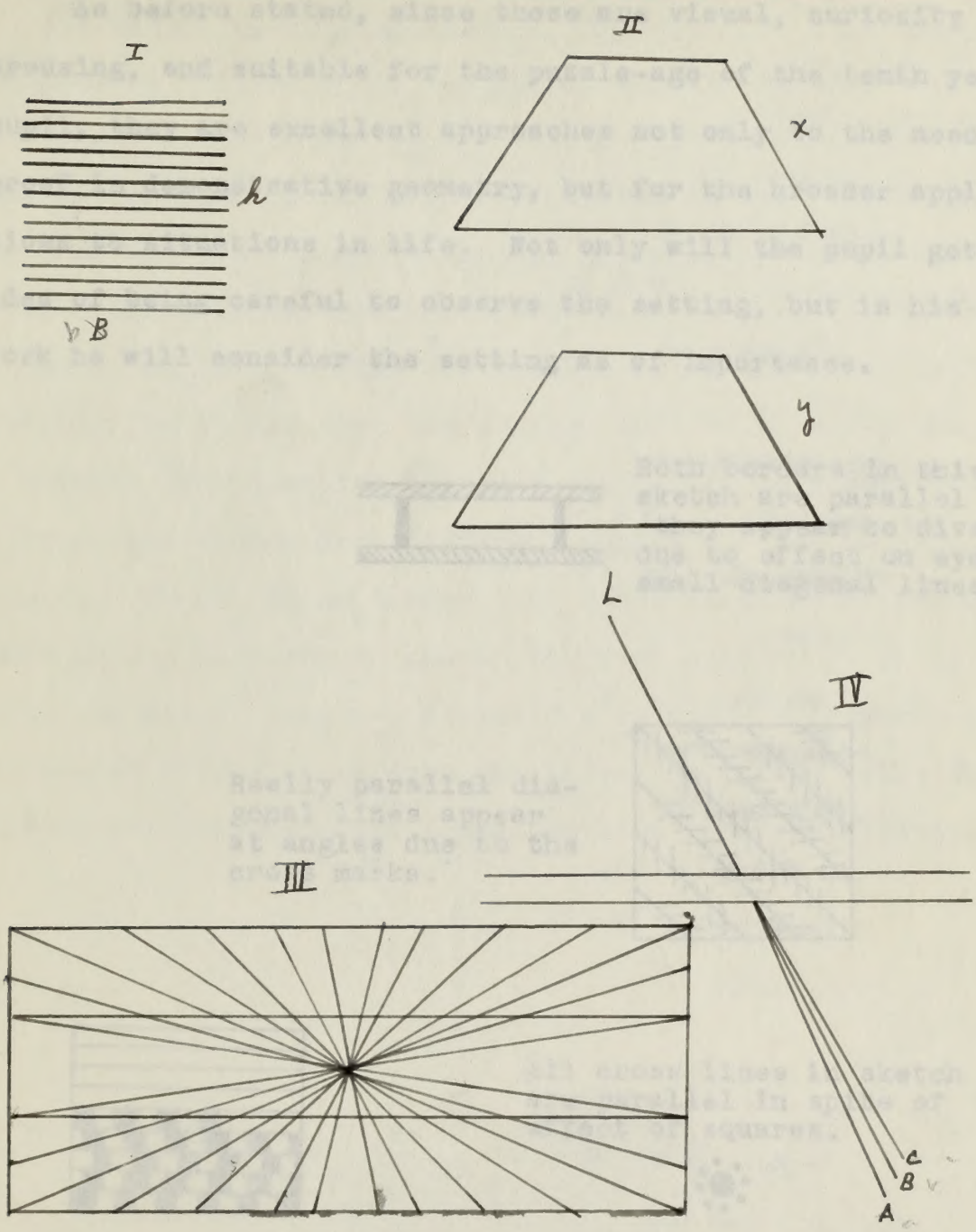
or readings they have made, in regard to their topics. Mirages would come under this topic. The advantage of all this is that there is a linking with the familiar, sometimes with the actual families, a clearing away of superstitions, a realization of the necessity for proof. When this ground work is properly laid, there will be less chance that later in geometry pupils will be looking at a figure and telling the teacher that it does not need proof because they can see by the figure that it is that way.

When one views an electric light on the street through a screen, there appears with it a cross of light. Until one knows the scientific explanation of this, the situation is weird. Here is another case of where the eyes are fooled.

The next procedure is to expose a large amount of illustrative material to the class, passing around the various items; starting a paper in each row.

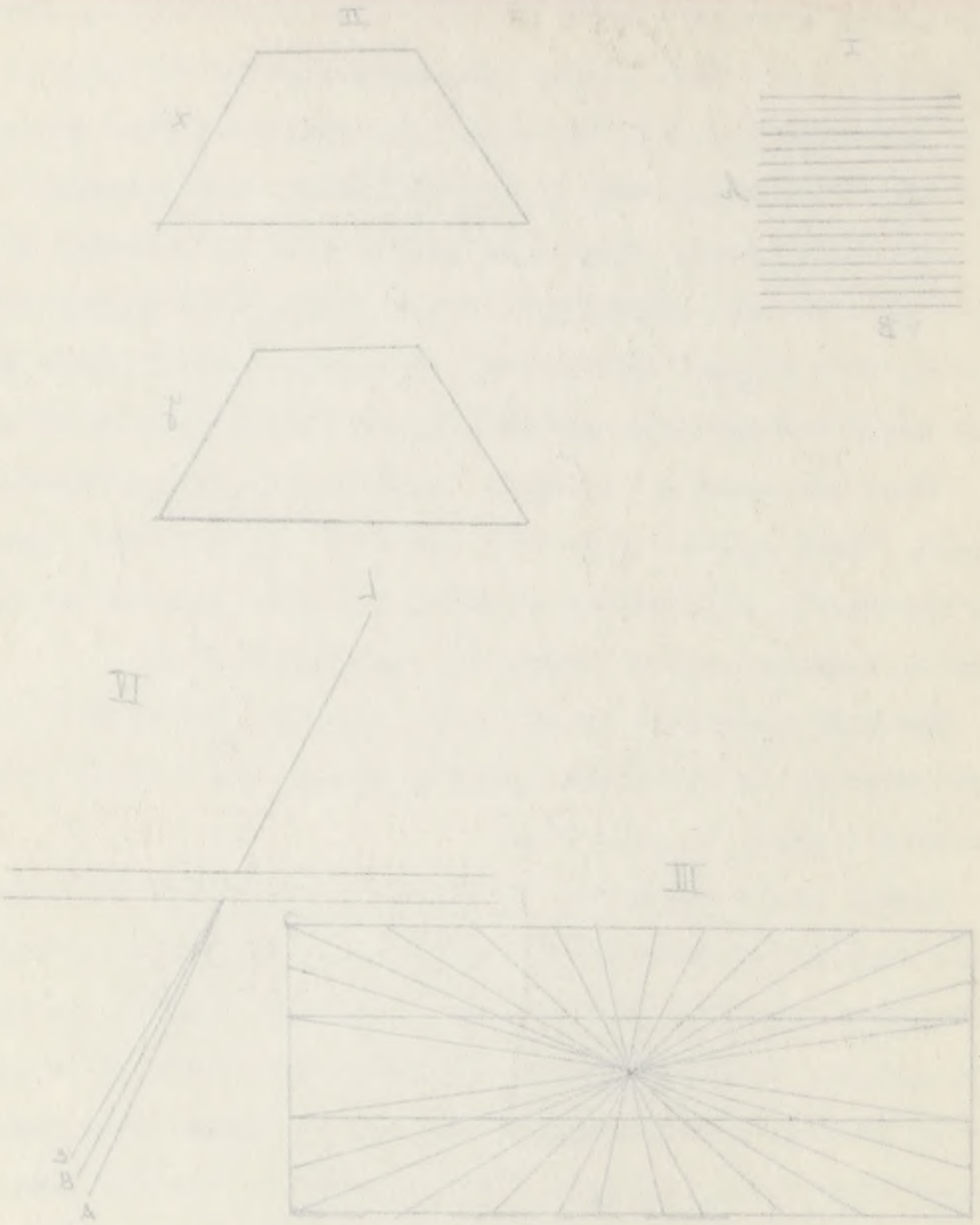
Reeve[1] calls attention to the value of optical illusions and gives four cases shown on the next page. His first illusions is based on the idea that the eye moves more easily and therefore more quickly sideways than up and down, and since the speed is slower looking up and down, to the eye the lines look higher rather than wider, although the measurements are the same in each case. It is for this reason that tall ladies do not wear dresses with lines running lengthwise; nor do stout ladies wear dresses with lines running horizontally. In each case these lines would accentuate the tallness or stoutness as the case may be.

1. W.D.Reeve. "Demonstrative Geometry for the Ninth Grade". The Mathematics Teacher. (March, 1933). p. 55.



Is b longer or shorter than h?
Which is larger x or y?
Are the parallel lines straight or curved?
Is it a, b, or c which is the prolongation of l?

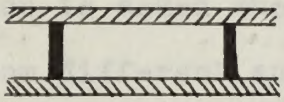
W. D. Reeve. "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher. (March 1933) p. 55.



Is b longer or shorter than b?
Which is larger x or y?
Are the parallel lines straight or curved?
Is it a, b, or c which is the prolongation of I?

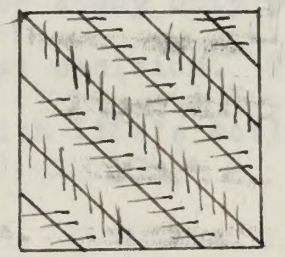
W. D. Reeve, "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher, (March 1932) p. 82.

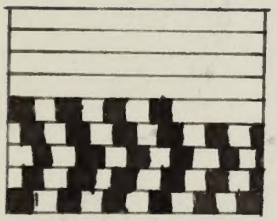
As before stated, since these are visual, curiosity arousing, and suitable for the puzzle-age of the tenth year pupil, they are excellent approaches not only to the need of proof in demonstrative geometry, but for the broader applications to situations in life. Not only will the pupil get the idea of being careful to observe the setting, but in his own work he will consider the setting as of importance.



Both borders in this sketch are parallel but they appear to diverge, due to effect on eye of small diagonal lines.

Really parallel diagonal lines appear at angles due to the cross marks.





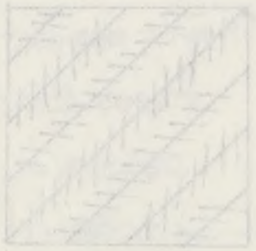
All cross lines in sketch are parallel in spite of effect of squares.

Which is the larger of the two center dots, the one below or the one above? As a matter of fact, they are exactly the same size.

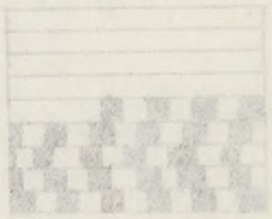
Literary Digest, April 26, 1930. "How Eyes Tell Lies" p. 28. from Popular Science, Ken. Murray.

As before stated, since these are visual, curiosity arousing, and suitable for the puzzle-age of the tenth year pupil, they are excellent approaches not only to the need of proof in demonstrative geometry, but for the broader applications to situations in life. Not only will the pupil get the idea of being careful to observe the setting, but in his own work he will consider the setting as of importance.

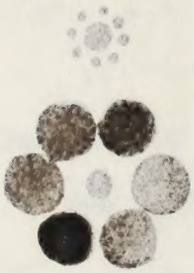
Both borders in this sketch are parallel but they appear to diverge, due to effect on eye of small diagonal lines.



Really parallel diagonal lines appear at angles due to the cross marks.



All cross lines in sketch are parallel in spite of effect of squares.

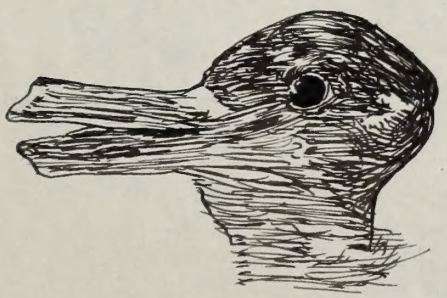


Which is the larger of the two center dots, the one below or the one above? As a matter of fact, they are exactly the same size.

Literary Digest, April 26, 1930. "How Eyes Tell Lies" p. 28.
From Popular Science. Jan. Murray.

d). Effect of looking at different parts of a picture.

Another bit of fun which may be had in driving home the idea of the need of demonstrating and not merely accepting the sensory results comes from the picture attached below. The method is to show one half of the class the picture, carefully keeping the finger over the extreme right hand side so that it will show as a duck, and then to show the other half of the class the picture, keeping the finger over the extreme left which makes the picture a rabbit. The question is asked whether it is possible out in life to see things from different angles and yet be perfectly honest. Should one be tolerant of other viewpoints then one's own? What is the best way to get at the best decision? It is seen that one cannot jump to conclusions or study only partially the situation if one is to make the wisest decision. Thus these visual presentations are linked up with life's activities.



W. B. Pillsbury, The Essentials of Psychology. p. 176

d). Effect of looking at different parts of a picture.

Another bit of fun which may be had in driving home the idea of the need of demonstrating and not merely accepting the sensory results comes from the picture attached below. The method is to show one half of the class the picture, carefully keeping the finger over the extreme right hand side so that it will show as a duck, and then to show the other half of the class the picture, keeping the finger over the extreme left which makes the picture a rabbit. The question is asked whether it is possible out in life to see things from different angles and yet be perfectly honest. Should one be tolerant of other viewpoints than one's own? What is the best way to get at the best decision? It is seen that one cannot jump to conclusions or study only partially the situation if one is to make the wisest decision. Thus these visual presentations are linked up with life's activities.

W. B. Pillsbury. The Essentials of Psychology. p. 196

e). The after image in the eye. The writer remembers an elderly relative who told of going up stairways and seeing in the bedroom, sitting in a chair, the lady she had just left downstairs. She returned downstairs, and there was the lady just as she had left her. This was a matter of much superstition but probably explainable by the after images in the eye. Thus superstitious fears are driven away by knowledge.

"The Literary Digest" quotes from "Popular Mechanics" (Chicago) Ken Murray for the following stories:[1]

> "According to ancient folklore, a peasant was once sitting by his cottage door, when he chanced to notice a white horse with rider dressed in black passing along a distant road.
>
> "The peasant watched the rider and horse for some minutes until they passed behind a small group of trees.
>
> "When they emerged into view again, he was startled to see a black horse and the rider dressed in white. To the peasant it was a spiritual token; represented the angel of death."

Mr. Murray then goes on to explain how you can take a black cross and look at it intently and then see a white cross. This is similar to the experiment with the red cross I described below. He gives a popular explanation of these effects and relates the story of the artist who had the power to keep in his mind exactly the picture, but who finally had hullucinations of people sitting in chairs when they were not there.

f). Complementary colors. The writer has for some time collected material from books on psychology and from other sources to illustrate optical illusions. Since this is visual

1. The Literary Digest, (April 26, 1930). p. 28. from Popular Science, Ken Murray.

e). The after image in the eye. The writer remembers an elderly relative who told of going up stairways and seeing in the bedroom, sitting in a chair, the lady she had just left downstairs. She returned downstairs, and there was the lady just as she had left her. This was a matter of much superstition but probably explainable by the after images in the eye. Thus superstitious fears are driven away by knowledge.

"The Literary Digest" quotes from "Popular Mechanics" (Chicago) Ken Murray for the following stories:[1]

> "According to ancient folklore, a peasant was once sitting by his cottage door, when he chanced to notice a white horse with rider dressed in black passing along a distant road.
>
> "The peasant watched the rider and horse for some minutes until they passed behind a small group of trees.
>
> "When they emerged into view again, he was startled to see a black horse and the rider dressed in white. To the peasant it was a spiritual token; represented the angel of death."

Mr. Murray then goes on to explain how you can take a black cross and look at it intently and then see a white cross. This is similar to the experiment with the red cross I described. He gives a popular explanation of these effects and relates the story of the artist who had the power to keep in his mind exactly the picture, but who finally had hallucinations of people sitting in chairs when they were not there.

f). Complementary colors. The writer has for some time collected material from books on psychology and from other sources to illustrate optical illusions. Since this is visual

1. The Literary Digest, (April 26, 1930). p. 28. from Popular Science, Ken Murray.

material and often colored, it is highly interesting to the geometry class and causes them to be quite excited. In this connection also the complementary colors are considered. The writer has a fairly large red cross which was obtained from the Red Cross. This is on a white background. The pupils are told to look steadily at this for a short time. It is then removed, and the pupils look at a blank white cardboard of fair size. Each one is then asked to write down on paper what he sees without telling his neighbor or the teacher. The papers are then collected, and,of course, the result is a green cross. It is then explained why this happens.

g). Other sensory illusions. Although this unit has focussed on optical illusions, there are illusions of all the senses. Extreme heat and extreme cold are not distinguished by the tactile sense. Dry ice against the hand has the feeling of burning. Initiation societies are fond of drawing a piece of ordinary ice over a part of the body which leaves a dribble of water and informing the blindfolded candidate that blood is pouring from a bad cut.

In the airplane, illusions play a vital part. Aviators have difficulty in judging by their own senses whether they are flying right side up or the reverse when in a cloud. Just recently an aviator flew for three hours upside down.

Another instance is the feeling of motion when one is seated at the station in a motionless train beside another train in actual motion. When the train next to you disappears from sight, the illusion is shattered.

material and often colored, it is highly interesting to the geometry class and causes them to be quite excited. In this connection also the complementary colors are considered. The writer has a fairly large red cross which was obtained from the Red Cross. This is on a white background. The pupils are told to look steadily at this for a short time. It is then removed, and the pupils look at a blank white cardboard of fair size. Each one is then asked to write down on paper what he sees without telling his neighbor or the teacher. The papers are then collected, and, of course, the result is a green cross. It is then explained why this happens.

g). Other sensory illusions. Although this unit has focussed on optical illusions, there are illusions of all the senses. Extreme heat and extreme cold are not distinguished by the tactile sense. Dry ice against the hand has the feeling of burning. Initiation societies are fond of drawing a piece of ordinary ice over a part of the body which leaves a dribble of water and informing the blindfolded candidate that blood is pouring from a bad cut.

In the airplane, illusions play a vital part. Aviators have difficulty in judging by their own senses whether they are flying right side up or the reverse when in a cloud. Just recently an aviator flew for three hours upside down.

Another instance is the feeling of motion when one is seated at the station in a motionless train beside another train in actual motion. When the train next to you disappears from sight, the illusion is shattered.

The writer was told recently of the passengers on the boat who were fearful that the boat could not pass beneath the bridge. Passengers in automobiles on the far side from the wheel misjudge the position of the auto on the road, placing it nearer the middle of the road than it really is. Everyone has heard of "backseat" driving. Much of the difficulty is caused by the misjudgment of the eye due to position.

2. Proof necessary in geometry to avoid sensory illusions.

The aim of the preceding demonstration is to show the need of careful attention to a situation before judgment is formed. It has been shown that the senses can be fooled. Aviation has told this when the flyer does not know whether he is upside down or not. When lost in the woods, one goes around in circles. One must rely on other guides than the senses.

Here the compass corrects the false impression of direction. At sea, also, this instrument is used. The aviator does blind flying by the use of instruments. The thermometor tells the correct room temperature when the senses are in error. The watch tells the time of day regardless of when darkness comes on, as in the various seasons of the year. The speedometer tells how fast one is going when one's own senses fail. This is shown in the start of an auto trip. At first one seems to be going faster than the speedometer reads, but after riding awhile, the senses record a slower apeed than that shown by the speedometer.

In life situations only the habit of careful thinking and complete thinking will rescue one from the maze of false ideas,

The writer was told recently of the passengers on the boat who were fearful that the boat could not pass beneath the bridge. Passengers in automobiles on the far side from the wheel misjudge the position of the auto on the road, placing it nearer the middle of the road than it really is. Everyone has heard of "bookeast" driving. Much of the difficulty is caused by the misjudgment of the eye due to position.

2. Proof necessary in geometry to avoid sensory illusions.

The aim of the preceding demonstration is to show the need of careful attention to a situation before judgment is formed. It has been shown that the senses can be fooled. Aviation has told this when the flyer does not know whether he is upside down or not. When lost in the woods, one goes around in circles. One must rely on other guides than the senses.

Here the compass corrects the false impression of direction. At sea, also, this instrument is used. The aviator does blind flying by the use of instruments. The thermometer tells the correct room temperature when the senses are in error. The watch tells the time of day regardless of when darkness comes on, as in the various seasons of the year. The speedometer tells how fast one is going when one's own senses fail. This is shown in the start of an auto trip. At first one seems to be going faster than the speedometer reads, but after riding awhile, the senses record a slower speed than that shown by the speedometer.

In life situations only the habit of careful thinking and complete thinking will rescue one from the maze of false ideas.

false emphases, and incomplete situations presented. In plane geometry the habit of logical demonstrations will build the habits, so that one may the more carefully think on every situation.

The visual presentations in the pages preceding have warned against taking any apparent equality or inequality for granted. The eyes may not be trusted to measure accurately, when supplementary data which distracts, is present. When one looks at a figure in plane geometry, one may not say, "Such and such is so because I can see it with my eyes."

Nevertheless, in spite of the necessity of proving things by demonstrations which depend on other aids than that of the senses, each is grateful for the ears which hear, the eyes which see, and the other sense organs which convey impression. It is only that one must not trust these alone to select the correct from the maze of incorrectness.

Demonstrations in geometry which are about to be considered will help to judge accurately and completely, and not let the senses be fooled by illusions or the reasoning be inaccurate because of incomplete information.

false emphases, and incomplete situations presented. In plane geometry the habit of logical demonstrations will build the habits, so that one may the more carefully think on every situation.

The visual presentations in the pages preceding have warned against taking any apparent equality or inequality for granted. The eyes may not be trusted to measure accurately, when supplementary data which distracts, is present. When one looks at a figure in plane geometry, one may not say, "Such and such is so because I can see it with my eyes."

Nevertheless, in spite of the necessity of proving things by demonstrations which depend on other aids than that of the senses, each is grateful for the sens which hear, the eyes which see, and the other sense organs which convey impression. It is only that one must not trust these alone to select the correct from the maze of incorrectness.

Demonstrations in geometry which are about to be considered will help to judge accurately and completely, and not let the senses be fooled by illusions or the reasoning be inaccurate because of incomplete information.

E. How to Distinguish the Hypothesis from the Conclusion in Plane Geometry.

With the pupil having the memory of the unit on illusions lingering in his consciousness, he is now in a receptive mood for the consideration of the problem of the demonstration in geometry. Thomas,[1] Parker,[2] and Wilson-Kyte-Lull[3] all have valuable hints to offer on the problem situation. One is not concerned in the thesis, however, with the solution of problems in general, so will proceed to a particular situation.

The first step, after making the problem a real and interesting task to the learner, is to segregate the known and the unknown. It is this second step that is being considered here, the segregating of the hypothesis and conclusion. The writer proceeds somewhat as follows:

1. Difficulty from the declarative sentence.

Since the declarative sentence is used more largely now in the plane geometry theorems, and since pupils during their previous school years have accepted declarative sentences in texts as true, there is not only the difficulty of an unknown field to be overcome, but this grammatical difficulty. In other words, there is a real difficulty in making the pupil see that there is a problem to be solved and another difficulty in

1. Frank W. Thomas, Principles and Techniques of Teaching, pp. 227-240.

2. Samuel C. Parker, Methods of Teaching in High Schools. pp. 169-229.

3. H. B. Wilson, G. C. Kyte, H. G. Lull, Modern Methods in Teaching, pp. 92-110.

3. How to Distinguish the Hypothesis from the Conclusion in Plane Geometry.

With the pupil having the memory of the unit on Illusions lingering in his consciousness, he is now in a receptive mood for the consideration of the problem of the demonstration in geometry. Thomas,[1] Parker,[2] and Wilson-Kyte-Luli[3] all have valuable hints to offer on the problem situation. One is not concerned in the thesis, however, with the solution of problems in general, so will proceed to a particular situation.

The first step, after making the problem a real and interesting task to the learner, is to segregate the known and the unknown. It is this second step that is being considered here, the segregating of the hypothesis and conclusion. The writer proceeds somewhat as follows:

1. Difficulty from the declarative sentence.

Since the declarative sentence is used more largely now in the plane geometry theorems, and since pupils during their previous school years have accepted declarative sentences in texts as true, there is not only the difficulty of an unknown field to be overcome, but this grammatical difficulty. In other words, there is a real difficulty in making the pupil see that there is a problem to be solved and another difficulty in

1. Frank W. Thomas, Principles and Techniques of Teaching, pp. 227-240.

2. Samuel C. Parker, Methods of Teaching in High Schools. pp. 169-229.

3. H. E. Wilson, G. C. Kyte, H. G. Luli, Modern Methods in Teaching, pp. 92-110.

interesting him in the solution. The latter difficulty is largely overcome by the lesson on optical illusions which has preceded this; reference is made to the pupils to that lesson.

To create for the pupil a realness to the problem and produce a felt need, the problem at hand is related to the pupil's experience.

2. The hypothesis and conclusion related to the activity of the pupil.

To form a link with the pupil's own experience, the teacher should make the initial contact by asking what is necessary to take in going fishing. This brings out the fact that certain materials are needed, such as fish hooks, line, stopper, pole, and bait. These are all for a certain purpose, to catch fish. For the girls, the picnic idea is developed. You have to start with something, take account of stock, collect it, know how to use it, recognize it, keep it in good order if you are to reach the goal of catching fish or having everything on hand at the picnic.

3. The relation to plane geometry.

It is the same way with geometry. You have to collect your material, which is the hypothesis, in order to achieve your purpose, which is the conclusion. You have to know the material well enough to recognize it; have to put it in order; keep it bright and shiny by frequent use; pay attention to it. If you are friendly with it, it will be friendly to you. When you pick up such a tool as "Mr. Transversal with his friends, the two parallel lines", if you have smiled on him in the past and given

interesting him in the solution. The latter difficulty is largely overcome by the lesson on optical illusions which has preceded this; reference is made to the pupils to that lesson.

To create for the pupil a realness to the problem and produce a felt need, the problem at hand is related to the pupil's experience.

2. The hypothesis and conclusion related to the activity of the pupil.

To form a link with the pupil's own experience, the teacher should make the initial contact by asking what is necessary to take in going fishing. This brings out the fact that certain materials are needed, such as fish hooks, line, stopper, pole, and bait. These are all for a certain purpose, to catch fish. For the girls, the picnic idea is developed. You have to start with something, take account of stock, collect it, know how to use it, recognize it, keep it in good order if you are to reach the goal of catching fish or having everything on hand at the picnic.

3. The relation to plane geometry.

It is the same way with geometry. You have to collect your material, which is the hypothesis, in order to achieve your purpose, which is the conclusion. You have to know the material well enough to recognize it; have to put it in order; keep it bright and shiny by frequent use; pay attention to it. If you are friendly with it, it will be friendly to you. When you pick up such a tool as "Mr. Transversal with his friends, the two parallel lines", if you have smiled on him in the past and given

him a hearty handshake, he will gladly give one the contents of his pocket book--namely, corresponding angles are equal. Reference is made to geometry facts as tools because so many boys take manual training where the tools have to be in order and are kept sharp and clean.

The above procedure aims to make the pupil recognize the difficulty at hand and experience a definite need for guidance in order to proceed toward the solution. The problem has now become a problem to all. The teacher's place here was to relate the abstract to the concrete within the pupil's capacity and within his realm of experience. The discussion has been a group discussion rather than a teacher dissertation. The teacher has contrived the situation.

It is well at this point to call attention to the fact that one has been accepting declarative statements in text books without question, and that, therefore, it is harder to realize that the declarative statements in geometry books when given as theorems are for one to prove. It is noted that this difficulty is somewhat enhanced by the removal of the "if" clause from the newer texts. It is difficult to wedge the pupil away from the acceptance of the declarative sentence because of the long years during which he has accepted it.

4. <u>The hypothesis as the subject of the sentence: the conclusion as the predicate.</u>

Several theorems are now examined, and the line-up of hypothesis and conclusion made by the text is noted. The theorem is examined to see what words gave the hypothesis and

him a hearty handshake, he will gladly give him the contents of his pocket book--namely, corresponding angles are equal.

Reference is made to geometry facts as tools because so many boys take manual training where the tools have to be in order and are kept sharp and clean.

The above procedure aims to make the pupil recognize the difficulty at hand and experience a definite need for guidance in order to proceed toward the solution. The problem has now become a problem to all. The teacher's place here was to relate the abstract to the concrete within the pupil's capacity and within his realm of experience. The discussion has been a group discussion rather than a teacher dissertation. The teacher has contrived the situation.

It is well at this point to call attention to the fact that one has been accepting declarative statements in text books without question, and that, therefore, it is harder to realize that the declarative statements in geometry books when given as theorems are for one to prove. It is noted that this difficulty is somewhat enhanced by the removal of the "if" clause from the newer texts. It is difficult to wedge the pupil away from the acceptance of the declarative sentence because of the long years during which he has accepted it.

4. <u>The hypothesis as the subject of the sentence;</u>

<u>the conclusion as the predicate.</u>

Several theorems are now examined, and the line-up of hypothesis and conclusion made by the text is noted. The theorem is examined to see what words gave the hypothesis and

what words gave the conclusion. A vertical line is then drawn after the last word in the hypothesis in the theorem writing on the board; this process is repeated for several theorems. A hint may here be given by asking the class if this were a lesson in grammar, what part of the sentence would this be, pointing to the first part, and then to the second part. Some of the group will see that the first part is the subject of the sentence, and the second part is the predicate. To verify whether this just happened, a number of theorems from the text book are now examined, and it is found that the subject contains the hypothesis, what is given, while the predicate contains the conclusion, what is to be proved.

To relate and give meaning to the word "hypothesis", contact is now made with the previous experiences in the recognition of the problem by means of drawings on the board of the materials for the fishing situation which one labels "Hypothesis", while at the end of the line the fish is labeled "Conclusion". It is here noted that if the hypothesis, the line, bait, etc. were not present, there could be no fish or no "conclusion". The same procedure is written up for the girls by presenting a girl in the picnic situation.

The parallel procedure is continued for the assembling of the materials, putting proper value on them, putting them in proper place, on the one hand, the fishing procedure and, on the other hand, the facts in the hypothesis. This may be made graphic by means of blocks labeled in the proper manner, each block representing a fact. The visual work may be done by a

what words gave the conclusion. A vertical line is then drawn after the last word in the hypothesis in the theorem writing on the board; this process is repeated for several theorems. A hint may here be given by asking the class if this were a lesson in grammar, what part of the sentence would this be, pointing to the first part, and then to the second part. Some of the group will see that the first part is the subject of the sentence, and the second part is the predicate. To verify whether this just happened, a number of theorems from the text book are now examined, and it is found that the subject contains the hypothesis, what is given, while the predicate contains the conclusion, what is to be proved.

To relate and give meaning to the word "hypothesis", contact is now made with the previous experiences in the recognition of the problem by means of drawings on the board of the materials for the fishing situation which one labels "Hypothesis", while at the end of the line the fish is labeled "Conclusion". It is here noted that if the hypothesis, the line, bait, etc. were not present, there could be no fish or no "conclusion". The same procedure is written up for the girls by presenting a girl in the picnic situation.

The parallel procedure is continued for the assembling of the materials, putting proper value on them, putting them in proper place, on the one hand, the fishing procedure and, on the other hand, the facts in the hypothesis. This may be made graphic by means of blocks labeled in the proper manner, each block representing a fact. The visual work may be done by a

group of pupils interested in drawing and may be in colors. This cares for individual differences, and besides presenting the matter more vividly to the group, gets the art pupil friendly and interested in the work.

5. Meaning of the word "hypothesis."

Finally, the meaning of "hypothesis" may be explained. The prefix "hypo" means under. "Hypo-dermic", under the skin; "hyp-notic", under the influence of another; "hypo-thesis", under the writing. That is, it is the platform on which you build,--it is under the building.

6. Drill

Verification of the determining of the hypothesis and conclusion will consist of actual practise in picking these out. The following are illustrative of possible methods of procedure:

a. Play devices by means of oak tag, on which may be printed some hypotheses and some conclusions. These are identified by the class while being presented by a member of the class.

b. Another method is to have pupils write theorems on the board and have the group decide the hypothesis and the conclusion.

The aim is to make the procedure somewhat automatic.

7. Test.

Application may consist of a test somewhat as follows:

a. Why is it difficult to pick out the hypothesis and conclusion from a declarative sentence?

B. Does the knowledge of grammar help you? Explain.

group of pupils interested in drawing and may be in colors. This cares for individual differences, and besides presenting the matter more vividly to the group, gets the art pupil friendly and interested in the work.

5. Meaning of the word "hypothesis."

Finally, the meaning of "hypothesis" may be explained. The prefix "hypo" means under. "Hypo-dermic", under the skin; "hypo-notic", under the influence of another; "hypo-thesis", under the writing. That is, it is the platform on which you build,--it is under the building.

6. Drill

Verification of the determining of the hypothesis and conclusion will consist of actual practice in picking these out. The following are illustrative of possible methods of procedure:

a. Play devices by means of oak tag, on which may be printed some hypotheses and some conclusions. These are identified by the class while being presented by a member of the class.

b. Another method is to have pupils write theorems on the board and have the group decide the hypothesis and the conclusion.

The aim is to make the procedure somewhat automatic.

7. Test.

Application may consist of a test somewhat as follows:

A. Why is it difficult to pick out the hypothesis and conclusion from a declarative sentence?

B. Does the knowledge of grammar help you? Explain.

c. What is the general meaning of the prefix "hypo"? What is the meaning of "thesis"?

d. Give an experience you have had which illustrates the necessity of having the hypothesis and conclusion clearly in mind.

e. In the following statements indicate by a vertical line the termination of the hypothesis.

(1). The base angles of an isosceles triangle are equal.

(2) The opposite sides of a parallelogram are equal.

(3). Equiangular triangles are similar.

(4). The sum of the interior angles of a polygon is equal to (n-2) straight angles.

(5). The line joining the mid-points of two sides of a triangle is parallel to the third side and equal to one-half the third side.

8. Colored chalk and special marking for the hypothesis.

Schultz[1] says:

"To teach students to think, we should in the early stages remove as far as possible all external difficulties. Students who can reason logically sometimes forget the hypothesis, or forget preceding parts of the proof, and hence are unable to continue. To make such forgetfulness almost impossible graphic methods may be employed. The hypothesis may be indicated by colors, equal colors representing equal lines or equal angles, arrows denoting parallel lines, a small colored square indicating a right angle.

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools, pp. 110-112.

c. What is the general meaning of the prefix "hypo"? What is the meaning of "thesis"?

d. Give an experience you have had which illustrates the necessity of having the hypothesis and conclusion clearly in mind.

e. In the following statements indicate by a vertical line the termination of the hypothesis.

(1). The base angles of an isosceles triangle are equal.

(2) The opposite sides of a parallelogram are equal.

(3). Equiangular triangles are similar.

(4). The sum of the interior angles of a polygon is equal to (n-2) straight angles.

(5). The line joining the mid-points of two sides of a triangle is parallel to the third side and equal to one half the third side.

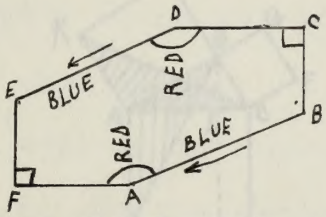
8. <u>Colored chalk and special marking for the hypothesis.</u>

Schultz[1] says:

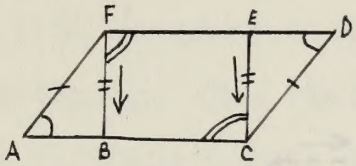
"To teach students to think, we should in the early stages remove as far as possible all external difficulties. Students who can reason logically sometimes forget the hypothesis, or forget preceding parts of the proof, and hence are unable to continue. To make such forgetfulness almost impossible graphic methods may be employed. The hypothesis may be indicated by colors, equal colors representing equal lines or equal angles, arrows denoting parallel lines, a small colored square indicating a right angle.

1. Arthur Schultz, <u>The Teaching of Mathematics in Secondary Schools</u>, pp. 110-112.

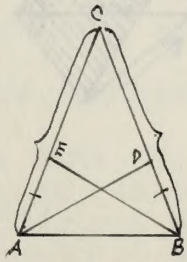
"Thus, in the annexed diagram the two blue lines indicate the equality of AB and DE, the two red arcs indicate the equality of angles A and D, the two colored arrows represent the parallelism of AB and DE, and the two small squares show that angles F and C are right angles.



"For the results obtained in the proof we use white crayon, equality of lines being indicated by equal cross marks, equality of angles by equal number of arcs, parallelism by arrows, etc. Thus the marks in the annexed diagram, which are supposed to be drawn in white crayon, indicate that we have proved: AF=CD, BF=CE, ∠A=∠D, ∠BFE=∠ECB, BF∥CE.



"If lines overlap, we use braces. Thus the marks of the annexed diagram represent the equality of AC and BC, and the equality of AE and BD.



"If colored crayons are not available, white cross marks,

Arthur Schultze, The Teaching of Mathematics in Secondary Schools. pp. 110-112.

"Thus, in the annexed diagram the two blue lines indicate the equality of AB and DE, the two red arcs indicate the equality of angles A and D, the two colored arrows represent the parallelism of AB and DE, and the two small squares show that angles F and C are right angles.

"For the results obtained in the proof we use white crayon, equality of lines being indicated by equal cross marks, equality of angles by equal number of arcs, parallelism by arrows, etc. Thus the marks in the annexed diagram, which are supposed to be drawn in white crayon, indicate that we have proved: AF=CD, BF= CE, ∠A= ∠D, ∠BFE= ∠ECB, BF=CE.

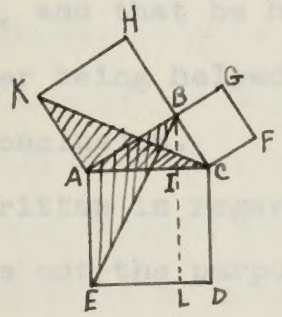
"If lines overlap, we use braces. Thus the marks of the annexed diagram represent the equality of AC and BC, and the equality of AE and BD.

"If colored crayons are not available, white cross marks,

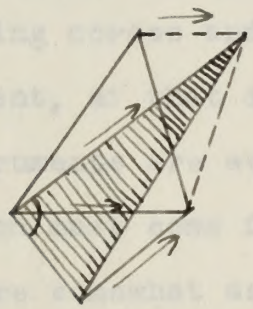
Arthur Schultze, The Teaching of Mathematics in Secondary Schools. pp. 110-112.

etc., may also be used to indicate the hypothesis, but for the beginner a distinction between hypothesis and proof is very helpful.

"To point out certain triangles or polygons to the students, either shade them or mark their perimeters by heavy lines, as △AEB and KAC in the annexed diagram. Thus, if we transform one figure into another, it is advisable to mark the perimeter of the given area heavy, and to shade the resulting area, as indicated in the annexed diagrams.



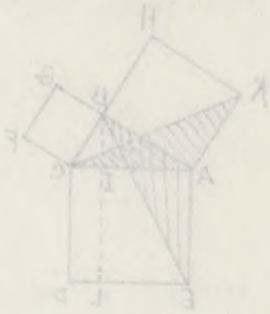
"In other constructions given lines may be drawn thin, lines of construction dotted, and resulting lines heavy. In complex constructions various colors may be used to distinguish among different lines."



Arthur Schultze, loc. cit.

etc., may also be used to indicate the hypothesis, but for the beginner a distinction between hypothesis and proof is very helpful.

"To point out certain triangles or polygons to the students, either shade them or mark their perimeters by heavy lines, as △AEB and KAC in the annexed diagram. Thus, if we transform one figure into another, it is advisable to mark the perimeter of the given area heavy, and to shade the resulting area, as indicated in the annexed diagrams.



"In other constructions given lines may be drawn thin, lines of construction dotted, and resulting lines heavy. In complex constructions various colors may be used to distinguish among different lines."

Arthur Schultze, loc. cit.

Part III.

PLANE GEOMETRY UNITS

A. The Teaching of Congruency in Plane Geometry.

With the stage prepared for logical demonstration by the various introductory units as previously written, one may assume that the pupil is in a favorable attitude for the actual beginnings of proofs, and that he has the ability to comprehend such work, this latter being helped by the unit on selecting the hypothesis and conclusion.

Much has been written in regard to the propositions on congruence, but it is not the purpose of this thesis to evaluate the various ideas, although at the end of the unit there have been placed various ideas from the writers.

1. The applied problem. How can you find the distance across the river? The current is such that you cannot estimate the distance from time consumed by a boat. The same objection is applicable to swimming. The river is too wide to permit any rope that you have reaching across and making direct measurement. No bridge is present, so that direct measure cannot be made. No surveying instruments are available which might give it directly. The solution must come from congruent triangles. The method and drawing are somewhat as follows:[1]

> "One boy drove a stake at B. John then placed a carpenter's square on stake B, in such a position that as he looked along one edge he could see a knot-hole in the trunk of tree A, on the opposite

1. Elizabeth B. Cowley, Plane Geometry, p. 61.

Part III.

PLANE GEOMETRY UNITS

A. The Teaching of Congruency in Plane Geometry.

With the stage prepared for logical demonstration by the various introductory units as previously written, one may assume that the pupil is in a favorable attitude for the actual beginnings of proofs, and that he has the ability to comprehend such work, this latter being helped by the unit on selecting the hypothesis and conclusion.

Much has been written in regard to the propositions on congruence, but it is not the purpose of this thesis to evaluate the various ideas, although at the end of the unit there have been placed various ideas from the writers.

1. The applied problem. How can you find the distance across the river? The current is such that you cannot estimate the distance from time consumed by a boat. The same objection is applicable to swimming. The river is too wide to permit any rope that you have reaching across and making direct measurement. No bridge is present, so that direct measure cannot be made. No surveying instruments are available which might give it directly. The solution must come from congruent triangles. The method and drawing are somewhat as follows:[1]

> "One boy drove a stake at B. John then placed a carpenter's square on stake B, in such a position that as he looked along one edge he could see a knot-hole in the trunk of tree A, on the opposite

---

1. Elizabeth B. Cowley, Plane Geometry, p. 81.

bank. By looking along the other edge, he was able to direct the boys in locating stakes C and D, so that points B, C, and D were in a straight line, and distances DC and BC were equal.

"Then they placed the square on a stake at D. They located E so that ∠CDE was a right angle and points A, C, and E were in a straight line. They measured the distance DE and concluded that the distance AB was equal to it. Was that conclusion correct? Why?"[1]

It will be remembered that in our introduction we could not trust our eyesight but must rely on proof. Therefore, we will have to devise some means of proving this proposition. The distances AB and DE would be equal if the triangles were congruent. They have a right angle and a vertical angle in each triangle and a side constructed equal. Do two angles and a side equal in each triangle make the two triangles congruent? It is noted that if we have two angles equal in each triangle, we always have the third because by paper cutting we can prove that the three angles make 180 degrees, so subtracting the sum of the two from 180 in each triangle, the third angle must be the same. So if we need an included angle, we have it.

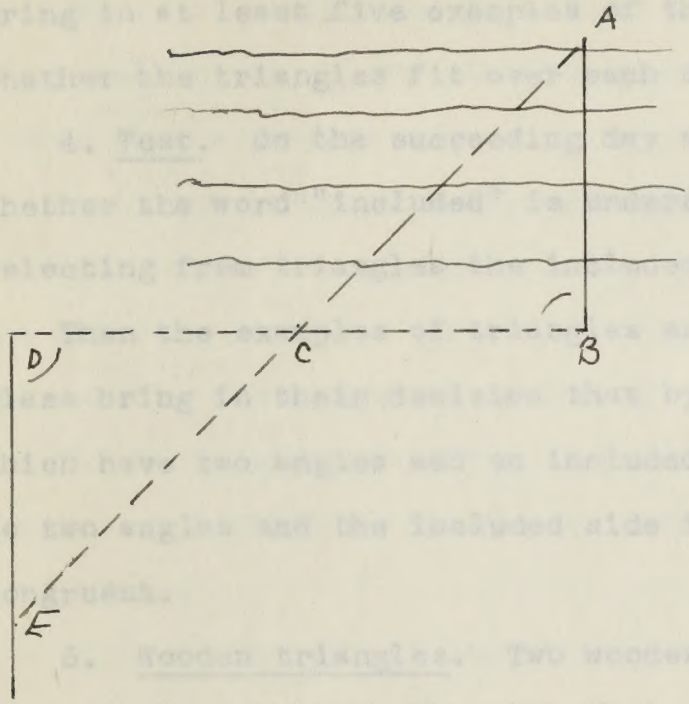
1. Elizabeth B. Cowley, loc. cit.

bank. By looking along the other edge, he was able to direct the boys in locating stakes C and D, so that points B, C, and D were in a straight line, and distances DC and BC were equal.

"Then they placed the square on a stake at D. They located E so that ∠CDE was a right angle and points A, C, and E were in a straight line. They measured the distance DE and concluded that the distance AB was equal to it. Was that conclusion correct? Why?"[1]

It will be remembered that in our introduction we could not trust our eyesight but must rely on proof. Therefore, we will have to devise some means of proving this proposition. The distances AB and DE would be equal if the triangles were congruent. They have a right angle and a vertical angle in each triangle and a side constructed equal. Do two angles and a side equal in each triangle make the two triangles congruent? It is noted that if we have two angles equal in each triangle, we always have the third because by paper cutting we can prove that the three angles make 180 degrees, so subtracting the sum of the two from 180 in each triangle, the third angle must be the same. So if we need an included angle, we have it.

1. Elizabeth B. Cowley, loc. cit.



This may be an actual trip in the vicinity of the school.

DC=CB

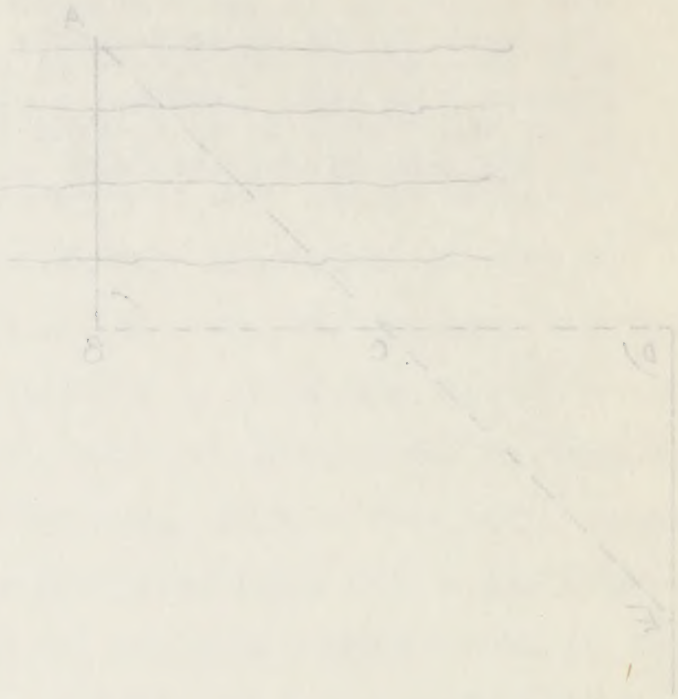
The challenge is to prove that this conclusion is correct why?

This creates the natural setting of a problem suggested by the teacher, but which the class has accepted. From this point on the task will be to prove that triangles are congruent, providing they have the equalities which exist in the case of these two triangles.

2. Drill on "included" angle. A short drill from figures on the blackboard may now be conducted in connection with the word "included", which is a somewhat bothersome word whether used in connection with the word "angle" or the word "side".

3. Assignment.

The assignment will consist of making triangles from paper, cardboard, or wood, which will fulfill the condition of two angles and the included side being equal in each triangle.



This may be an actual trip in the vicinity of the school.

DC=CB

The challenge is to prove that this conclusion is correct why?

This creates the natural setting of a problem suggested by the teacher, but which the class has accepted. From this point on the task will be to prove that triangles are congruent, providing they have the equalities which exist in the case of these two triangles.

2. Drill on "included" angle. A short drill from figures on the blackboard may now be conducted in connection with the word "included", which is a somewhat bothersome word whether used in connection with the word "angle" or the word "side".

3. Assignment.

The assignment will consist of making triangles from paper, cardboard, or wood, which will fulfill the condition of two angles and the included side being equal in each triangle.

In each case the triangle is placed over the second triangle. Bring in at least five examples of these and your report on whether the triangles fit over each other.

4. Test. On the succeeding day a short test is given on whether the word "included" is understood. This is by means of selecting from triangles the included angle or included side.

Then the examples of triangles are exhibited, and the class bring in their decision that by measurement triangles which have two angles and an included side equal respectively to two angles and the included side in the other triangle are congruent.

5. Wooden triangles. Two wooden models of triangles are exhibited in class, and the triangles are superimposed. Drawings are made on the board which show that, given the same slope, the sides of the hill must meet at the same peak in each triangle.

6. Two lines intersecting in one point. A little bit of humor is introduced by having two bugs crawl up the hill.

In the case of the writer, since most of the children have been to the lookout station and seen the maps, this becomes a vital method of approach to the fact that two lines can cross in only one point. The diagram is on page 43 .

Figures are drawn on the board, one with yellow lines for sides and one white. The yellow figure is then pictured as being lifted and placed over the white figure, and the yellow line is drawn over the white to strengthen the impression of congruity.

In each case the triangle is placed over the second triangle. Bring in at least five examples of these and your report on whether the triangles fit over each other.

4. Test. On the succeeding day a short test is given on whether the word "included" is understood. This is by means of selecting from triangles the included angle or included side.

Then the examples of triangles are exhibited, and the class bring in their decision that by measurement triangles which have two angles and an included side equal respectively to two angles and the included side in the other triangle are congruent.

5. Wooden triangles. Two wooden models of triangles are exhibited in class, and the triangles are superimposed. Drawings are made on the board which show that, given the same slope, the sides of the hill must meet at the same point in each triangle.

6. Two lines intersecting in one point. A little bit of humor is introduced by having two bugs crawl up the hill.

In the case of the writer, since most of the children have been to the lookout station and seen the maps, this becomes a vital method of approach to the fact that two lines can cross in only one point. The diagram is on page 43.

Figures are drawn on the board, one with yellow lines for sides and one white. The yellow figure is then pictured as being lifted and placed over the white figure, and the yellow line is drawn over the white to strengthen the impression of congruity.

7. Congruence a special case of similarity. Later it will be noted that congruence is a special case of similarity; that congruent triangles are similar triangles but a particular rather than a general case.

It is expressed, but not emphasized, that in the beginning theorems have to use congruence for proof. Thus, in areas where the start is from the rectangle which assumes that the area is the base by the height by method of dividing into units, we assume the congruence of the identical units. And again in the section on circles we start with angles equal by the method of congruence.

Drill on the actual superposition or the technical points should not be permitted. The chief aim is to link congruence up with life situations to enable the pupil to get a proper concept.

8. Individual assignments. The next assignment will be varied and individual. To a group of pupils interested in graph work and algebraic work Swenson's[1] development will be given. It will be their task to work out this idea, so that one of their members may give a short talk on it, and another member be responsible for putting the graph on the board, ready for the next day. Too much class time may not be taken with this. The aim will be to present an audience situation for this smaller group and to acquaint the others in the class with the existence of such work, rather than a mastery of it. The group, however,

---

[1] John A. Swenson, Graphic Methods of Teaching Congruence in Geometry, 1930, pp. 96-97, 99-100. The National Council of Teachers of Mathematics, The Fifth Yearbook.

7. Congruence a special case of similarity. Later it will be noted that congruence is a special case of similarity; that congruent triangles are similar triangles but a particular rather than a general case.

It is expressed, but not emphasized, that in the beginning theorems have to use congruence for proof. Thus, in areas where the start is from the rectangle which assumes that the area is the base by the height by method of dividing into units, we assume the congruence of the identical units. And again in the section on circles we start with angles equal by the method of congruence.

Drill on the actual superposition or the technical points should not be permitted. The chief aim is to link congruence up with life situations to enable the pupil to get a proper concept.

8. Individual assignments. The next assignment will be varied and individual. To a group of pupils interested in graph work and algebraic work Swenson's[1] development will be given. It will be their task to work out this idea, so that one of their members may give a short talk on it, and another member be responsible for putting the graph on the board, ready for the next day. Too much class time may not be taken with this. The aim will be to present an audience situation for this smaller group and to acquaint the others in the class with the existence of such work, rather than a mastery of it. The group, however,

---

[1] John A. Swenson, Graphic Methods of Teaching Congruence in Geometry, 1930, pp. 96-97, 99-100. The National Council of Teachers of Mathematics. The Ninth Yearbook.

which is studying it will be held to the mastery of the idea through individual written lessons. A second group interested in manual training will be asked to make wooden models for congruence. Those interested in art will be asked to draw congruent designs of their own choosing. Still others in the class will contribute by bringing in congruent things such as milk bottles, lasts, etc. The literary group will be asked to study the matter of congruence as it refers to life and write compositions on what phases of life it affects.

The succeeding assignments on this topic will involve pupil activity as above suggested with the teacher in the background as a rather silent chairman.

The final written lesson on this topic will involve questions similar to the following:

1. Of what use is the knowledge of how to get congruency? Give a practical use in life.
2. Why do we not accept measurement as a proof of congruency?
3. How do you prove congruency?
4. Give four examples of congruency in life.
5. Would machines be useful if we were not interested in congruent things in life?

9. Use of congruence. The powerfulness of the propositions on congruence should be called to the attention of the pupils by showing that if figures are congruent, the corresponding parts are equal. Drill should be on this. Thus one has but to get one side in one triangle to get the corresponding side in the other as in the stream problem and the same thing is true of

which is studying it will be held to the mastery of the idea through individual written lessons. A second group interested in manual training will be asked to make wooden models for congruence. Those interested in art will be asked to draw congruent designs of their own choosing. Still others in the class will contribute by bringing in congruent things such as milk bottles, laets, etc. The literary group will be asked to study the matter of congruence as it refers to life and write compositions on what phases of life it affects.

The succeeding assignments on this topic will involve pupil activity as above suggested with the teacher in the background as a rather silent chairman.

The final written lesson on this topic will involve questions similar to the following:

1. Of what use is the knowledge of how to get congruency? Give a practical use in life.

2. Why do we not accept measurement as a proof of congruency?

3. How do you prove congruency?

4. Give four examples of congruency in life.

5. Would machines be useful if we were not interested in congruent things in life?

9. <u>Use of congruence</u>. The powerfulness of the propositions on congruence should be called to the attention of the pupils by showing that if figures are congruent, the corresponding parts are equal. Drill should be on this. Thus one has but to get one side in one triangle to get the corresponding side in the other as in the stream problem and the same thing is true of

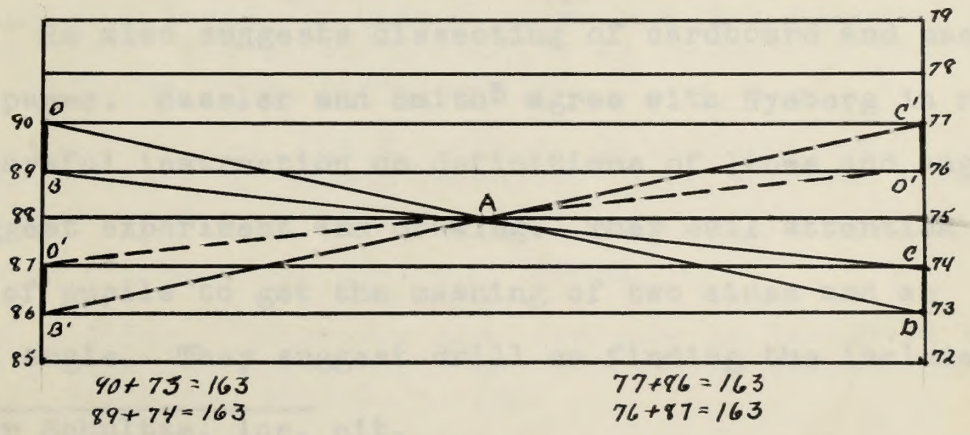
the corresponding angles.

Schlauch[1] gives an interesting application of the congruence. Some pupil may be made to achieve better his ego by being given this matter, so that before the class he may place the chart on the board and have the matter well enough in mind to given an application to the class.

In the accompanying chart it will be observed that as triangles OAB and DAC swing up and down through A, so that the end lines are always one space wide, that the sum of the numbers at the two extremeties of each line is the same, and that the triangles are congruent. Thus we have automatic addition.

The same author writes of a chart for multiplication and for extra hours involved.[2]

10. Application to wage chart. The chief purpose here is to show the pupils that arithmetic links with geometry, so that processes become automatic readings for results.



1. W. S. Schlauch, "Mathematics as an Interpreter of Life". The National Council of Teachers of Mathematics, The Third Yearbook. 1928, p. 28.

2. W. S. Schlauch, loc. cit.

the corresponding angles.

Schlauch[1] gives an interesting application of the congruence. Some pupil may be made to achieve better his ego by being given this matter, so that before the class he may place the chart on the board and have the matter well enough in mind to give an application to the class.

In the accompanying chart it will be observed that as triangles OAB and DAC swing up and down through A, so that the end lines are always one space wide, that the sum of the numbers at the two extremeties of each line is the same, and that the triangles are congruent. Thus we have automatic addition.

The same author writes of a chart for multiplication and for extra hours involved.[2]

10. Application to wage chart. The chief purpose here is to show the pupils that arithmetic lines with geometry, so that processes become automatic readings for results.

1. W. S. Schlauch, "Mathematics as an Interpreter of Life". The National Council of Teachers of Mathematics, The Third Year-book. 1928, p. 28.

2. W. S. Schlauch, loc. cit.

The pupil will be told to prove the two triangles involved as congruent, a process which is done by means of two angles and a side, the side being the distance between the division lines, and the angles being made equal by the parallel lines and also the vertical angles.

Schultze[1] urges the concept of equality of triangles by means of concrete illustrations. He would avoid frequent repetition of proof. Rather he would pass over the topic hastily. He suggests one polygon in color, so that when the figure is superimposed, the color may show up over the white. Nyeberg[2] feels careful instructional work should be done along the line of getting clear in the pupil's mind the idea of equality of angles and lines.

Reeve[3] suggests tracing on paper, cutting and placing over. Young[4] advises the use of models which the pupil would make for himself. He also suggests dissecting of cardboard and use of tracing paper. Hassler and Smith[5] agree with Nyeberg in regard to the careful instruction on definitions of lines and angles. They suggest experiment and drawing. They call attention to the failure of pupils to get the meaning of two sides and an included angle. They suggest drill on finding the included

1. Arthur Schultze, loc. cit.
2. Joseph Nyeberg, The Mathematics Teacher, XXIV (October, 1931) pp. 395-399.
3. William D. Reeve, "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher, XXVI, (March, 1933). p. 158.
4. J.W.A.Young, The Teaching of Mathematics in the Elementary and Secondary School, p. 263, 284.
5. Hassler and Smith, The Teaching of Secondary Mathematics. pp. 317, 331-332.

The pupil will be told to prove the two triangles involved as congruent, a process which is done by means of two angles and a side, the side being the distance between the division lines, and the angles being made equal by the parallel lines and also the vertical angles.

Schultze[1] urges the concept of equality of triangles by means of concrete illustrations. He would avoid frequent repetition of proof. Rather he would pass over the topic hastily. He suggests one polygon in color, so that when the figure is superimposed, the color may show up over the white. Myeberg[2] feels careful instructional work should be done along the line of getting clear in the pupil's mind the idea of equality of angles and lines.

Reeve[3] suggests tracing on paper, cutting and placing over. Young[4] advises the use of models which the pupil would make for himself. He also suggests dissecting of cardboard and use of tracing paper. Hassler and Smith[5] agree with Myeberg in regard to the careful instruction on definitions of lines and angles. They suggest experiment and drawing. They call attention to the failure of pupils to get the meaning of two sides and an included angle. They suggest drill on finding the included

1. Arthur Schultze, loc. cit.
2. Joseph Myeberg, The Mathematics Teacher, XXIV (October, 1931) pp. 395-399.
3. William D. Reeve, "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher, XXVI, (March, 1933), p. 158.
4. J.W.A. Young, The Teaching of Mathematics in the Elementary and Secondary School, p. 203, 204.
5. Hassler and Smith, The Teaching of Secondary Mathematics, pp. 317, 321-322.

angle and adjacent angles, etc. Orleans[1] suggests cutting out paper or cardboard. Miss Allen[2] has the children bring in congruent things from the world, a wide assortment, such as quart milk bottles, linoleum patterns, etc. Swenson[3] uses a graphic method of showing how the angle and side vary when the conditions are not for rigidity; he uses the idea of showing what takes place when the conditions for rigidity are not given. His graphic method is of value because it links up with algebra.

Beatley[4] would leave "equal" undefined. Betz[5] urges the use of very simple original exercises. He agrees with Miss Allen in the use of bringing in figures from the world to show congruence; they are the products of our machine age. Reprints, lasts for shoes, castings, all involve congruence.

---

1. Joseph B. Orleans, "A Unit of Demonstrative Geometry for the Ninth Year", The National Council of Teachers of Mathematics The Fifth Yearbook. (1930). p. 45.

2. Gertrude E. Allen, "An Experiment in Redistribution of Material for High School Geometry." The National Council of Teachers of Mathematics, The Fifth Yearbook.(1930) p. 73.

3. John A. Swenson, op. cit. pp. 99-100.

4. Ralph Beatley, "Coherence and Diversity in Secondary Mathematics", The National Council of Teachers of Mathematics, The Eighth Yearbook, (1933). p. 198.

5. William Betz, "The Teaching of Intuitive Geometry", The National Council of Teachers of Mathematics, The Eighth Yearbook, (1933). p. 155.

angle and adjacent angles, etc. Orleans[1] suggests cutting out paper or cardboard. Miss Allen[2] has the children bring in congruent things from the world, a wide assortment, such as quart milk bottles, linoleum patterns, etc. Swenson[3] uses a graphic method of showing how the angle and side vary when the conditions are not for rigidity; he uses the idea of showing what takes place when the conditions for rigidity are not given. His graphic method is of value because it links up with algebra.

Beatley[4] would leave "equal" undefined. Betz[5] urges the use of very simple original exercises. He agrees with Miss Allen in the use of bringing in figures from the world to show congruence; they are the products of our machine age. Re-prints, lasts for shoes, castings, all involve congruence.

---

1. Joseph B. Orleans, "A Unit of Demonstrative Geometry for the Ninth Year", The National Council of Teachers of Mathematics The Fifth Yearbook. (1930). p. 45.

2. Gertrude E. Allen, "An Experiment in Redistribution of Material for High School Geometry." The National Council of Teachers of Mathematics, The Fifth Yearbook.(1930) p. 73.

3. John A. Swenson, op. cit. pp. 99-100.

4. Ralph Beatley, "Coherence and Diversity in Secondary Mathematics", The National Council of Teachers of Mathematics, The Eighth Yearbook. (1933). p. 158.

5. William Betz, "The Teaching of Intuitive Geometry", The National Council of Teachers of Mathematics, The Eighth Yearbook, (1933). p. 185.

B. The Base Angles of an Isosceles Triangle are Equal. Continuing the work on formal demonstrations, the next step is from congruence to the isosceles triangle which again furnishes a fertile field for various proofs which will be noted briefly at the end of the unit. For the present, the greatest concern is the motivation of the task, and again one turns to the applied problem for a felt need.

1. The applied problem. You are building a house and have decided that the peak angle shall be 80 degrees. You have a timber that you wish to use for a brace and wish to know at what angle you must saw this timber in order to have it fit snugly against the roof. That is, you wish to find angle 2, figure 1. Angle 2 could be found if angle 1, figure 1 could be found, because 2 and 1 are supplementary, their sum equaling 180 degrees. You could find angle 1 if you knew the relationship between 1 and 3, since angle 4 has been decided as 80 degrees, and you have found by paper cutting in a triangle that the three angles always equal a straight line, 180 degrees. The task is to find angle 1. The information you have is that the triangle has equal sides down the slope of the roof and you know the vertex, or peak angle.

a). The brace. Keep in mind the task that you cannot saw the brace haphazardly and still have a snug fitting joint, but you must saw it at a certain angle. This angle is determined from the fact that base angles of an isosceles triangle are equal.

B. The Base Angles of an Isosceles Triangle are Equal.

Continuing the work on formal demonstrations, the next step is from congruence to the isosceles triangle which again furnishes a fertile field for various proofs which will be noted briefly at the end of the unit. For the present, the greatest concern is the motivation of the task, and again one turns to the applied problem for a felt need.

1. The applied problem. You are building a house and have decided that the peak angle shall be 80 degrees. You have a timber that you wish to use for a brace and wish to know at what angle you must saw this timber in order to have it fit snugly against the roof. That is, you wish to find angle 2, figure 1. Angle 2 could be found if angle 1, figure 1 could be found, because 2 and 1 are supplementary, their sum equaling 180 degrees. You could find angle 1 if you knew the relationship between 1 and 3, since angle 4 has been decided as 80 degrees, and you have found by paper cutting in a triangle that the three angles always equal a straight line, 180 degrees. The task is to find angle 1. The information you have is that the triangle has equal sides down the slope of the roof and you know the vertex, or peak angle.

a). The brace. Keep in mind the task that you cannot saw the brace haphazardly and still have a snug fitting joint, but you must saw it at a certain angle. This angle is determined from the fact that base angles of an isosceles triangle are equal.

Below are the various parts of the figure:

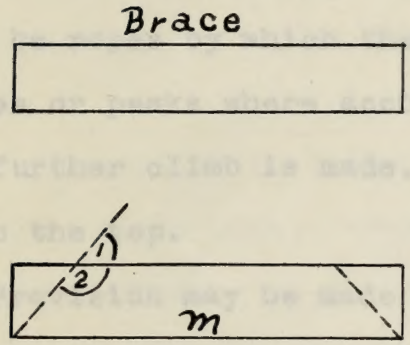


Figure 1.

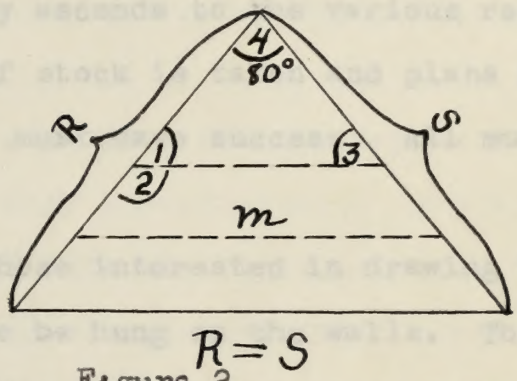


Figure 2.

The brace on the roof can be extended to any brace, such as under a shelf. In addition, the size of the angle on the brace will vary. In putting the roof of a garage together, the workman must know at what angle the beams will be cut, because, as in a lamp shade, four pieces come together to form the peak. A knowledge of plane geometry aids in accurate cutting and close fitting.

2. Graphic analysis. One may now attack the task of finding each of the base angles of the triangle by proving the base angles equal. Since the goal is "Base angles are equal" and the start is an isosceles triangle which gives two sides equal, the task is to discover the intervening peaks. Discussion brings out these peaks, and they are presented here in graphic form and held constantly before the class during the entire procedure. This aids in keeping the problem definitely in mind and also in avoiding confusion and insuring easy surveys of progress, and

Below are the various parts of the figure:

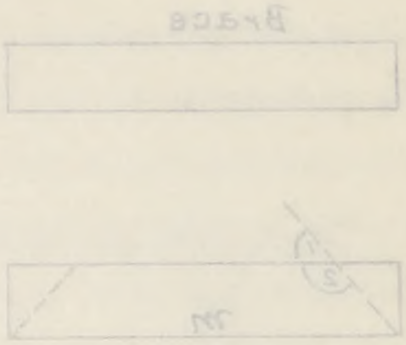


Figure 1.

Figure 2.

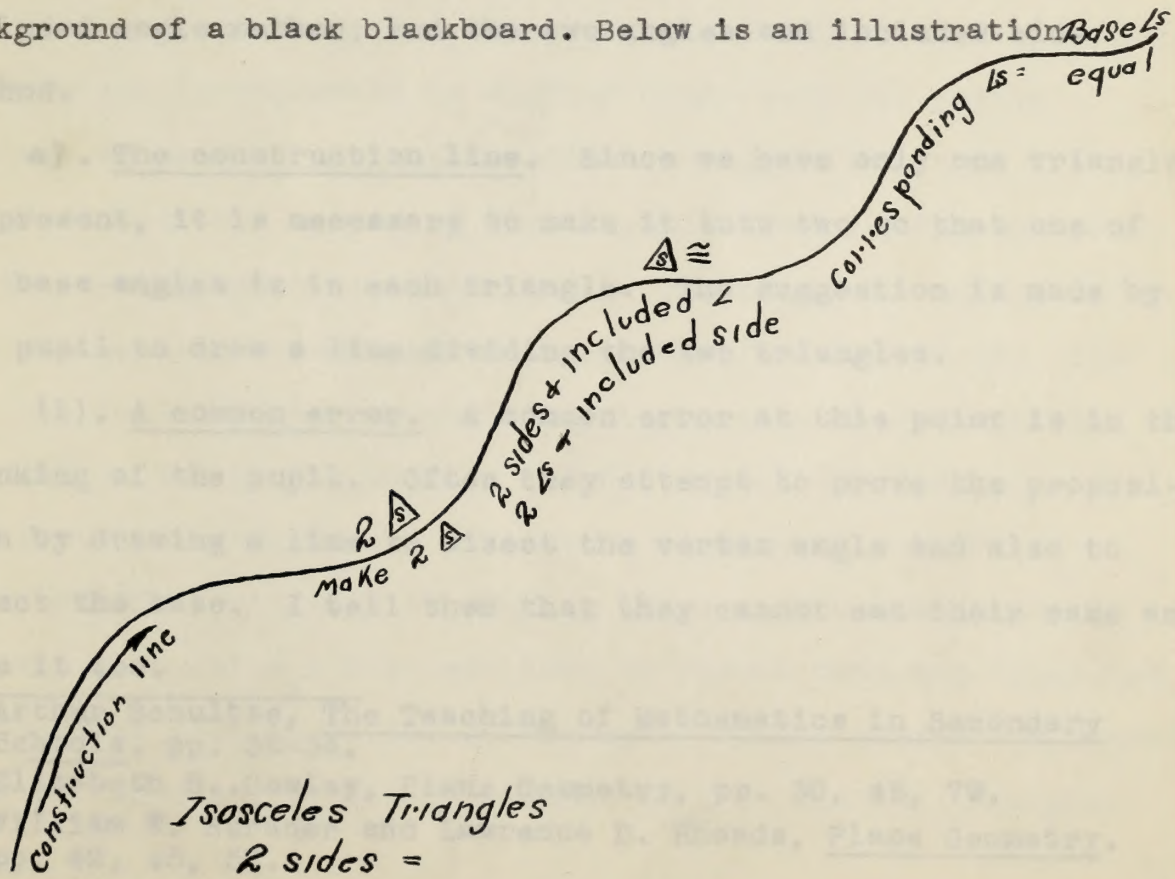
The brace on the roof can be extended to any brace, such as under a shelf. In addition, the size of the angle on the brace will vary. In putting the roof of a garage together, the workman must know at what angle the beams will be cut, because, as in a lamp shade, four pieces come together to form the peak. A knowledge of plane geometry aids in accurate cutting and close fitting.

2. Graphic analysis. One may now attack the task of finding each of the base angles of the triangle by proving the base angles equal. Since the goal is "Base angles are equal" and the start is an isosceles triangle which gives two sides equal, the task is to discover the intervening peaks. Discussion brings out these peaks, and they are presented here in graphic form and held constantly before the class during the entire procedure. This aids in keeping the problem definitely in mind and also in avoiding confusion and insuring easy surveys of progress, and

and organizing of material.

a). <u>The mountain.</u> The mountain peak graph may be endowed with life by referring to the progress of the proposition as the climbers ascending the Alps. Like the actual climbers there must be ropes by which the party ascends to the various resting places or peaks where account of stock is taken and plans for the further climb is made. All must have success. All must reach the top.

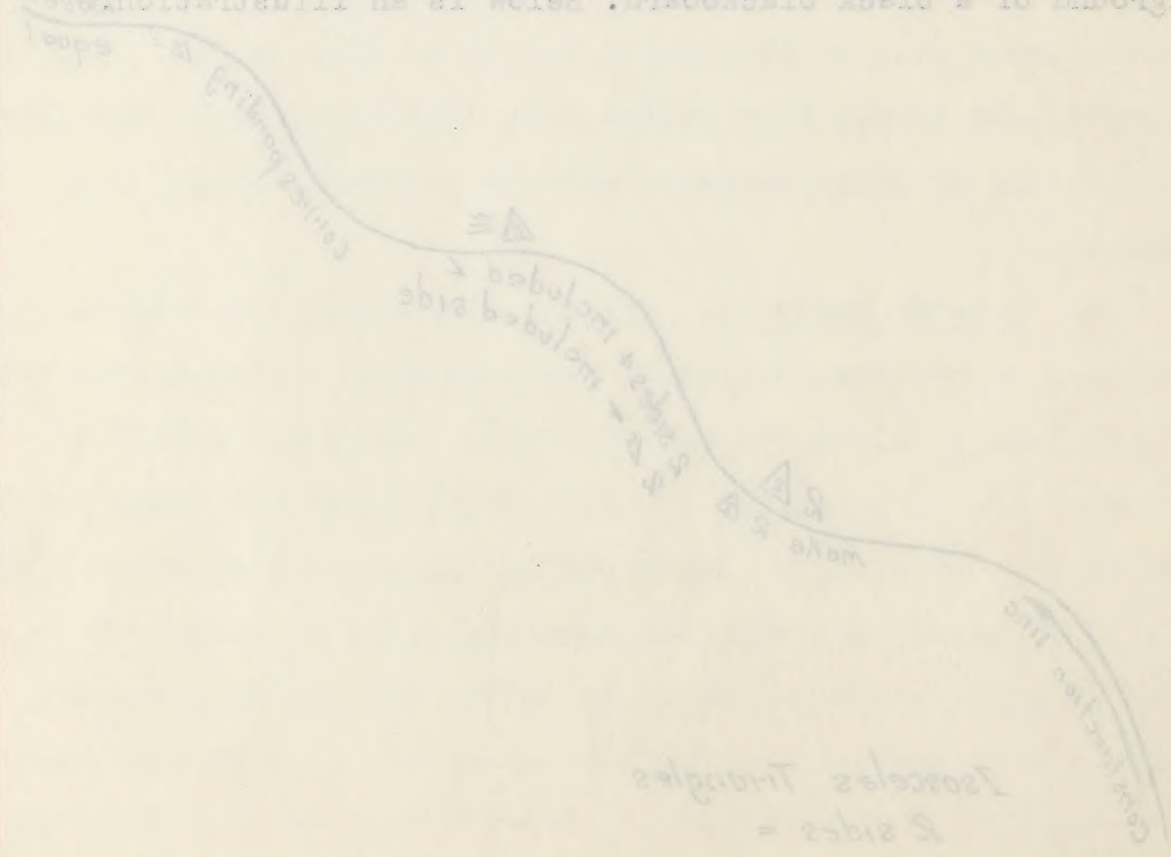
Provision may be made for those interested in drawing by making this diagram in colors to be hung on the walls. The peaks may be made of green or white, while orange or yellow is a good color to use for the ropes in this picture. In all cases, colors should be used which are discernible from the background of a black blackboard. Below is an illustration.



and organizing of material.

8). The mountain. The mountain peak graph may be endowed with life by referring to the progress of the proposition as the climbers ascending the Alps. Like the actual climbers there must be ropes by which the party ascends to the various resting places or peaks where account of stock is taken and plans for the further climb is made. All must have success. All must reach the top.

Provision may be made for those interested in drawing by making this diagram in colors to be hung on the walls. The peaks may be made of green or white, while orange or yellow is a good color to use for the ropes in this picture. In all cases, colors should be used which are discernible from the background of a black blackboard. Below is an illustration.



Schultze[1] discusses the analytic method, but does not connect it with the graphic method.

3. Class discussion. The class secretary under the tutelage of the class constructs an isosceles triangle which involves constructing two sides equal, an exercise in making line segments equal.

Discussion is then held as to what methods may be used to get the angles equal. The theorems preceding will vary according to the text. In the Cowley[2] text the three methods of congruence of triangles have been accepted without formal proof and parallels have been studied. The material here would be quite complete. In the Strader and Rhoads[3] text which is being used here, the only two methods available are the two sides and included angle method, and the two angles and included side method.

a). The construction line. Since we have only one triangle at present, it is necessary to make it into two so that one of the base angles is in each triangle. The suggestion is made by the pupil to draw a line dividing the two triangles.

(1). A common error. A common error at this point is in the thinking of the pupil. Often they attempt to prove the proposition by drawing a line to bisect the vertex angle and also to bisect the base. I tell them that they cannot eat their cake and have it too.

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools. pp. 32-34.
2. Elizabeth B..Cowley, Plane Geometry, pp. 30, 45, 79.
3. William W. Strader and Lawrence D. Rhoads, Plane Geometry. pp. 42, 45, 52.

Schultze[1] discusses the analytic method, but does not connect it with the graphic method.

3. Class discussion. The class secretary under the tutelage of the class constructs an isosceles triangle which involves constructing two sides equal, an exercise in making line segments equal.

Discussion is then held as to what methods may be used to get the angles equal. The theorems preceding will vary according to the text. In the Cowley[2] text the three methods of congruence of triangles have been accepted without formal proof and parallels have been studied. The material here would be quite complete. In the Stradar and Rhoads[3] text which is being used here, the only two methods available are the two sides and included angle method, and the two angles and included side method.

a). The construction line. Since we have only one triangle at present, it is necessary to make it into two so that one of the base angles is in each triangle. The suggestion is made by the pupil to draw a line dividing the two triangles.

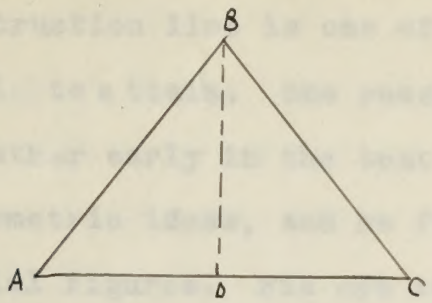
(1). A common error. A common error at this point is in the thinking of the pupil. Often they attempt to prove the proposition by drawing a line to bisect the vertex angle and also to bisect the base. I tell them that they cannot get their cake and have it too.

1. Arthur Schultze. The Teaching of Mathematics in Secondary Schools. pp. 32-34.
2. Elizabeth H. Cowley. Plane Geometry. pp. 30, 45, 79.
3. William W. Stradar and Lawrence D. Rhoads. Plane Geometry. pp. 42, 43, 52.

(2). Correction of the error by construction. Instead of reviewing with them the fact that a line is determined by two points (the position which it takes is so determined), the suggestion is made to draw a good sized scalene triangle in which the sides differ considerably in size. The pupil who made the suggestion to draw a line bisecting the vertex angle in the isosceles triangle and bisecting the base is called to the board and asked to construct a bisector of the vertex angle. Upon completion of this he is then asked if it bisects the base. Then he is asked to bisect the base and the question then comes as to whether that line bisects the vertex angle. Finally, attention is called by means of a pencil touching the end of a finger and the end of an eraser, that the position of the pencil is determined by two points, the finger end and the eraser point. The question is raised as to whether there were two points involved in each of the constructions. Suggestions from the class point to the vertex of the triangle always being one point and the second point is determined in bisecting the angle, or in case of the line bisection, the point of bisection of the line. Thus the pupil by his own activity, by graphic representation, and finally by linking up to a known fact and the discovery of that fact in the drawings, is made to acquire the important notion of the limitation in drawing lines. In a conclusion it is noted that while a line may look to bisect both the angle and the opposite side in isosceles triangle, yet in general triangles it is not true and cannot be assumed as true in this triangle.

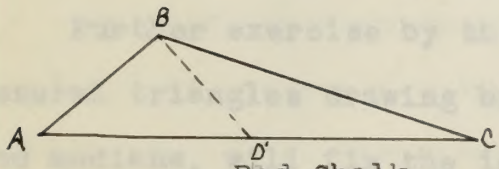
(2). Correction of the error by construction. Instead of revolving with them the fact that a line is determined by two points (the position which it takes is so determined), the suggestion is made to draw a good sized scalene triangle in which the sides differ considerably in size. The pupil who made the suggestion to draw a line bisecting the vertex angle in the isosceles triangle and bisecting the base is called to the board and asked to construct a bisector of the vertex angle. Upon completion of this he is then asked if it bisects the base. Then he is asked to bisect the base and the question then comes as to whether that line bisects the vertex angle. Finally, attention is called by means of a pencil touching the end of a finger and the end of an eraser, that the position of the pencil is determined by two points, the finger end and the eraser point. The question is raised as to whether there were two points involved in each of the constructions. Suggestions from the class point to the vertex of the triangle always being one point and the second point is determined in bisecting the angle, or in case of the line bisection, the point of bisection of the line. Thus the pupil by his own activity, by graphic representation, and finally by linking up to a known fact and the discovery of that fact in the drawings, is made to acquire the important notion of the limitation in drawing lines. In a conclusion it is noted that while a line may look to bisect both the angle and the opposite side in isosceles triangle, yet in general triangles it is not true and cannot be assumed as true in this triangle.

The drawings to illustrate the previous section follow:



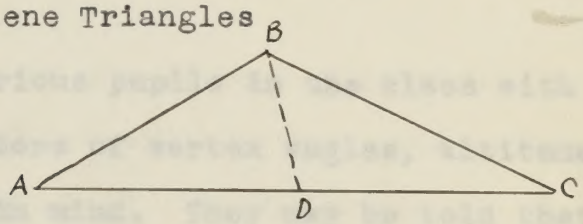
Isosceles triangle--Theorem

Scalene Triangles



Red Chalk

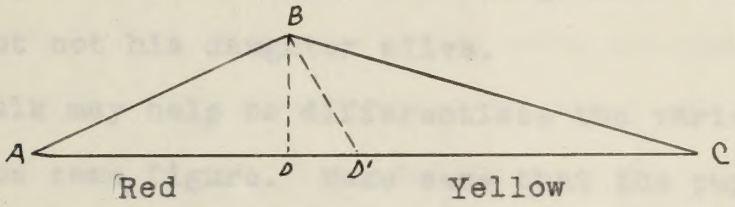
Bisecting side



Yellow chalk

Bisecting angle.

For the bisectors, colored chalk.



Showing B D as bisector of B
Showing B D as bisector of opposite side.

Self-discovery of error by construction.

In the preceding figures, the scalene triangles are all of the same size and the method is to draw the two construction lines in the different figures in order to make clear to the class the construction and finally to combine them in the single

The drawings to illustrate the previous section follow:

Isosceles triangle--Theorem

Scalene Triangles

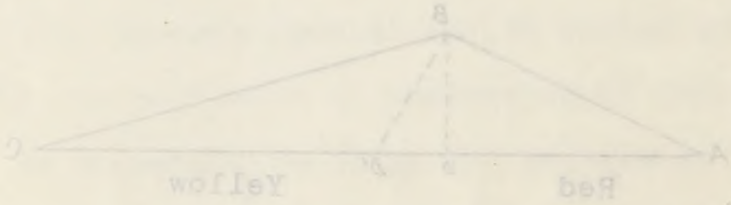
Yellow chalk

Bisecting angle.

Red Chalk

Bisecting side

For the bisectors, colored chalk.



Showing B D as bisector of B
Showing B D as bisector of opposite side.

Self-discovery of error by construction.

In the preceding figures, the scalene triangles are all of the same size and the method is to draw the two construction lines in the different figures in order to make clear to the class the construction and finally to combine them in the single

figure to show that they are not the same line. This comprehension of the construction line is one of the most difficult points for the pupil to attain. One reason is that the proposition comes rather early in the text before the pupil is well grounded in geometric ideas, and he falls an easy prey to fallacies from special figures. His eye tells him it is true from the visual standpoint and he does not yet readily grasp the idea of general figures. The method already outlined of drawing a scalene triangle will give opportunity for the realization of the mistake in reasoning.

Further exercise by the various pupils in the class with general triangles drawing bisectors of vertex angles, altitudes, and medians, will fix the idea in mind. They may be told that construction of lines is like the boy in the story to whom the fairies gave three wishes, that only one wish could be had at a time. Another illustration would be of King Midas who could have his gold but not his daughter alive.

Colored chalk may help to differentiate the various lines when drawn in the same figure. Make sure that the pupil understands that the isosceles and equilateral triangles are special cases.

If the constructions are made automatic, and if the idea that a construction line can fulfill only two conditions is thoroughly instilled, the pupils will have an easier time in proving the propositions.

This is an excellent stopping place also for a discussion of the necessity of selection in life in general. If we have

figure to show that they are not the same line. This comprehension of the construction line is one of the most difficult points for the pupil to attain. One reason is that the proposition comes rather early in the text before the pupil is well grounded in geometric ideas, and he falls an easy prey to fallacies from special figures. His eye tells him it is true from the visual standpoint and he does not yet readily grasp the idea of general figures. The method already outlined of drawing a scalene triangle will give opportunity for the realization of the mistake in reasoning.

Further exercise by the various pupils in the class with general triangles drawing bisectors of vertex angles, altitudes, and medians, will fix the idea in mind. They may be told that construction of lines is like the boy in the story to whom the fairies gave three wishes, that only one wish could be had at a time. Another illustration would be of King Midas who could have his gold but not his daughter alive.

Colored chalk may help to differentiate the various lines when drawn in the same figure. Make sure that the pupil understands that the isosceles and equilateral triangles are special cases.

If the constructions are made automatic, and if the idea that a construction line can fulfill only two conditions is thoroughly instilled, the pupils will have an easier time in proving the propositions.

This is an excellent stopping place also for a discussion of the necessity of selection in life in general. If we have

such and such, we cannot have such and such: that person is most successful who can choose wisely and lead a well-balanced life.

(3). The right road to the "Land of Congruent Triangles". The next question is--"Which construction line shall we choose?" It is noted that there is the possibility of the perpendicular in addition to two lines just discussed, the median and the bisector of the vertex angle. This third line was not introduced in the previous section because it was desired to make the difficulty at hand as simple as possible by drawing a line only to two conditions.

The highway from the "Land of the Isosceles Triangle" to the "Land of Congruent Triangles" branches in three directions with the signs "Altitude-Perpendicular", "Median-Bisector of Opposite Side", and "Bisector of Vertex Angle".

Each road is traversed by the class until some sign tells them that they are on the wrong road. If the first road they choose is the correct one, they are then asked whether one of the other roads would bring them out to the right place, and it is accordingly traversed. In this particular proposition, the only correct method in a certain arrangement of propositions in the text is the "Bisector of the Vertex Angle", but often there are two or three methods by which a proposition may be solved.

*The word "included" comes in for attention at this time because one of the roads leads to two sides but not the included angle. It is wise to begin to emphasize also that whenever two angles of a triangle have been proved equal to two other angles, you never have to worry as to whether the two angles involve the

such and such, we cannot have such and such: that person is most successful who can choose wisely and lead a well-balanced life.

(3). The right road to the "Land of Congruent Triangles".

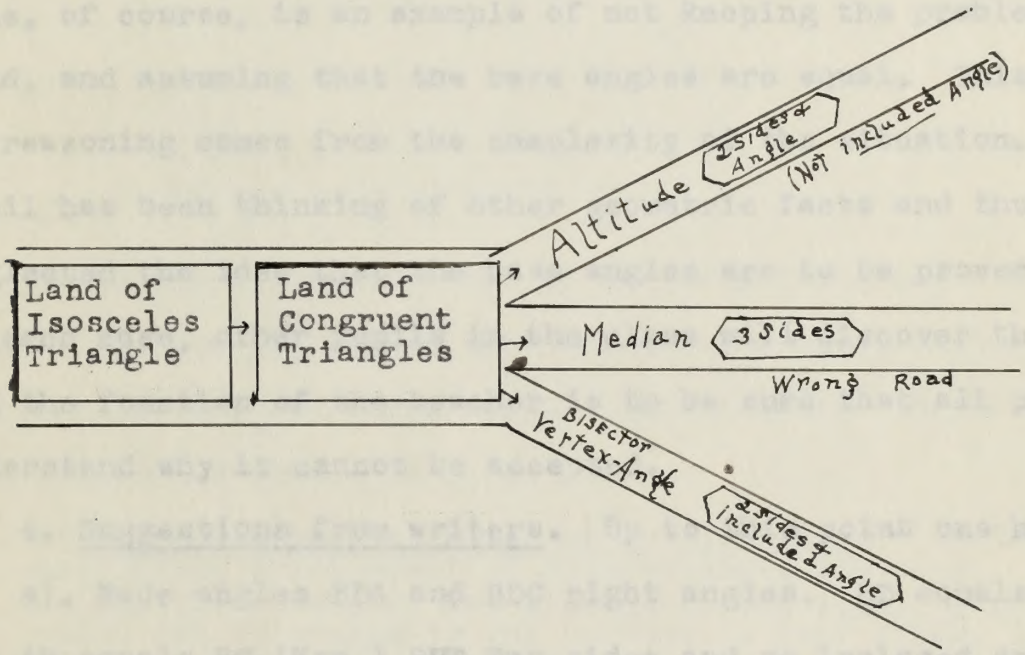
The next question is--"Which construction line shall we choose?" It is noted that there is the possibility of the perpendicular in addition to two lines just discussed, the median and the bisector of the vertex angle. This third line was not introduced in the previous section because it was desired to make the difficulty at hand as simple as possible by drawing a line only to two conditions.

The highway from the "Land of the Isosceles Triangle" to the "Land of Congruent Triangles" branches in three directions with the signs "Altitude-Perpendicular", "Median-Bisector of Opposite Side", and "Bisector of Vertex Angle".

Each road is traversed by the class until some sign tells them that they are on the wrong road. If the first road they choose is the correct one, they are then asked whether one of the other roads would bring them out to the right place, and it is accordingly traversed. In this particular proposition, the only correct method in a certain arrangement of propositions in the text is the "Bisector of the Vertex Angle", but often there are two or three methods by which a proposition may be solved.

*The word "included" comes in for attention at this time because one of the roads leads to two sides but not the included angle. It is wise to begin to emphasize also that whenever two angles of triangle have been proved equal to two other angles, you never have to worry as to whether the two angles involve the

included angle, because the third angle is equal by a proposition to be proved shortly. That is, two angles equal to two other angles in two triangles always mean that the three angles in each triangle are equal, respectively, but not so when two sides are equal. The sides may not be extended to three equal sides without additional equalities.

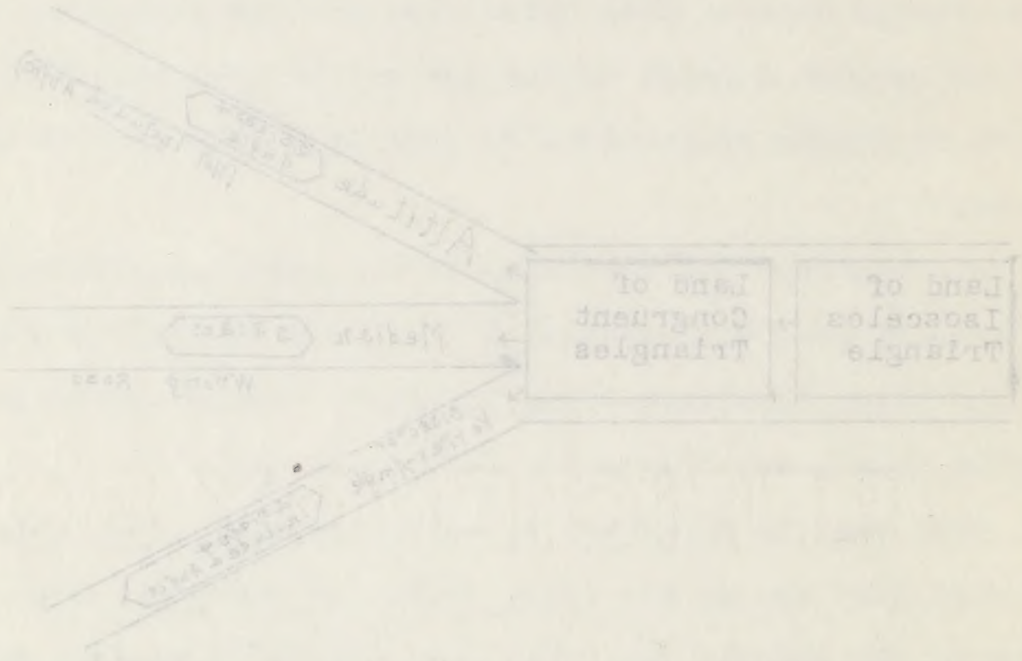


The above diagram is built by the pupil at the board as the discussion progresses. Colored chalk, red, yellow, white, may be used for the three branches.

It will be noted that this idea savors of Bunyan's Pilgrim Progress, where abstract ideas were made clear by relating to concrete. Bunyan's sin was a load carried on his back. Here a construction line becomes a road.

There are enough points of similarity along each of the roads to give the careless pupils difficulty. For instance,

included angle, because the third angle is equal by a proposition to be proved shortly. That is, two angles equal to two other angles in two triangles always mean that the three angles in each triangle are equal, respectively, but not so when two sides are equal. The sides may not be extended to three equal sides without additional equalities.



The above diagram is built by the pupil at the board as the discussion progresses. Colored chalk, red, yellow, white, may be used for the three branches.

It will be noted that this idea savors of Bunyan's Pilgrim Progress, where abstract ideas were made clear by relating to concrete. Bunyan's sin was a load carried on his back. Here a construction line becomes a road.

There are enough points of similarity along each of the roads to give the careless pupils difficulty. For instance,

taking the "Altitude" road will be sure to bring forth from some pupil that that is the correct road for it makes two sides and an angle. The *paragraph on the second page back takes care of this situation. The next construction, the "Median" road brings from some pupil the remark that it is the right road because it makes two sides and the included angle at the base. This, of course, is an example of not keeping the problem in mind, and assuming that the base angles are equal. This fallacy of reasoning comes from the complexity of the situation. The pupil has been thinking of other geometric facts and thus has neglected the idea that the base angles are to be proved equal. In each case, other pupils in the class will discover this error and the function of the teacher is to be sure that all pupils understand why it cannot be accepted.

4. Suggestions from writers. Up to this point one has:

a). Made angles BDA and BDC right angles. BD equals BD and AB equals BC (Hyp.) BUT Two sides and no included angle.

b). Made AD equal DC. AB equals BC (Hyp.). BD equals BD (Id.). Three sides but no angle. In Cowley[1] text this would be acceptable according to that sequence of propositions.

A third possible construction of BD is to bisect the vertex angle. This is done and it is found that AB and BD surround the angle 1 and BD and BC surround angle 2, so that triangles 1 and II are congruent because two sides and the included angle are respectively equal.

---

1. Elizabeth B. Cowley, Plane Geometry, loc. cit.

taking the "Altitude" road will be sure to bring forth from some pupil that that is the correct road for it makes two sides and an angle. The "paragraph on the second page back makes sure of this situation. The next construction, the "Median" road brings from some pupil the remark that it is the right road because it makes two sides and the included angle at the base. This, of course, is an example of not keeping the problem in mind, and assuming that the base angles are equal. This fallacy of reasoning comes from the complexity of the situation. The pupil has been thinking of other geometric facts and thus has neglected the idea that the base angles are to be proved equal. In each case, other pupils in the class will discover this error and the function of the teacher is to be sure that all pupils understand why it cannot be accepted.

4. Suggestions from writers. Up to this point one has:

a). Made angles BDA and BDC right angles. BD equals BD and AB equals BC (Hyp.) BUT Two sides and no included angle.

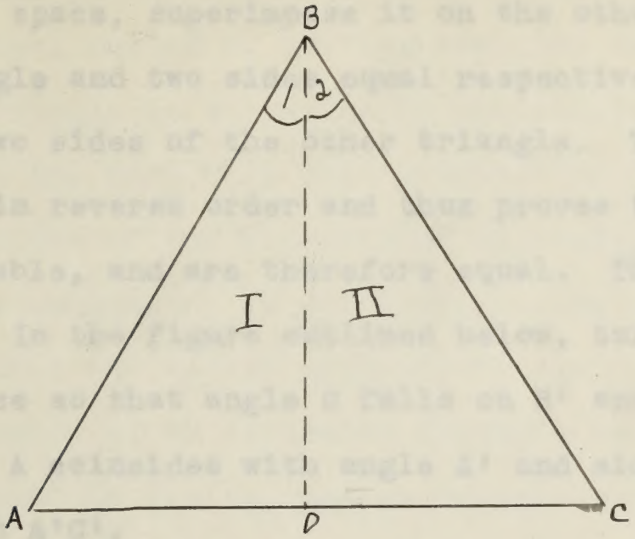
b). Made AD equal DC. AB equals BC (Hyp.). BD equals BD (Id.). Three sides but no angle. In Cowley[1] text this would be acceptable according to that sequence of propositions.

A third possible construction of BD is to bisect the vertex angle. This is done and it is found that AB and BD surround the angle 1 and BD and BC surround angle 2, so that triangles I and II are congruent because two sides and the included angle are respectively equal.

---

1. Elizabeth S. Cowley, Plane Geometry, loc. cit.

Below is the figure.[1]



Since the triangles I and II are congruent, angle A and angle C are equal, because they are corresponding angles of congruent triangles; these are the base angles. Therefore, the base angles are equal. Q. E. D.

a) Self-discovery of error by construction. "In the prevailing arrangement of propositions, the only method that leads to two triangles, the equality of which can be proved by the student at this stage, is based upon the bisection of the vertex angle. As the student, however, cannot foresee this result, it would be advisable to let him try to obtain two equal triangles by means of the median or the altitude, in order that he may discover for himself why the bisector alone can be used."[2]

b) Proof of proposition by turning a congruent triangle through space.--Superimposing. To avoid drawing the bisector,

---

1. William W. Strader and Lawrence D. Rhoads, loc. cit.

2. Arthur Schultze, op. cit. p. 152.

Below is the figure.[1]

Since the triangles I and II are congruent, angle A and angle C are equal, because they are corresponding angles of congruent triangles; these are the base angles. Therefore, the base angles are equal. Q. E. D.

a) Self-discovery of error by construction. "In the prevailing arrangement of propositions, the only method that leads to two triangles, the equality of which can be proved by the student at this stage, is based upon the bisection of the vertex angle. As the student, however, cannot foresee this result, it would be advisable to let him try to obtain two equal triangles by means of the median or the altitude, in order that he may discover for himself why the bisector alone can be used."[2]

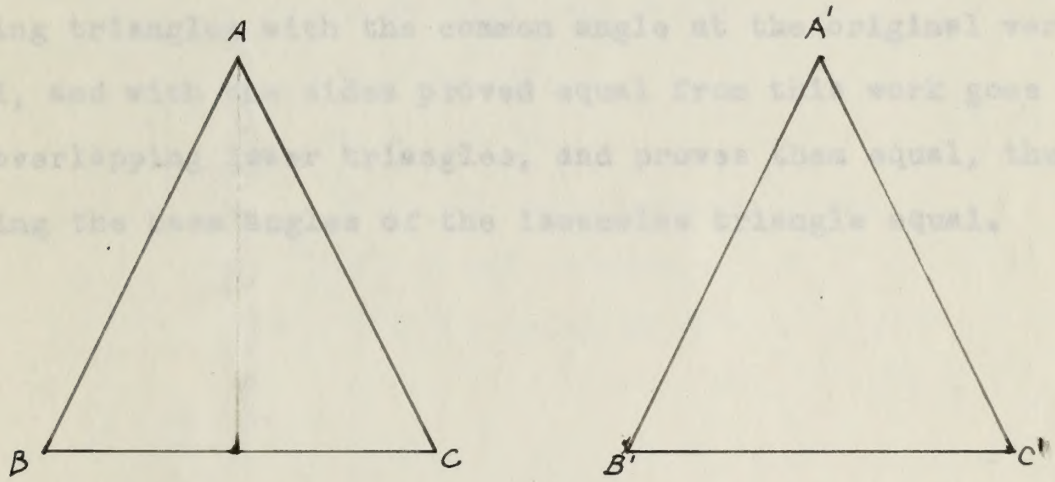
b) Proof of proposition by turning a congruent triangle through space.--Superimposing. To avoid drawing the bisector,

---

1. William W. Strader and Lawrence D. Rhoads, loc. cit.

2. Arthur Schultze, op. cit. p. 182.

Nyeberg[1] suggests drawing two congruent triangles, and turning one through space, superimpose it on the other, since it has an included angle and two sides equal respectively to the included angle and two sides of the other triangle. This makes the base angles fit in reverse order and thus proves that they are interchangeable, and are therefore equal. That is, base angles are equal. In the figure outlined below, turning Triangle ABC through space so that angle C falls on B' and angle B on C', since angle A coincides with angle A' and side AC with AB' and side AB with A'C'.



1. Joseph A. Nyeberg, "Proving the Equality of the Base Angles of the Isosceles Triangle". The Mathematics Teacher, XXII, (October, 1929), p. 318.

Myeberg[1] suggests drawing two congruent triangles, and turning one through space, superimpose it on the other, since it has an included angle and two sides equal respectively to the included angle and two sides of the other triangle. This makes the base angles fit in reverse order and thus proves that they are interchangeable, and are therefore equal. That is, base angles are equal. In the figure outlined below, turning Triangle ABC through space so that angle C falls on B' and angle B on C', since angle A coincides with angle A' and side AC with AB' and side AB with A'C'.

1. Joseph A. Myeberg, "Proving the Equality of the Base Angles of the Isosceles Triangle". The Mathematics Teacher, XXII, (October, 1929), p. 318.

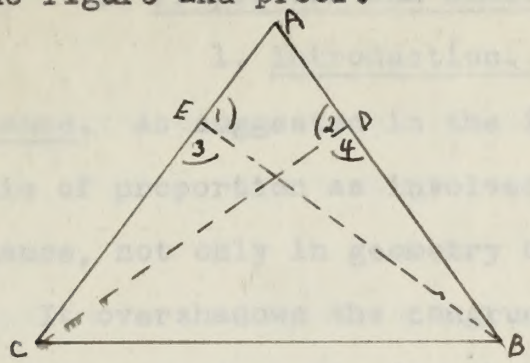
c). Perpendiculars drawn from base angles to the opposite sides. Another method of proving that the base angles of an isosceles triangle is suggested by Christofferson[1], who calls attention to the fact that this proposition is based on proving the construction of the bisector of an angle, which in turn rests on the congruency of two triangles by means of three sides, and that the latter proposition again rests on the proof that base angles of an isosceles triangle are equal. In other words the proof is faulty, unless one assumes one or the other of the propositions. He avoids drawing the bisector of the vertex angle, and draws a line instead from the two base angles perpendicular to the equal sides. He then proves the two overlapping triangles with the common angle at the original vertex equal, and with the sides proved equal from this work goes to the overlapping lower triangles, and proves them equal, thus proving the base angles of the isosceles triangle equal.

1. H. C. Christofferson, "A Fallacy in Geometric Reasoning". The Mathematics Teacher, XXIII (January, 1930) pp. 19-22.

c). Perpendiculars drawn from base angles to the opposite sides. Another method of proving that the base angles of an isosceles triangle is suggested by Christofferson[1], who calls attention to the fact that this proposition is based on proving the construction of the bisector of an angle, which in turn rests on the congruency of two triangles by means of three sides, and that the latter proposition again rests on the proof that base angles of an isosceles triangle are equal. In other words the proof is faulty, unless one assumesone or the other of the propositions. He avoids drawing the bisector of the vertex angle, and draws a line instead from the two base angles perpendicular to the equal sides. He then proves the two overlapping triangles with the common angle at the original vertex equal, and with the sides proved equal from this work goes to the overlapping lower triangles, and proves them equal, thus proving the base angles of the isosceles triangle equal.

1. H. C. Christofferson, "A Fallacy in Geometric Reasoning", The Mathematics Teacher, XXIII (January, 1930) pp. 19-22.

Below is the figure and proof.



$\triangle ABE \cong \triangle ADC$

$\angle A = \angle A$ Id.

$\angle 1 = \angle 2$ Rt. $\angle$s

$AC = AB$ Hyp.

$EB = CD$ Corresponding sides.

$\triangle ECB \cong \triangle DBC$ Leg. & Hyp.

$\angle 3 = \angle 4$ Rt. $\angle$s

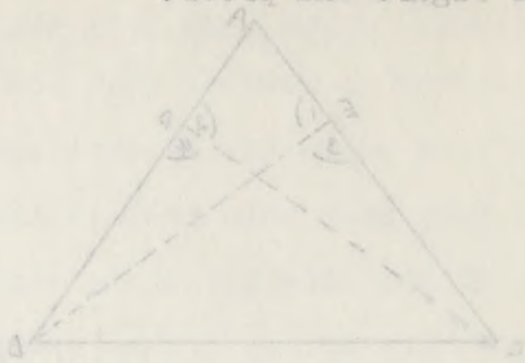
$CB = CB$ Id.

$EB = CD$ Corresponding sides.

$\angle DBC = \angle ECB$ Corresp. $\angle$s

$\angle B = \angle C$ above $\cong \triangle$s

Below is the figure and proof.



△ ABE ≅ △ ADC

∠ A = ∠ A Id.

∠ 1 = ∠ 2 Rt. ∠s

AC = AB Hyp.

EB = CD Corresponding sides.

△ ECB ≅ △ DBC Leg.s & Hyp.

∠ 3 = ∠ 4 Rt. ∠s

CB = CB Id.

EB = CD Corresponding sides.

∠ DBC = ∠ ECB Corresp. ∠s

∠ B = ∠ C above ≅ △s

C. Proportion and Similarity

1. Introduction.

a). Importance. As suggested in the introduction to the thesis, the topic of proportion as involved with similarity is of great importance, not only in geometry but in all phases of human activity. It overshadows the congruence unit in that the latter is a special case of similarity where the ratio is a 1:1 ratio. Writers seem to agree that the topic of proportion and similarity is one of the main threads which runs through geometry. Orleans[1] names it as one of three main threads that run throughout the plane geometry of the tenth year.

This unit will begin with general suggestions in regard to the teaching of proportion. Various ways of introducing the subject are noted which include the applied problem, and hand work such as manual training work in constructing a pantograph and drawing, and for more verbal-minded, a discussion.

The topic is further motivated by postponing the algebraic processes until close contact with the geometric need is made. The importance of the topic is shown through its relationship with trigonometry, physics, and chemistry.

Clarity is tried for by graphical work, drill on important facts, pre-work on definitions, and the attachment to the school subjects as noted above.

b). General suggestions. Let us note first the general suggestions.

1. Joseph B. Orleans, "A Unit of Demonstrative Geometry for the Ninth Year", The National Council of Teachers of Mathematics, The Fifth Yearbook. (1930), pp. 51-52.

C. Proportion and Similarity

1. Introduction.

a). Importance. As suggested in the introduction to the thesis, the topic of proportion as involved with similarity is of great importance, not only in geometry but in all phases of human activity. It overshadows the congruence unit in that the latter is a special case of similarity where the ratio is a 1:1 ratio. Writers seem to agree that the topic of proportion and similarity is one of the main threads which runs through geometry. Orleans[1] names it as one of three main threads that run throughout the plane geometry of the tenth year.

This unit will begin with general suggestions in regard to the teaching of proportion. Various ways of introducing the subject are noted which include the applied problem, and hand work such as manual training work in constructing a pantograph and drawing, and for more verbal-minded, a discussion.

The topic is further motivated by postponing the algebraic processes until close contact with the geometric need is made. The importance of the topic is shown through its relationship with trigonometry, physics, and chemistry.

Clarity is tried for by graphical work, drill on important facts, pre-work on definitions, and the attachment to the school subjects as noted above.

b). General suggestions. Let us note first the general suggestions.

1. Joseph E. Orleans, "A Unit of Demonstrative Geometry for the Ninth Year", The National Council of Teachers of Mathematics, The Fifth Yearbook. (1930), pp. 51-52.

Drill on unit of finding corresponding parts in similar figures.

Teach by meaningful situations; illustrations; the concrete; practical problems, rather than through verbal statements or algebraic symbols.

Teach proportion in connection with beauty.

Teach with applications to physics and other subjects.

Connect with study of congruent triangles.

Study proportion of the human body as made by artists.

Make a gradual introduction. Teach ratio first.

Connect with numerical measure.

Give a new method of proving lines equal; if three quantities in each proportion the same the fourth must be.

Group the various book propositions together to show that they come under the fundamental propositions for similar triangles.

Show mode of notation in diagramming similar parts.

Describe two methods of copying pictures:
a. Squared paper.
b. Pantograph.

Provide for individual differences:
a. Construction of models.
b. History of art.
c. Artistry.
d. Verbal delivery; must be in proportion.
e. Difficult problems on proportion.
f. Relation to physics and chemistry.
Illustration and talk on.

c). Introductory discussion.

(1). Applications to life activities. In discussing proportion, instead of making the pupils do much mathematical

Drill on unit of finding corresponding parts in similar figures.

Teach by meaningful situations; illustrations; the concrete; practical problems, rather than through verbal statements or algebraic symbols.

Teach proportion in connection with beauty.

Teach with applications to physics and other subjects.

Connect with study of congruent triangles.

Study proportion of the human body as made by artists.

Make a gradual introduction. Teach ratio first.

Connect with numerical measure.

Give a new method of proving lines equal; if three quantities in each proportion the same the fourth must be.

Group the various book propositions together to show that they come under the fundamental propositions for similar triangles.

Show mode of notation in diagramming similar parts.

Describe two methods of copying pictures:
a. Squared paper.
b. Pantograph.

Provide for individual differences:
a. Construction of models.
b. History of art.
c. Artistry.
d. Verbal delivery; must be in proportion.
e. Difficult problems on proportion.
f. Relation to physics and chemistry.
Illustration and talk on.

c). Introductory discussion.

(1). Applications to life activities. In discussing proportion, instead of making the pupils do much mathematical

work in changing the form of the proportion, or remembering definitions, the aim is to develop an interest and realization of the importance of proportion. This is done by showing the practical applications proportion has to life. It will be the chief purpose of this unit to suggest connections with other subjects in school and in life in general. As the pupil sees the importance, and directly with the need involved, the necessary algebraic manipulations may be taught readily.

(a). All calculations based upon it. In showing the interrelation with life, it is noted that all true samples involve proportion. As the quality of the sample so the quality of the whole, if it be an honest sample. As the price, or amount of the unit, so we can figure the price or amount of the whole. All calculations come under proportion. The amount of paint for a building, or the cost may be figured from a unit surface, or unit cost. The same is true with lumber which involves volumes.

(b). Industry uses it. Industry makes all its estimates on it, does all its buying on it. To illustrate a little more in detail, in the human body there exist certain relations between measurements well known to dressmakers. Every purchase made on sample involves the implication that as the quality is in this sample so the quality will be in all samples. Every piece of work involving estimates of material used over a common denominator of space involves proportion. As so much paint to such a unit area, then so much paint for the total area. In chemistry so much sulphuric acid for this amount of

work in changing the form of the proportion, or remembering definitions, the aim is to develop an interest and realisation of the importance of proportion. This is done by showing the practical applications proportion has to life. It will be the chief purpose of this unit to suggest connections with other subjects in school and in life in general. As the pupil sees the importance, and directly with the need involved, the necessary algebraic manipulations may be taught readily.

(a). All calculations based upon it. In showing the interrelation with life, it is noted that all true samples involve proportion. As the quality of the sample so the quality of the whole, if it be an honest sample. As the price, or amount of the unit, so we can figure the price or amount of the whole. All calculations come under proportion. The amount of paint for a building, or the cost may be figured from a unit surface, or unit cost. The same is true with lumber which involves volumes.

(b). Industry uses it. Industry makes all its estimates on it, does all its buying on it. To illustrate a little more in detail, in the human body there exist certain relations between measurements well known to dressmakers. Every purchase made on sample involves the implication that as the quality is in this sample so the quality will be in all samples. Every piece of work involving estimates of material used over a common denominator of space involves proportion. As so much paint to such a unit area, then so much paint for the total area. In chemistry so much sulphuric acid for this amount of

nitric acid, then so much sulphuric acid for the new amount.

(c). Good photography considers it. In good photographs the following proportion holds:

$$\frac{\text{Size of picture head}}{\text{Size of real head}} = \frac{\text{Size of picture foot.}}{\text{Size of real foot.}}$$

One of the errors of life, however, is to take a small sample of human conduct and judge the entire conduct of the individual by it. Of course one knows that human beings in their conduct toward one individual may be entirely different from their conduct toward another human being. This is one place where proportion has other complications.

(d). Literature recognizes it. And in its larger aspects every presentation whether delivered orally or in written form before an audience is based on proportion. The balance is what is continually wished for. Not an outline of a debate or a thesis may be written down without regard to proportion. As the emphasis on this particular portion is to the whole, so in another situation emphasis must play the same relation to the whole. The mind has accepted the Golden Ratio not only to the rectangle but in affairs of life.

nitric acid, then so much sulphuric acid for the new amount.

(c). Good photography considers it. In good photography the following proportion holds:

$$\frac{\text{Size of picture head}}{\text{Size of real head}} = \frac{\text{Size of picture foot.}}{\text{Size of real foot.}}$$

One of the errors of life, however, is to take a small sample of human conduct and judge the entire conduct of the individual by it. Of course one knows that human beings in their conduct toward one individual may be entirely different from their conduct toward another human being. This is one place where proportion has other complications.

(d). Literature recognizes it. And in its larger aspects every presentation whether delivered orally or in written form before an audience is based on proportion. The balance is what is continually wished for. Not an outline of a debate or a thesis may be written down without regard to proportion. As the emphasis on this particular portion is to the whole, so in another situation emphasis must play the same relation to the whole. The mind has accepted the Golden Ratio not only to the rectangle but in affairs of life.

d). Early assignments.

(1). Enlarging pictures. Engle[1] suggests an approach to the study of proportion which involves the activity of all the pupils in a simple drawing situation within reach of the entire class, which is attached to a concrete situation, involves material easily obtainable, and which finally has an audience situation. His procedure is as follows:

> "Young artists in copying masterpieces use the following method: Across the photograph of the original painting small squares are drawn. Then on a canvas which is to contain the reproduction larger squares are drawn in faint charcoal lines. Later these lines can be easily erased. By locating points on the canvas in squares corresponding to squares on the photograph the main outline of the pictures are obtained in correct proportion.
>
> "Two or three weeks in advance of the formal work on proportion I tell about the method in the art gallery, and make an assignment requiring each pupil to secure some picture and reproduce it by the method.
>
> "For class work the Sunday comic supplements and daily comic strips make excellent masterpieces and add a little fun to the subject. Small line drawings are best.
>
> "Squares for example of 1/4 inch are made on the original picture and then pencil squares say of 1/2 inch, are drawn on a sheet of drawing paper. Lines of the original drawings are reproduced in the corresponding squares.
>
> "Colors may be added.
>
> "Post some of the pictures on the bulletin board, where the other high school pupils can see them thus making mathematics popular. Enlargement has been found easiest, although of course pictures may be reduced or copied the same size.
>
> "Art drawings from magazines are good. Bring in the idea of proportion in photography."

(2). Making a pantograph. It is a good plan also to have some pupil interested in manual training make a simple pantograph which involves a more scientific method in copying certain

1. B. T. Engle, Proportion, School Science and Mathematics, (March, 1932), pp. 268-271.

d). Early assignments.

(1). Enlarging pictures. Engle[1] suggests an approach to the study of proportion which involves the activity of all the pupils in a simple drawing situation within reach of the entire class, which is attached to a concrete situation, involves material easily obtainable, and which finally has an audience situation. His procedure is as follows:

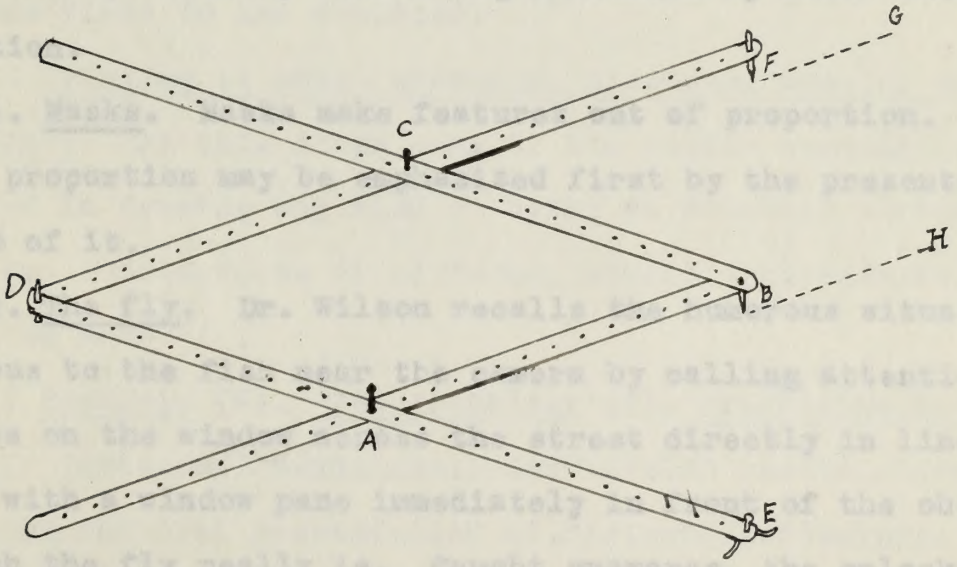
> "Young artists in copying masterpieces use the following method: Across the photograph of the original painting small squares are drawn. Then on a canvas which is to contain the reproduction larger squares are drawn in faint charcoal lines. Later these lines can be easily erased. By locating points on the canvas in squares corresponding to squares on the photograph the main outline of the pictures are obtained in correct proportion.
>
> "Two or three weeks in advance of the formal work on proportion I tell about the method in the art gallery, and make an assignment requiring each pupil to secure some pictures and reproduce it by the method.
>
> "For class work the Sunday comic supplements and daily comic strips make excellent masterpieces and add a little fun to the subject. Small line drawings are best.
>
> "Squares for example of 1\4 inch are made on the original picture and then pencil squares say of 1\2 inch, are drawn on a sheet of drawing paper. Lines of the original drawings are reproduced in the corresponding squares.
>
> "Colors may be added.
>
> "Post some of the pictures on the bulletin board, where the other high school pupils can see them thus making mathematics popular. Enlargement has been found easiest, although of course pictures may be reduced or copied the same size.
>
> "Art drawings from magazines are good. Bring in the idea of proportion in photography."

(2). Making a pantograph. It is a good plan also to have some pupil interested in manual training make a simple pantograph which involves a more scientific method in copying certain

1. B. T. Engle. Proportion. School Science and Mathematics, (March, 1932), pp. 268-271.

types of drawings which requires the tracing of lines, or making a map to any desired scale. A description of this instrument is given by Cowley as follows:[1]

> "A pantograph is an instrument used for making enlarged or reduced pictures. The four bars, parallel in pairs, are jointed at points A, B, C, and D. The lengths of the sides of parallelogram ABCD can be adjusted at points A and C, but points E, B, and F must be in a straight line. The instrument is fastened to the drawing board at E. While a dull point at B traces the original figure, the pencil point at F draws a similar figure.
>
> "If pencil point F draws line segment FG while the dull point traces line segment BH, why is $\frac{BH}{FG} = \frac{EA}{ED}$?
>
> "In adjusting the pantograph at points A and C, what would be the ratio of EA to ED if you wished each line segment in the enlargement to be three times its original line segment? Why?



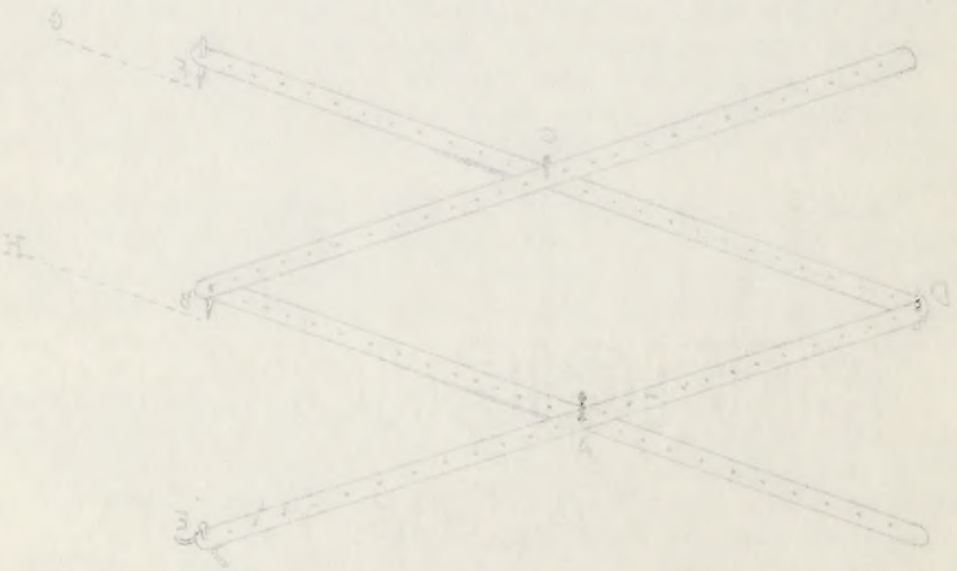
1. Elizabeth B. Cowley, Plane Geometry, pp. 283-284.

types of drawings which requires the tracing of lines, or making a map to any desired scale. A description of this instrument is given by Cowley as follows:[1]

"A pantograph is an instrument used for making enlarged or reduced pictures. The four bars, parallel in pairs, are jointed at points A, B, C, and D. The lengths of the sides of parallelogram ABCD can be adjusted at points A and C, but points E, B, and F must be in a straight line. The instrument is fastened to the drawing board at E. While a dull point at B traces the original figure, the pencil point at F draws a similar figure.

"If pencil point F draws line segment FG while the dull point traces line segment BH, why is $\frac{BH}{FG} = \frac{EA}{ED}$?

"In adjusting the pantograph at points A and C, what would be the ratio of EA to ED if you wished each line segment in the enlargement to be three times its original line segment? Why?



1. Elizabeth B. Cowley, Plane Geometry, pp. 283-284.

e). Humor from lack of proportion. Following the hand activity of the pupils in making copies of pictures may come activity in observation of proportion in life. This can be emphasized by showing what happens when objects are out of proportion.

(1). Big feet in the photograph. The photograph of the individual sitting down with the foot too near the camera is a good example of lack of proportion.

(2). The fish you caught. Humor is introduced also by showing a picture of the fish you caught which seems to be as large as the man holding it because the picture is out of focus.

(3). Stilts. Men on stilts with big advertising signs walk the city streets, attracting attention by their lack of proportion.

(4). Masks. Masks make features out of proportion. In short, proportion may be emphasized first by the presentation of lack of it.

(5). The fly. Dr. Wilson recalls the humorous situation analogous to the fish near the camera by calling attention to a fly as on the window across the street directly in line of vision with a window pane immediately in front of the observer on which the fly really is. Caught unawares, the onlooker thinks of the fly as across the street until the absurdity of the supposition dawns on him.

f). Individual assignments.

(1). Sketching pictures of the human body. The idea of

e). Humor from lack of proportion. Following the hand activity of the pupils in making copies of pictures may come activity in observation of proportion in life. This can be emphasized by showing what happens when objects are out of proportion.

(1). Big feet in the photograph. The photograph of the individual sitting down with the foot too near the camera is a good example of lack of proportion.

(2). The fish you caught. Humor is introduced also by showing a picture of the fish you caught which seems to be as large as the man holding it because the picture is out of focus.

(3). Stilts. Men on stilts with big advertising signs walk the city streets, attracting attention by their lack of proportion.

(4). Masks. Masks make features out of proportion. In short, proportion may be emphasized first by the presentation of lack of it.

(5). The fly. Dr. Wilson recalls the humorous situation analogous to the fish near the camera by calling attention to a fly as on the window across the street directly in line of vision with a window pane immediately in front of the observer on which the fly really is. Caught unawares, the onlooker thinks of the fly as across the street until the absurdity of the supposition dawns on him.

f). Individual assignments.

(1). Sketching pictures of the human body. The idea of

beauty and proportion being linked together has been suggested by Wolff.[1] He says:

> "The theory of proportion affords a very good example of practical use according to the methodical principles of "concentration", as proportions were used by artists for drawings of men and women in the days of the Renaissance when man was regarded as the measure of all, and when the chief aim of artists was to model the human body in an ideal form. We know that Leonardo da Vinci (1452-1519) diligently studied proportion in order to create beautiful bodies. These proportions of the human body were analyzed in the time of the ancients. Great tables were drawn up showing the relations of all parts to one another and especially of all parts to the whole. Such a table was called a "canon".
>
> "One sketch by Leonardo represents a head. In it one can see that the part above the forehead is equal to the breadth of eyes and to the distance from nose to mouth. Likewise we find the same distance from eye to point of nose and from mouth to chin. Moreover, we see that the shorter distance is contained twice in the space from mouth to chin and three times in the forehead...."

(2). Bringing to class drawings, pictures, models, and photographs. At this stage some of the pupils especially interested in drawing may wish to bring in drawings showing proportion. Collections of pictures, models, photographs, etc. may also be made.

(3). Reading; (4). Photographing; (5). Presenting historical findings. Individual assignments may involve reading, drawing, photography and oral presentation of historical findings.

(6). Choosing widths of tucks and hems. For the girls, the problem becomes one such as choosing widths of tucks and hems and distances between them.

1. George Wolff, The Mathematical Collection, The National Council of Teachers of Mathematics, The Fifth Yearbook. (1930), p. 229.

beauty and proportion being linked together has been suggested by Wolff.[1] He says:

> "The theory of proportion affords a very good example of practical use according to the methodical principles of "concentration". as proportions were used by artists for drawings of men and women in the days of the Renaissance when man was regarded as the measure of all, and when the chief aim of artists was to model the human body in an ideal form. We know that Leonardo da Vinci (1452-1519) diligently studied proportion in order to create beautiful bodies. These proportions of the human body were analyzed in the time of the ancients. Great tables were drawn up showing the relations of all parts to one another and especially of all parts to the whole. Such a table was called a "canon".
>
> "One sketch by Leonardo represents a head. In it one can see that the part above the forehead is equal to the breadth of eyes and to the distance from nose to mouth. Likewise we find the same distance from eye to point of nose and from mouth to chin. Moreover, we see that the shorter distance is contained twice in the space from mouth to chin and three times in the forehead...."

(2). Bringing to class drawings, pictures, models, and photographs. At this stage some of the pupils especially interested in drawing may wish to bring in drawings showing proportion. Collections of pictures, models, photographs, etc. may also be made.

(3). Reading; (4). Photographing; (5). Presenting historical findings. Individual assignments may involve reading, drawing, photography and oral presentation of historical findings.

(6). Choosing widths of tucks and hems. For the girls, the problem becomes one such as choosing widths of tucks and hems and distances between them.

---

1. George Wolff, The Mathematical Collection, The National Council of Teachers of Mathematics, The Fifth Yearbook. (1930), p. 229.

(7). Learning the golden ratio. The algebraic method of discovering the so-called "Divine Proportion" or golden section may be taught at this time.

When this introductory work has been accomplished in proportion one may next move on to actual work with proportion.

2. The Demonstration.

a). The approach to work on demonstrations.

(1). Defining "corresponding" sides. Before considering complex figures in proportion propositions it is well to have the pupil get clearly in mind the meaning of the word "corresponding". Schnell[1] advises putting in an entire unit in finding corresponding parts of geometrical figures so that that particular part of the work will not bother in the solution of the various problems.

These parts may be located by noting the sides opposite equal angles as corresponding sides. Instead of lines simply being placed in ratio at first the ratio may read somewhat as follows:

$$\frac{\text{Longer leg Tri. 1}}{\text{Longer leg Tri. 2}} = \frac{\text{Shorter leg Tri. 1}}{\text{Shorter leg Tri. 2}} = \frac{\text{hyp. Tri. 1}}{\text{hyp. Tri. 2}}$$

in the case of a right triangle, while in the case of a general triangle, the ratios may read:

$$\frac{\text{Longest side Tri. 1}}{\text{Longest side Tri. 2}} = \frac{\text{Shortest side Tri. 1}}{\text{Shortest side Tri. 2}} = \frac{\text{Med. side Tri. 1}}{\text{Med. side Tri. 2}}$$

Considerable activity may be engaged in picking these various

1. Leroy H. Schnell, "An Attempt to Apply the Principles of Progressive Education to the Teaching of Geometry." The Mathematics Teacher, XXVI, (March, 1933) pp. 163-175.

(7). Learning the golden ratio. The algebraic method of discovering the so-called "Divine proportion" or golden section may be taught at this time.

When this introductory work has been accomplished in proportion one may next move on to actual work with proportion.

2. The Demonstration.

a). The approach to work on demonstrations.

(1). Defining "corresponding" sides. Before considering complex figures in proportion propositions it is well to have the pupil get clearly in mind the meaning of the word "corresponding". Sonnell[1] advises putting in an entire unit in finding corresponding parts of geometrical figures so that that particular part of the work will not bother in the solution of the various problems.

These parts may be located by noting the sides opposite equal angles as corresponding sides. Instead of lines simply being placed in ratio at first the ratio may read somewhat as follows:

$$\frac{\text{Longer leg Tri. 1}}{\text{Longer leg Tri. 2}} = \frac{\text{Shorter leg Tri. 1}}{\text{Shorter leg Tri. 2}} = \frac{\text{hyp. Tri. 1}}{\text{hyp. Tri. 2}}$$

In the case of a right triangle, while in the case of a general triangle, the ratios may read:

$$\frac{\text{Longest side Tri. 1}}{\text{Longest side Tri. 2}} = \frac{\text{Shortest side Tri. 1}}{\text{Shortest side Tri. 2}} = \frac{\text{Med. side Tri. 1}}{\text{Med. side Tri. 2}}$$

Considerable activity may be engaged in picking these various

1. Leroy H. Sonnell, "An Attempt to Apply the Principles of Progressive Education to the Teaching of Geometry." The Mathematics Teacher, XXVI, (March, 1933) pp. 163-173.

parts out from simple to imcreasingly complex figures.

(2). Study of ratio by numerical quantities measured by pupils. Emphasis should be on the gradual approach through a study of ratio by means of numerical quantities measured by the pupils. Relation to the concrete activity of the pupil with familiar numerical quantities will help to fix the idea in mind.

(3). Close linking of algebraic manipulation with geometric needs. The various exercises which are required of proportions should be performed only in close connection with the geometry involved. Thus, in proving the law of inversion algebraically, the pupil should be shown the figure in geometry where it is needed and the two processes, algebraic and geometric, should be kept side by side. The same is true in establishing the proportion between the sides of a polygon and the perimeter. When the algebraic proof is given that the sum of the antecedents is to the sum of the consequents as any antecedent is to any consequent, the geometric figure should be immediately drawn, and the picture and use of the proof should be shown at the same time. Thus the pupil sees the reason for the abstract algebraic process.

(4). Congruence as a special case of similarity. Meaning may also be obtained by showing relationship to other parts of geometry. Thus, as mentioned heretofore, congruence is a special case of similarity where the ratio is a 1:1 ratio. Contrast equal corresponding sides of congruent triangles and proportional corresponding sides of similar triangles. Relate this work to the solution of the right triangle in trigonometry.

parts out from simple to increasingly complex figures.

(2). Study of ratio by numerical quantities measured by pupils. Emphasis should be on the gradual approach through a study of ratio by means of numerical quantities measured by the pupils. Relation to the concrete activity of the pupil with familiar numerical quantities will help to fix the idea in mind.

(3). Close linking of algebraic manipulation with geometric needs. The various exercises which are required of proportions should be performed only in close connection with the geometry involved. Thus, in proving the law of inversion algebraically, the pupil should be shown the figure in geometry where it is needed and the two processes, algebraic and geometric, should be kept side by side. The same is true in establishing the proportion between the sides of a polygon and the perimeter. When the algebraic proof is given that the sum of the antecedents is to the sum of the consequents as any antecedent is to any consequent, the geometric figure should be immediately drawn, and the picture and use of the proof should be shown at the same time. Thus the pupil sees the reason for the abstract algebraic process.

(4). Congruence as a special case of similarity. Meaning may also be obtained by showing relationship to other parts of geometry. Thus, as mentioned heretofore, congruence is a special case of similarity where the ratio is a 1:1 ratio. Contrast equal corresponding sides of congruent triangles and proportional corresponding sides of similar triangles. Relate this work to the solution of the right triangle in trigonometry.

Schultze[1] devotes considerable space in showing the relationship between the various propositions on similarity. It may be noted that when the words "mean proportional" are used that there is similarity involved. Also in the case where the product of two line segments equal the product of two other line segments. Schultze[2] calls attention also to the fact that proportion is another method by which lines may be proved equal.

b). Value of proportion.

(1). A method of proving lines equal. In case three quantities of a proportion are equal to three quantities in another proportion, the fourth must be equal. Thus, if a:x as b:c and a:x as d:c, "b" and "d" must be equal. Constant reference to arithmetical quantities and to the fact that a ratio is a fraction brings the pupil back to the familiar. Give each letter a value and write in the numerical values in case of difficulty by the pupil.

(2). Trigonometry built on it. Relation to trigonometric tables may be made by the fact that these are written in terms of decimals, but the decimals really have a denominator of one and are therefore fractions. They are fractions, the ratio of the two sides involved being called a function such as sine or cosine. Whole numbers are also fractions with the number one as the denominator. Thus weights and heights take on meanings as ratio. To say that a person weighs 125 lbs. means that he is 125 times a one-pound unit of weight; the same idea holds in

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools, pp. 197-206.

2. Arthur Schultze, loc. cit.

Schultze[1] devotes considerable space in showing the relationship between the various propositions on similarity. It may be noted that when the words "mean proportional" are used that there is similarity involved. Also in the case where the product of two line segments equal the product of two other line segments. Schultze[2] calls attention also to the fact that proportion is another method by which lines may be proved equal.

b). Value of proportion.

(1). A method of proving lines equal. In case three quantities of a proportion are equal to three quantities in another proportion, the fourth must be equal. Thus, if a:x as b:c and a:x as d:c, "b" and "d" must be equal. Constant reference to arithmetical quantities and to the fact that a ratio is a fraction brings the pupil back to the familiar. Give each letter a value and write in the numerical values in case of difficulty by the pupil.

(2). Trigonometry built on it. Relation to trigonometric tables may be made by the fact that these are written in terms of decimals, but the decimals really have a denominator of one and are therefore fractions. They are fractions, the ratio of the two sides involved being called a function such as sine or cosine. Whole numbers are also fractions with the number one as the denominator. Thus weights and heights take on meanings as ratio. To say that a person weighs 125 lbs. means that he is 125 times a one-pound unit of weight; the same idea holds in

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools, pp. 197-208.

2. Arthur Schultze, loc. cit.

lengths.

(3). Time-saver in measurements. The use of proportion may gain added emphasis when shown that it may save much time in measurements. Areas of two similar figures are in equal ratio to the squares of any two corresponding lines, and their volumes are in the same ratio to the cubes of any two corresponding sides or altitudes. When it is shown that the area or the volume of the smaller of the two objects may be found, together with a certain line in the smaller and the corresponding line in the larger figure, which will thus give the area or volume of the larger object, the pupil is instantly interested in the labor saving involved. In fact, constant reference to the shorthand of various subjects is always interesting. Notation made simpler always proves not only interesting but a valuable aid to clarity. For instance, instead of triangle ABC being similar to triangle DEF, $\triangle I \sim \triangle II$. In the same way a single letter for a line in the triangle instead of two letters aids in simplicity. Reference to volume leads, of course, into solid geometry which has a beneficial effect in showing the interrelationship of the subjects.

c). Proportion theorems.

(1). Hints for procedure. Motivation is always obtained also by a clear idea of the goal, the birds-eye viewpoint. This may be not only stated in verbal language but may be expressed graphically perhaps as follows:

In order to cross the stream you have to use certain stones

lengths.

(3). Time-saver in measurements. The use of proportion may gain added emphasis when shown that it may save much time in measurements. Areas of two similar figures are in equal ratio to the squares of any two corresponding lines, and their volumes are in the same ratio to the cubes of any two corresponding sides or altitudes. When it is shown that the area or the volume of the smaller of the two objects may be found, together with a certain line in the smaller and the corresponding line in the larger figure, which will thus give the area or volume of the larger object, the pupil is instantly interested in the labor saving involved. In fact, constant reference to the shorthand of various subjects is always interesting. Notation made simpler always proves not only interesting but a valuable aid to clarity. For instance, instead of triangle ABC being similar to triangle DEF, $\triangle I \sim \triangle II$. In the same way a single letter for a line in the triangle instead of two letters aids in simplicity. Reference to volume leads, of course, into solid geometry which has a beneficial effect in showing the interrelationship of the subjects.

c). Proportion theorems.

(1). Hints for procedure. Motivation is always obtained also by a clear idea of the goal, the bird-eye viewpoint. This may be not only stated in verbal language but may be expressed graphically perhaps as follows:

In order to cross the stream you have to use certain stones

as steps. Given: Your starting point. To prove four lines proportional. Step 1. Try to find a pair of triangles each having two of these lines as sides. Mark these lines.

Step 2. Prove the triangles similar.

a. Angles equal.

b. Two sides in proportion and the included angle equal.

Step 3. If necessary, transform the proportion by the various algebraic procedures of alternation, inversion, etc. For instance, take the proposition:

The product of the segments of one of two intersecting chords equals the product of the segments of the other.

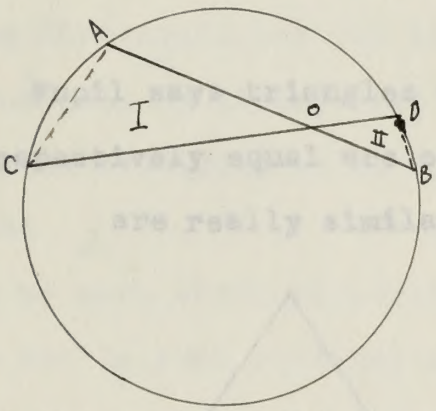
In the figure below:

Product is true if.....(AO)(OB)=(CO)(OD).

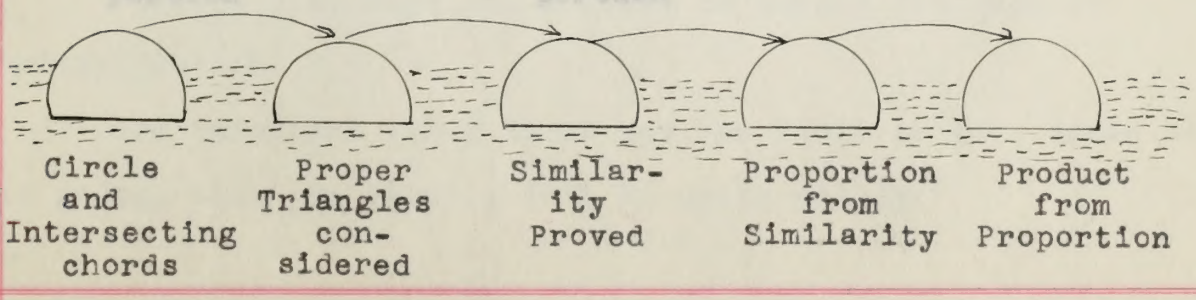
Proportion is true if.....$\frac{AO}{OC} = \frac{OD}{OB}$

Triangles are similar if.....$\triangle I \sim \triangle II$ $\angle s$ =

Proper triangles are considered. $\frac{\triangle AOC + \triangle DOB}{\triangle I + \triangle II}$



(2). <u>Graphic representation</u>. The graphic representation may be as follows: Hopping across the stream from stone to stone.



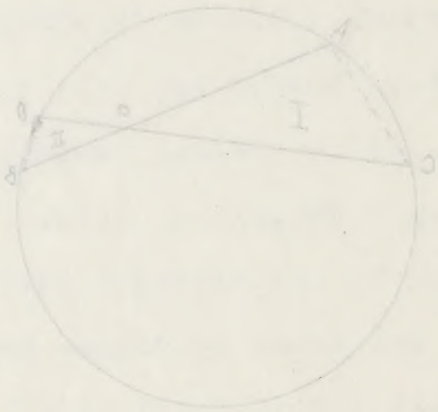
as steps. Given: Your starting point. To prove four lines

proportional. Step 1. Try to find a pair of triangles each
having two of these lines as sides. Mark these lines.
Step 2. Prove the triangles similar.
a. Angles equal.
b. Two sides in proportion and the
included angle equal.
Step 3. If necessary transform the proportion
by the various algebraic procedures of alternation,
inversion, etc. For instance, take the proposition:

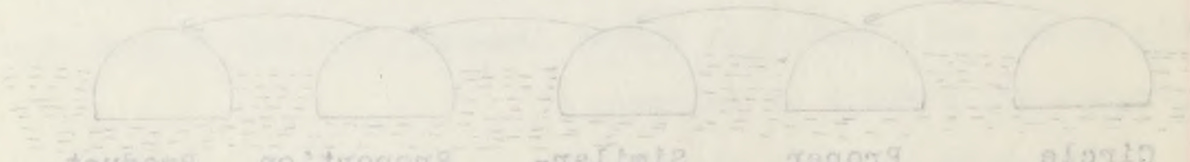
The product of the segments of one of two intersecting
chords equals the product of the segments of the other.

In the figure below:

Product is true if.....(AO)(OB)=(CO)(OD).
Proportion is true if.....$\frac{AO}{OC}=\frac{OD}{OB}$
Triangles are similar if.....$\triangle I \sim \triangle II$ ∠ =
Proper triangles are considered. $\triangle AOC + \triangle DOB$
$\triangle I + \triangle II$



(S). <u>Graphic representation</u>. The graphic representation may
be as follows: Hopping across the stream from stone to stone.

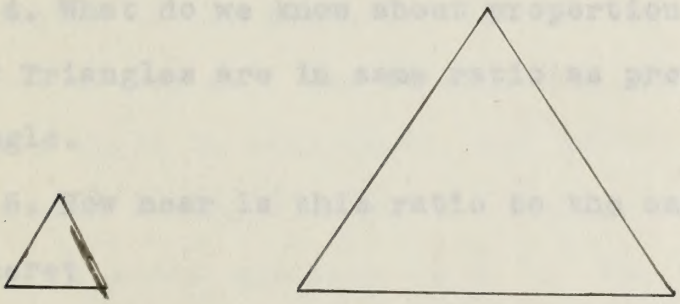


(3). Indentation of steps. The mountain graph idea may be used or the index idea, thus:

Triangles similar if sides in proportion; since
Angles are equal by hypothesis, then the
Task is to prove the sides proportional.
Since angles equal, product of sides in proportion.
Reduce the product to a single side.

(4). Cure for a common error. Sonner or later in the class someone will give the reason for congruency as equal angles in each triangle. The cure may be made by the following drawing at the board representing ice cream portions in which you tell the pupil that he may have the left-hand one and must not grumble since the angles in it are equal to the angles in the right-hand one. Smiles come from the class.

Pupil says triangles with angles respectively equal are congruent. They are really similar.



Pupil's portion Teacher's portion

(3). Indentation of steps. The mountain graph idea may be used or the index idea, thus:

Triangles similar if sides in proportion; since
Angles are equal by hypotheses, then the
Task is to prove the sides proportional.
Since angles equal, product of sides in proportion.
Reduce the product to a single side.

(4). Cure for a common error. Sooner or later in the class someone will give the reason for congruency as equal angles in each triangle. The cure may be made by the following drawing at the board representing ice cream portions in which you tell the pupil that he may have the left-hand one and must not grumble since the angles in it are equal to the angles in the right-hand one. Smiles come from the class.

Pupil says triangles with angles
respectively equal are congruent. They
are really similar.

Teacher's portion

Pupil's portion

d). The test.

(1). Significant questions. Both orally and by written work, the important points to be kept in mind may be stressed. This may be over a number of periods, obeying the law of distributed and diminishing repetitions. A completion test takes but a short time and serves as an extra repetition and supplies a motive for keeping in mind the necessary facts. Part of the period is now left for the inductive developments of the new proposition. A frequent review of anchor points helps.

Thus in developing the proposition that:

"Triangles are similar if they are equiangular", the following questions may be asked:

1. What are the conditions for similarity?

2. What condition is given? What condition to prove?

3. How can the condition to be proved be met, if sides are in proportion?

4. What do we know about proportion if the angles are equal? Triangles are in same ratio as product of sides including the angle.

5. How near is this ratio to the one desired? How can you get there?

On board draw graph of stream. Hint that algebra is used.

Someone discovers the process to change division to multiplication.

The important thing is to get the pupil to know where he is going, then the details fill in quite easily.

d). The test.

(1). Significant questions. Both orally and by written work the important points to be kept in mind may be stressed. This may be over a number of periods, obeying the law of distributed and diminishing repetitions. A completion test takes but a short time and serves as an extra repetition and supplies a motive for keeping in mind the necessary facts. Part of the period is now left for the inductive developments of the new proposition. A frequent review of anchor points helps.

Thus in developing the proposition that:

"Triangles are similar if they are equiangular", the following questions may be asked:

1. What are the conditions for similarity?

2. What condition is given? What condition to prove?

3. How can the condition to be proved be met, if sides are in proportion?

4. What do we know about proportion if the angles are equal? Triangles are in same ratio as product of sides including the angle.

5. How near is this ratio to the one desired? How can you get there?

On board draw graph of stream. Hint that algebra is used. Someone discovers the process to change division to multiplication.

The important thing is to get the pupil to know where he is going, then the details fill in quite easily.

3. The applied problem.

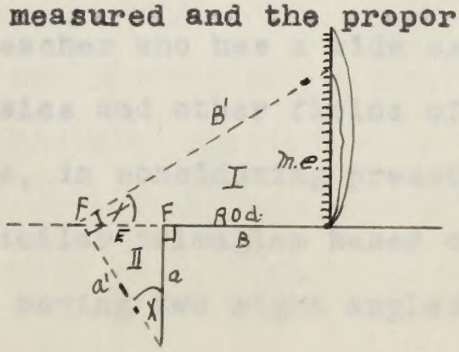
a). Applications of proportion; similar triangles.

(1). Expansion of rod in physics; two kinds of apparatus. The general idea may be presented that all magnification and reducing involves the idea of proportion. Interest may be increased by borrowing the microscope from the biological department and the telescope from the astronomical department, and showing magnification. From this idea it is an easy step to the reading of the expansion of a rod in the heat experiment in physics, based on similar triangles, or in another type of apparatus, based on an enlarged reading from the circular measure. The figures for each of these applications appear on the following page. It pays to take time to explain them to the pupils.

In the apparatus below for magnifying the expansion of a rod by heat, as the vertical rod "a" moves back through distance "e" because of the expansion of rod, the pointer moves through a distance of "m. e." The triangles I and II are similar, having an acute angle in each equal, and having right angles. Angle "x" in each triangle is equal as sides are perpendicular each to each. The sides opposite angle "x" are the actual, and magnified expansion. "a" and "b" are measured and the proportion reads:

$$\frac{a}{b} = \frac{e}{m.e.}$$

"a", "b" and "m.e." are measured and "e" computed.



3. The applied problem.

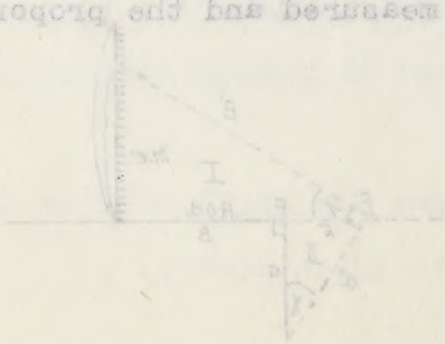
a). Applications of proportion; similar triangles.

(1). Expansion of rod in physics; two kinds of apparatus.

The general idea may be presented that all magnification and reducing involves the idea of proportion. Interest may be increased by borrowing the microscope from the biological department and the telescope from the astronomical department, and showing magnification. From this idea it is an easy step to the reading of the expansion of a rod in the best experiment in physics, based on similar triangles, or in another type of apparatus, based on an enlarged reading from the circular measure. The figures for each of these applications appear on the following page. It pays to take time to explain them to the pupils.

In the apparatus below for magnifying the expansion of a rod by heat, as the vertical rod "a" moves back through distance "e" because of the expansion of rod, the pointer moves through a distance of "m. e." The triangles I and II are similar, having an acute angle in each equal, and having right angles. Angle "x" in each triangle is equal as sides are perpendicular each to each. The sides opposite angle "x" are the actual, and magnified expansion. "a" and "b" are measured and the proportion reads:

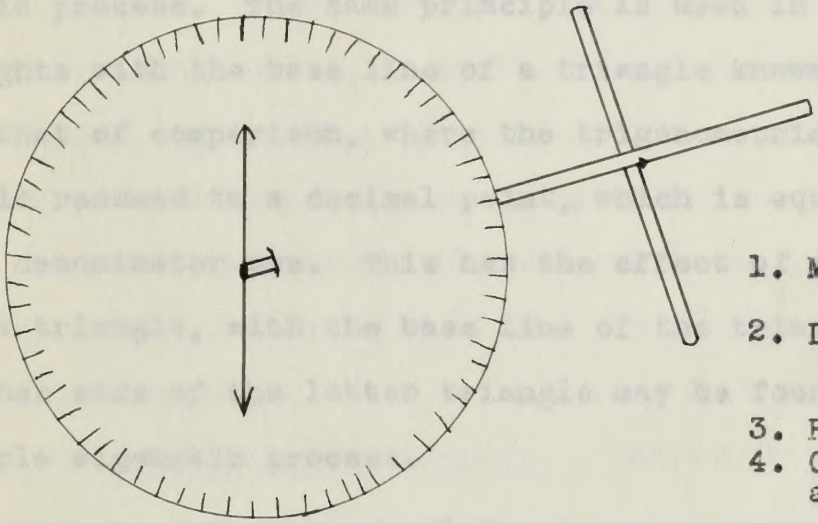
$$\frac{a}{b} = \frac{e}{m.e.}$$



"a", "b" and "m.e." are measured and "e" computed.

In the circular expansion apparatus, as the rod rolls over the small rod holding the needle, each revolution of the small rod is shown by the needle rotation, and the dial to which it points. The proportion reads:

$$\frac{1\ \text{circum. of rod}}{2\ \text{circum. described by needle on dial.}} = \frac{4\ \text{actual expansion}}{3\ \text{magnified expansion on dial.}}$$



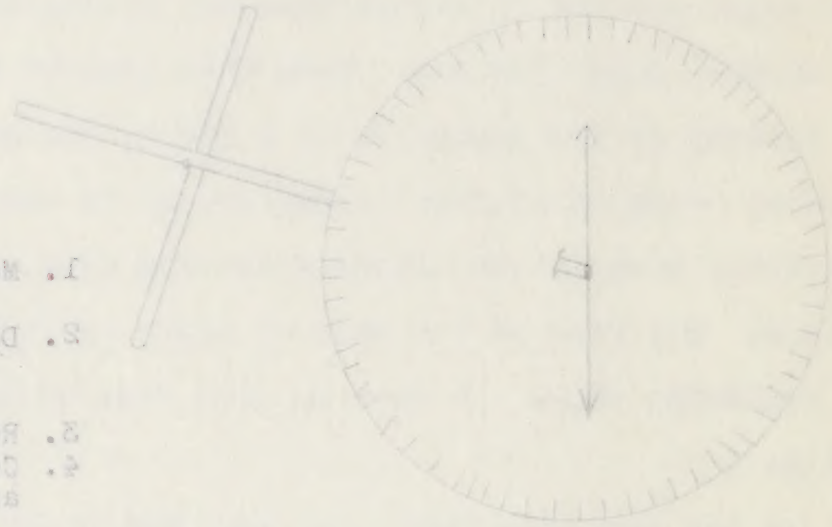
1. Measured by micrometer.
2. Distance computed from diameter of dial.
3. Read from dial.
4. Computed from 1, 2, and 3.

This practical application to physics shows the need and importance of proportion and its interrelation with other subjects of the curriculum.

(2). <u>Pressure on the inclined plane</u>. The applied problem will suggest the value of the interrelationship with other subjects within the school curriculum and also with life outside the school. Here is where the teacher who has a wide experience and an adequate knowledge of physics and other fields of interest is at an advantage. For instance, in considering pressure on an inclined plane, use is made of similar triangles based on triangles which are equiangular, having two right angles, and

In the circular expansion apparatus, as the rod rolls over the small rod holding the needle, each revolution of the small rod is shown by the needle rotation, and the dial to which it points. The proportion reads:

$$\frac{\text{circum. of rod}}{\text{circum. described by needle on dial.}} = \frac{\text{actual expansion}}{\text{magnified expansion on dial.}}$$

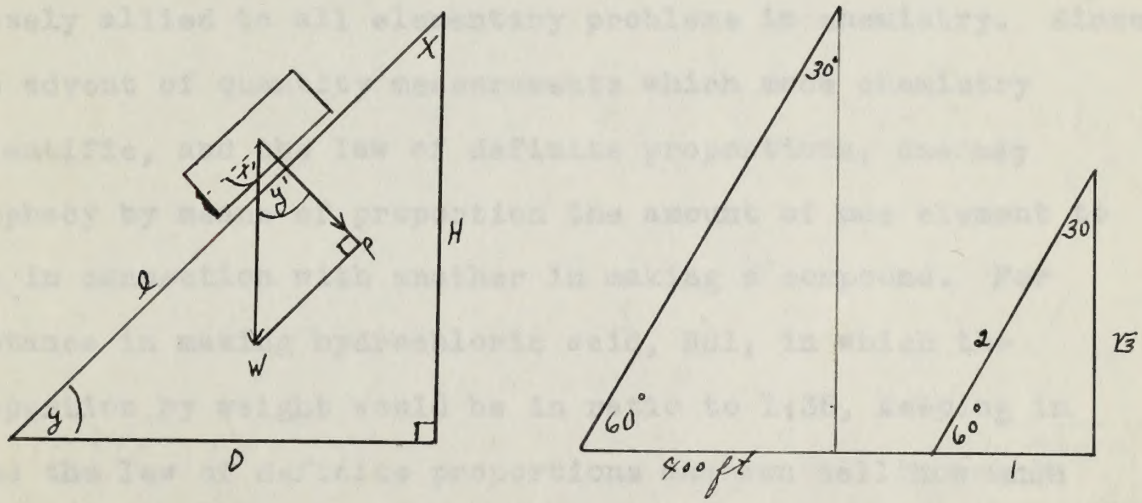


1. Measured by micrometer.
2. Distance computed from diameter of dial.
3. Read from dial.
4. Computed from 1, 2, and 3.

This practical application to physics shows the need and importance of proportion and its interrelation with other subjects of the curriculum.

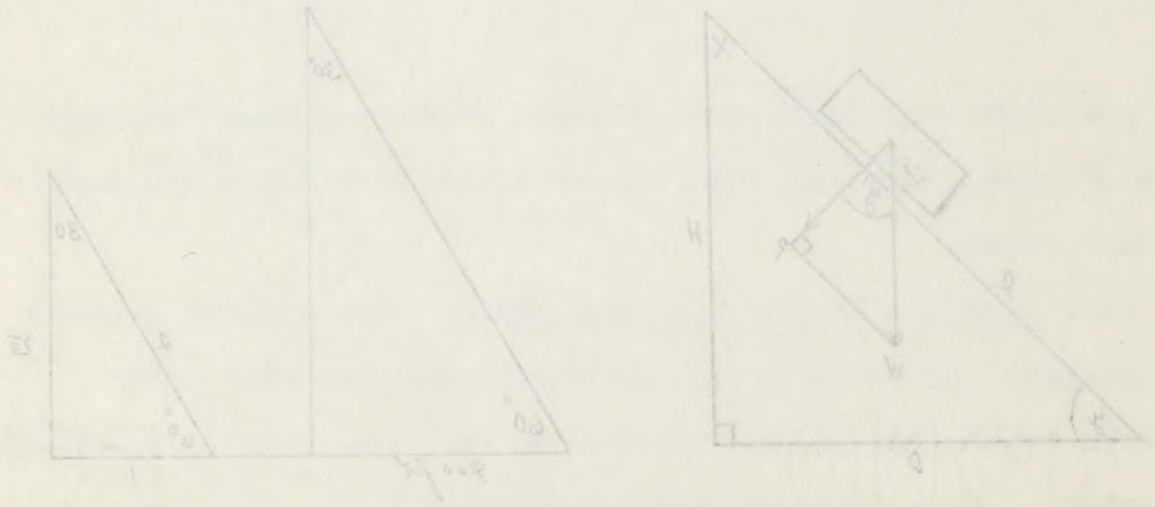
(2). Pressure on the inclined plane. The applied problem will suggest the value of the interrelationship with other subjects within the school curriculum and also with life outside the school. Here is where the teacher who has a wide experience and an adequate knowledge of physics and other fields of interest is at an advantage. For instance, in considering pressure on an inclined plane, use is made of similar triangles based on triangles which are equiangular, having two right angles, and

two acute angles equal. The acute angles are equal because the sides are perpendicular each to each. The problem of pressure on an inclined plane is solved by a measure of the length of the actual plane and by the weight of the body involved. This always gives three quantities of the proportion, and the unknown which is difficult to measure, is calculated by a simple algebraic process. The same principle is used in solving for heights with the base line of a triangle known. The principle is that of comparison, where the trigonometric tables give the ratio reduced to a decimal point, which is equivalent to making the denominator one. This has the effect of giving two sides of a triangle, with the base line of the triangle known, so that either side of the latter triangle may be found by the same simple algebraic process.



Preceding this paragraph is the drawing of the included plane with the block, with the triangle represented by a line drawn to length as per the unit of lengths for a specified number of pounds, perhaps ten pounds to an inch or whatever

two acute angles equal. The acute angles are equal because the sides are perpendicular each to each. The problem of pressure on an inclined plane is solved by a measure of the length of the actual plane and by the weight of the body involved. This always gives three quantities of the proportion, and the unknown which is difficult to measure, is calculated by a simple algebraic process. The same principle is used in solving for height with the base line of a triangle known. The principle is that of comparison, where the trigonometric tables give the ratio reduced to a decimal point, which is equivalent to making the denominator one. This has the effect of giving two sides of a triangle, with the base line of the triangle known, so that either side of the latter triangle may be found by the same simple algebraic process.



Preceding this paragraph is the drawing of the included plane with the block, with the triangle represented by a line drawn to length as per the unit of lengths for a specified number of pounds, perhaps ten pounds to an inch or whatever

suitable scale might be chosen. The "l", "d", and "h" of the triangle can be measured, the "w" can be weighed and the "p" can be solved from the simple algebraic equation l:w::d:p. These are corresponding sides of similar triangles, and the triangles are proved similar by observing that "x^1" which is equal to x (corresponding angles of parallel lines) is the complement of "y" and that "x^1" is the complement of "y^1", and since complements of equal angles are equal, "y" and "y^1" are equal, which makes an acute angle in each right triangle equal, thus making the respective triangles similar. The writer has found that it has increased the interest to develop this idea in the geometry class. It gives meaning to similarity in triangles.

(3). <u>Chemical calculations.</u> The subject of proportion is closely allied to all elementary problems in chemistry. Since the advent of quantity measurements which made chemistry scientific, and the law of definite proportions, one may prophecy by means of proportion the amount of one element to use in connection with another in making a compound. For instance in making hydrochloric acid, HCl, in which the proportion by weight would be in ratio to 1:35, keeping in mind the law of definite proportions one can tell how much chlorine would be required to make a ton of the acid, the ratio of hydrogen to chlorine used still being 1:35. The proportion would read:

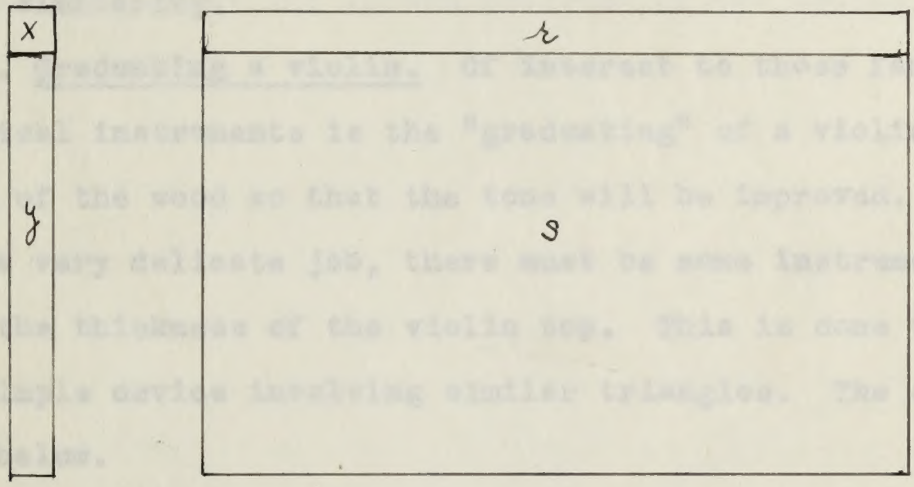
$$\frac{\text{Amount of chlorine in a molecule of HCl}}{\text{The molecule of HCl}} = \frac{\text{Wt. of chlorine in a ton of HCl}}{\text{Ton of HCl}}$$

suitable scale might be chosen. The "l", "d", and "h" of the triangle can be measured, the "w" can be weighed and the "p" can be solved from the simple algebraic equation l:w::d:p. These are corresponding sides of similar triangles, and the triangles are proved similar by observing that "x_l" which is equal to x (corresponding angles of parallel lines) is the complement of "y" and that "x_l" is the complement of "y_l", and since complements of equal angles are equal, "y" and "y_l" are equal, which makes an acute angle in each right triangle equal, thus making the respective triangles similar. The writer has found that it has increased the interest to develop this idea in the geometry class. It gives meaning to similarity in triangles.

(3). <u>Chemical calculations.</u> The subject of proportion is closely allied to all elementary problems in chemistry. Since the advent of quantity measurements which made chemistry scientific, and the law of definite proportions, one may prophecy by means of proportion the amount of one element to use in connection with another in making a compound. For instance in making hydrochloric acid, HCl, in which the proportion by weight would be in ratio to 1:35, keeping in mind the law of definite proportions one can tell how much chlorine would be required to make a ton of the acid, the ratio of hydrogen to chlorine used still being 1:35. The proportion would read:

$$\frac{\text{Amount of chlorine in a molecule of HCl}}{\text{The molecule of HCl}} = \frac{\text{Wt. of chlorine in a ton of HCl}}{\text{Ton of HCl}}$$

This can also be illustrated graphically by means of diagrams somewhat as follows:



$$\frac{x}{y} = \frac{r}{s}$$

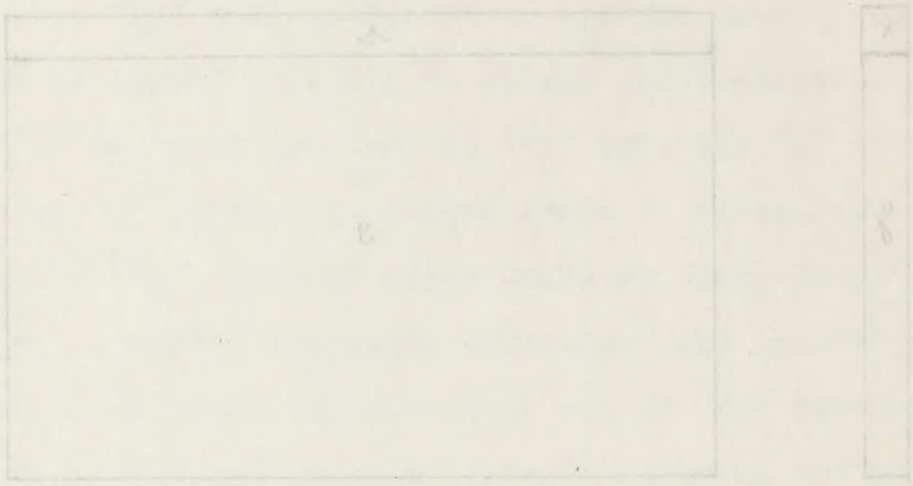
$$\frac{\text{Amount hydrogen in molecule of HCl}}{\text{Molecule of HCl}} = \frac{\text{Amount hydrogen in a ton of HCl}}{\text{Ton of HCl}}$$

The important thing is to show the pupils the usefulness of proportion in chemistry and where it links up with that subject.

To be sure, some of the applications in physics and chemistry of proportion are more algebraic than geometric in nature, yet it does no harm and enhances the value of this unit in the mind of the pupil if attention is called to both the geometric and the algebraic applications.

(4). <u>Mixing medicines in hospital training</u>. Girls who enter hospital training report that the study of proportion is one of the most important of all their high school mathematics as they are called upon to dilute certain medicines at times which involves a knowledge of proportion.

This can also be illustrated graphically by means of diagrams somewhat as follows:



$$\frac{\text{Amount hydrogen in molecule of HCl}}{\text{Molecule of HCl}} = \frac{\text{Amount hydrogen in a ton of HCl}}{\text{Ton of HCl}}$$

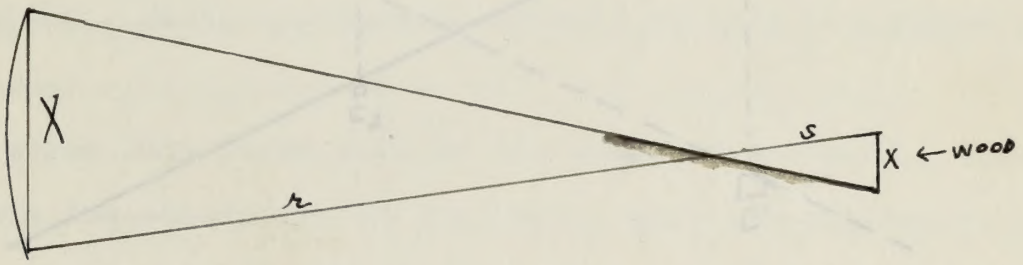
The important thing is to show the pupils the usefulness of proportion in chemistry and where it links up with that subject.

To be sure, some of the applications in physics and chemistry of proportion are more algebraic than geometric in nature, yet it does no harm and enhances the value of this unit in the mind of the pupil if attention is called to both the geometric and the algebraic applications.

(4). Making medicines in hospital training. Girls who enter hospital training report that the study of proportion is one of the most important of all their high school mathematics as they are called upon to dilute certain medicines at times which involves a knowledge of proportion.

(5)**Trade school** . Trade schools, also, report that the similarity and proportion, as applied to triangles, has direct practical application in various trades. Of course all molding involves similarity.

(6). Graduating a violin. Of interest to those familiar with musical instruments is the "graduating" of a violin, the thinning of the wood so that the tone will be improved. Since this is a very delicate job, there must be some instrument to magnify the thickness of the violin top. This is done through a very simple device involving similar triangles. The drawing appears below.



Between the ends of the small line "x" is the wood to be thinned: this measurement is magnified at "X". The magnification is in ratio of "r" to "s".

$$\frac{r}{s} = \frac{X}{x}$$

The three quantities r, s and X may be measured, and thus x computed. The two triangles involved are similar because they are equiangular; the two end lines being parallel.

(7). The teeter board. The teeter board may be given as a good example of the similar triangle situation. As the

(5) Trade school. Trade schools, also, report that the similarity and proportion, as applied to triangles, has direct practical application in various trades. Of course all molding involves similarity.

(6). Graduating a violin. Of interest to those familiar with musical instruments is the "graduating" of a violin, the thinning of the wood so that the tone will be improved. Since this is a very delicate job, there must be some instrument to magnify the thickness of the violin top. This is done through a very simple device involving similar triangles. The drawing appears below.

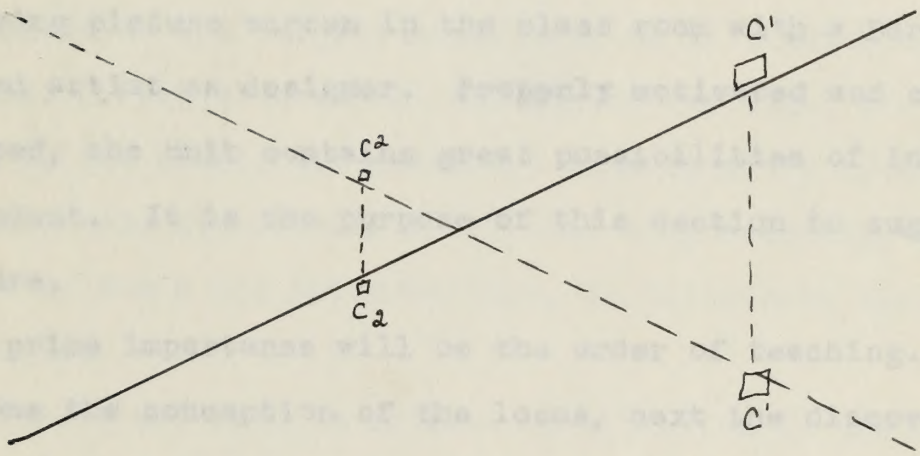
Between the ends of the small line "x" is the wood to be thinned: this measurement is magnified at "X". The magnification is in ratio of "r" to "s".

$$\frac{X}{x} = \frac{r}{s}$$

The three quantities r, s and X may be measured, and thus x computed. The two triangles involved are similar because they are equiangular; the two end lines being parallel.

(7). The teeter board. The teeter board may be given as a good example of the similar triangle situation. As the

smaller child sits nearer the fulcrum, thus making a side of the triangle smaller, the larger child describes a larger arc, sitting farther away from the fulcrum. In the diagram below, the dotted line shows the second position of the teeter board.



smaller child sits nearer the fulcrum, thus making a side of the triangle smaller, the larger child describes a larger arc, sitting farther away from the fulcrum. In the diagram below, the dotted line shows the second position of the teeter board.

D. Loci.

Even though the subject has its difficulties, there is probably no more interesting a unit in geometry than that of the study of the loci. Here, and here along, geometry is truly in motion and dynamic. One might well recall Bert Green's cartoons in the moving pictures as one receives the idea of a moving point making a line on a surface, although Green obeys the laws of humor, whereas in geometry, geometrical laws must be followed. One wonders about the future possibilities of this unit on the moving picture screen in the class room with a Bert-Green-talented artist as designer. Properly motivated and carefully developed, the unit contains great possibilities of interesting the student. It is the purpose of this section to suggest such procedure.

Of prime importance will be the order of teaching. First will come the conception of the locus, next the discovery, and finally the proof. First consider then, the development of the conception of the locus.

1. Conception. Hassler and Smith[1] call attention to the above three parts in the teaching of the locus, and adds that much of the difficulty with this topic can be attributed to an impatient desire to get to the third part before the first two parts are thoroughly considered.

---

1. J. O. Hassler, and R. R. Smith, The Teaching of Secondary Mathematics, pp. 360-365.

D. Loci.

Even though the subject has its difficulties, there is probably no more interesting a unit in geometry than that of the study of the loci. Here, and here alone, geometry is truly in motion and dynamic. One might well recall Bert Green's cartoons in the moving pictures as one receives the idea of a moving point making a line on a surface, although Green obeys the laws of humor, whereas in geometry, geometrical laws must be followed.

One wonders about the future possibilities of this unit on the moving picture screen in the class room with a Bert-Green-talented artist as designer. Properly motivated and carefully developed, the unit contains great possibilities of interesting the student. It is the purpose of this section to suggest such procedure.

Of prime importance will be the order of teaching. First will come the conception of the locus, next the discovery, and finally the proof. First consider then, the development of the conception of the locus.

1. Conception. Hassler and Smith[1] call attention to the above three parts in the teaching of the locus, and adds that much of the difficulty with this topic can be attributed to an impatient desire to get to the third part before the first two parts are thoroughly considered.

---

1. J. O. Hassler, and R. R. Smith, The Teaching of Secondary Mathematics, pp. 380-386.

J. Sibli[1] shows that there is real difficulty in teaching the locus by noting that the questions on loci, which have not been especially hard, have caused the college entrance candidates more worry than any other questions, and have been treated more unsuccessfully.

a). General discussion.

(1). Neighborhood examples. Since loci problems and activities are constantly in the life of the pupil, it is not difficult to get familiar examples to illustrate the same. The introduction to the subject might well be to discuss the neighborhood loci situations: the dog on the line; the cow grazing in the field; all machinery in motion; or levers. There are innumerable situations, which will vary according to the locality in which the children live, to illustrate loci applications.

Interest may also be aroused by attention to the fly wheel in the local pumping station. What is the path of a particular point on the wheel at various places? What is the speed of each particular spot at the center, and at the extreme outer edge? This may involve the reason for the fly wheel and lead a little away from discussion of loci, but will be none the less valuable by linking to familiar and moving things. Which tieing is more advantageous for the dog? At the end of a rope attached to an iron bar, or at the end of a rope attached to a line around which the rope is looped, so that the dog may run from one end to the other end of the line? What is the locus in each case?

1. Recent developments in the Teaching of Geometry, p. 223.

J. Sibli[1] shows that there is real difficulty in teaching the locus by noting that the questions on loci, which have not been especially hard, have caused the college entrance candidates more worry than any other questions, and have been treated more unsuccessfully.

a). General discussion.

(I). Neighborhood examples. Since local problems and activities are constantly in the life of the pupil, it is not difficult to get familiar examples to illustrate the same. The introduction to the subject might well be to discuss the neighborhood local situations: the dog on the line; the cow grazing in the field; all machinery in motion; or levers. There are innumerable situations, which will vary according to the locality in which the children live, to illustrate loci applications.

Interest may also be aroused by attention to the fly wheel in the local pumping station. What is the path of a particular point on the wheel at various places? What is the speed of each particular spot at the center, and at the extreme outer edge? This may involve the reason for the fly wheel and lead a little away from discussion of loci, but will be none the less valuable by linking to familiar and moving things. Which tieing is more advantageous for the dog? At the end of a rope attached to an iron bar, or at the end of a rope attached to a line around which the rope is looped, so that the dog may run from one end to the other end of the line? What is the locus in each case?

1. Recent developments in the Teaching of Geometry, p. 223.

(2). Tracing. Another place for the tactile approach is the learning of the locus exercises through finger tracing. With classroom material such as pen, pencil, eraser, pointer, the door, string and chalk, the paths of moving points may be illustrated. For example with pencil and string, or with chalk and string, the path of a point always at a given distance from the center, the end of the pencil held stationary, is a circle. Similarly the bottom point of a door describes a circle as it opens. Finger tracing may also be used to make real to the pupil the congruency of angles and triangles. Wooden models may be shown the class and the pupil may superimpose one triangle over the other and trace with his fingers the lines and angles which are equal. The point to bear in mind is that, in the locus picture, certain parts are to be still, and others moving.

All these exercises offer the additional sensory channels for learning, thus giving the pupil a greater chance of success, not only because of the addition of a sensory approach but because the tactile sensory approach may be the approach which is best for the pupil. The importance of learning through the hands has long been recognized. Maria Montessori[1] noted it in her book in teaching the children to write; it is one of her teaching activities which is generally accepted by educators as being a real contribution.

---

1. The Montessori Method, p. 276.

(2). Tracing. Another place for the tactile approach is the learning of the locus exercises through finger tracing. With classroom material such as pen, pencil, eraser, pointer, the door, string and chalk, the paths of moving points may be illustrated. For example with pencil and string, or with chalk and string, the path of a point always at a given distance from the center, the end of the pencil held stationary, is a circle. Similarly the bottom point of a door describes a circle as it opens. Finger tracing may also be used to make real to the pupil the congruency of angles and triangles. Wooden models may be shown the class and the pupil may superimpose one triangle over the other and trace with his fingers the lines and angles which are equal. The point to bear in mind is that, in the locus picture, certain parts are to be still, and others moving.

All these exercises offer the additional sensory channels for learning, thus giving the pupil a greater chance of success, not only because of the addition of a sensory approach but because the tactile sensory approach may be the approach which is best for the pupil. The importance of learning through the hands has long been recognized. Maria Montessori[1] noted it in her book in teaching the children to write: it is one of her teaching activities which is generally accepted by educators as being a real contribution.

---

1. The Montessori Method, p. 276.

(3). Classroom objects. The writer has used the swinging door, the opening book, and the disc eraser revolving around a fixed line as concrete examples in the classroom.

(4). Loci and the movies. One writer[1] notes a likeness to taking pictures for the movies; there must be many pictures a short time apart and some of the objects in each picture are in action and others are still. Thus, in the movies the houses are still, and the hero active.

(5). Loci and the algebraic graph. In geometry the line may be still, and the path or locus of points at a certain distance from the line will be moving. The teacher can dot these points as they are determined. These may be linked up to the graph in algebra which is really a locus of answers as the conditions vary according to the horizontal and vertical axes.

(6). Loci and solid geometry. Mary C. Wilt[2] notes that this conception of loci can also be strengthened by entering the realm of solid geometry in the finger tracings of the locus in room problems. Thus, the problem of finding the path of a point equidistant from two parallel lines becomes, in solid geometry,

---

1. Hassler and Smith, loc. cit.
2. Teaching Plane and Solid Geometry Simultaneously, The National Council of Teachers of Mathematics, The Fifth Yearbook, 1932. p. 66.

(3). Classroom objects. The writer has used the swinging door, the opening book, and the disc eraser revolving around a fixed line as concrete examples in the classroom.

(4). Loci and the movies. One writer[1] notes a likeness to taking pictures for the movies; there must be many pictures a short time apart and some of the objects in each picture are in action and others are still. Thus, in the movies the houses are still, and the hero active.

(5). Loci and the algebraic graph. In geometry the line may be still, and the path or locus of points at a certain distance from the line will be moving. The teacher can dot these points as they are determined. These may be linked up to the graph in algebra which is really a locus of answers as the conditions vary according to the horizontal and vertical axes.

(6). Loci and solid geometry. Mary C. Wilt[2] notes that this conception of loci can also be strengthened by entering the realm of solid geometry in the finger tracings of the locus in room problems. Thus, the problem of finding the path of a point equidistant from two parallel lines becomes, in solid geometry,

---

1. Hassler and Smith, loc. cit.

2. Teaching Plane and Solid Geometry Simultaneously, The National Council of Teachers of Mathematics, The Fifth Yearbook, 1932. p. 66.

the problem of finding the path of a point equidistant from two parallel planes. Similarly, the bisector of the angle becomes, in solid geometry, the plane which bisects the dihedral angle.

All the introduction in this loci unit may be concrete not only in the classroom constructions, discussions, and demonstrations, but also in the assignments which may consist of constructing loci, with no attempt at proof.

b). Assignment. Urging the pupils to bring in their own loci problems, with constructions, helps to get interest. In the teeter board, both riders follow the path of an arc of a circle either descending or ascending. All weight pans follow a similar path. The lever with the weight and force again both follow circular motions. Draw all these suggestions out on the board at the order of the class. The teacher suggests the situation, and the pupils direct the solution.

(1). Objects and materials along lines of individual interests. Here is a chance for individual interests. The boy interested in aviation may bring in examples and make drawings, illustrating the locus of points traced by the end of the airplane propellor, both when the airplane is at rest, or in motion in a straight line. The boy interested in mechanics may bring in loci examples in connection with wheels of automobiles and trains. Any example suitable to the interest, up-to-date, and familiar, will be worth while.

Here, as in other places in geometry, an orderly setting down of the work will go far toward making success. The unit is

the problem of finding the path of a point equidistant from two parallel planes. Similarly, the bisector of the angle becomes, in solid geometry, the plane which bisects the dihedral angle.

All the introduction in this local unit may be concrete not only in the classroom constructions, discussions, and demonstrations, but also in the assignments which may consist of constructing local, with no attempt at proof.

b). Assignment. Urging the pupils to bring in their own local problems, with constructions, helps to get interest. In the teeter board, both riders follow the path of an arc of a circle either descending or ascending. All weight pans follow a similar path. The lever with the weight and force again both follow circular motions. Draw all these suggestions out on the board at the order of the class. The teacher suggests the situation, and the pupils direct the solution.

(1). Objects and materials along lines of individual interests. Here is a chance for individual interests. The boy interested in aviation may bring in examples and make drawings, illustrating the loops of points traced by the end of the airplane propellor, both when the airplane is at rest, or in motion in a straight line. The boy interested in mechanics may bring in local examples in connection with wheels of automobiles and trains. Any example suitable to the interest, up-to-date, and familiar, will be worth while.

Here, as in other places in geometry, an orderly setting down of the work will go far toward making success. The unit is

a fascinating one, especially for those interested in art work. Designs in colors may be encouraged and hung up on the bulletin board or placed on exhibition. The pupil's originality and ingenuity may have free play. For those who are book-minded, more difficult special problems may be found, and this group may present these problems to the class as their contribution to the unit.

(2). Loci and mechanical devices. Manual training boys may rig up and bring in apparatus which will illustrate the locus, such as the lever, balance, wheel, etc. The physics laboratory will generally have a model engine with an eccentric which may be brought down and studied. Automobile gears, milk separators, almost any apparatus which may be put in motion, may be contributed to the class. Thus, the subject of loci may become a live, understandable, and liked topic.

2. Discovery.

a). Loci and design work. Provision for individual differences may be in the providing of more complicated loci drawings involving an increasing number of conditions to satisfy. Thus, the requirements may be, to be a certain distance from two parallel lines and to lie on the perpendicular bisector of another line (equidistant from two points) and to be a certain distance from another line. Teach the pupils to think about only one difficulty at a time in construction.

b). Loci and chemistry. Linking to the sciences may be accomplished through analogy by noting the various tests. Iodine

a fascinating one, especially for those interested in art work. Designs in colors may be encouraged and hung up on the bulletin board or placed on exhibition. The pupil's originality and ingenuity may have free play. For those who are book-minded, more difficult special problems may be found, and this group may present these problems to the class as their contribution to the unit.

(2). Tool and mechanical devices. Manual training boys may rig up and bring in apparatus which will illustrate the tools, such as the lever, balance, wheel, etc. The physics laboratory will generally have a model engine with an eccentric which may be brought down and studied. Automobile gears, milk separators, almost any apparatus which may be put in motion, may be contributed to the class. Thus, the subject of tool may become a live, understandable, and liked topic.

2. Discovery.

a). Tool and design work. Provision for individual differences may be in the providing of more complicated tool drawings involving an increasing number of conditions to satisfy. Thus, the requirements may be, to be a certain distance from two parallel lines and to lie on the perpendicular bisector of another line (equidistant from two points) and to be a certain distance from another line. Teach the pupils to think about only one difficulty at a time in construction.

b). Tool and chemistry. Linking to the sciences may be accomplished through analogy by noting the various tests. Iodine

is a test for starch used in biology or chemistry. All starches react to iodine; all points are good points. No other substance except starches react to give a purple with iodine; no other point is a good point. Various tests may be cited and related to give as illustrations.

c). Significant questions on applied loci. William Betz[1] suggests the following questions:

"Where are all the houses that are located one mile from your school building?
What is the path described by the door knob as you open the door? By one corner of the cover of your book as you open it?
When you open the drawer of a desk, what is the path of one of its moving corners? Of its edges?
When a street car travels along the tracks, what is the path of the center of one of the wheels?"

Arthur Schultze suggests:[2]

"What is the locus of one end of a stretched string, if the other end is fixed, and both ends are on the surface of a table? If only the moving end is on the table?"

Hassler and Smith[3] suggest:

"What is the path of the tip of a revolving airplane propeller; engine going; plane stationary; plane going forward in a straight line?
What is the path of the spoke of a car wheel moving along a straight level track?
What is the surface over which a cow grazes?"

William Betz[4]:

"A study of machinery in action can be made to suggest

1. The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook (1933). p. 114.

2. The Teaching of Mathematics in Secondary Schools. pp. 218-224

3. Hassler and Smith, loc cit.

4. William Betz, loc, cit.

is a test for starch used in biology or chemistry. All starches react to iodine; all points are good points. No other substance except starches react to give a purple with iodine; no other point is a good point. Various tests may be cited and related to give as illustrations.

c). Significant questions on applied loci. William Betz[1] suggests the following questions:

"Where are all the houses that are located one mile from your school building?
What is the path described by the door knob as you open the door? By one corner of the cover of your book as you open it?
When you open the drawer of a desk, what is the path of one of its moving corners? Of its edges?
When a street car travels along the tracks, what is the path of the center of one of the wheels?"

Arthur Schultze suggests:[2]

"What is the locus of one end of a stretched string, if the other end is fixed, and both ends are on the surface of a table? If only the moving end is on the table?"

Hassler and Smith[3] suggest:

"What is the path of the tip of a revolving airplane propeller; engine going; plane stationary; plane going forward in a straight line?
What is the path of the spoke of a car wheel moving along a straight level track?
What is the surface over which a cow grazes?"

William Betz[4]:

"A study of machinery in action can be made to suggest

1. The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook (1935), p. 114.

2. The Teaching of Mathematics in Secondary Schools, pp. 218-224

3. Hassler and Smith, loc cit.

4. William Betz, loc. cit.

"functional ideas to any pupil of average intelligence. For more ambitious minds, it is perfectly possible to include a few of the higher plane curves. Thus, as a wheel travels along the ground, what is the path described by a point on its rim?...."[1]

3. The proof.

a). Hand work precedes the abstract. Teach the pupil to think of only one difficulty at a time in the locus proof. Separate the problem into the various parts as suggested heretofore, making the pupils aware that this separation can be made. A practice and discussion on these parts are helpful. Pupils will vary in their power to visualize, but all will be helped by the constructions.

Needless to say the formal definitions and the abstract part should follow the concrete and hand work. Learning through muscular movement, through finger tracing, through eye stimuli vivified by colors, or various dotted lines, should all precede the proof of loci.

Perhaps the more practical applications may come, from the discovery by the pupil, that graphs may be traced by obeying the rules, and that various figures such as circles, parabolas, hyperbolas, and ellipses are derived from paths of moving points, which represent loci problems. The aim is to make the subject understandable, interesting, and of practical use to the pupil. Newspapers sometimes in the Sunday supplements have dots to be joined together to make pictures. These may be suggested to the pupils.

1. William Betz, loc. cit.

"Functional ideas to any pupil of average intelligence. For more ambitious minds, it is perfectly possible to include a few of the higher plane curves. Thus, as a wheel travels along the ground, what is the path described by a point on its rim?...."[1]

3. The proof.

a). Hand work precedes the abstract. Teach the pupil to think of only one difficulty at a time in the locus proof. Separate the problem into the various parts as suggested heretofore, making the pupils aware that this separation can be made. A practice and discussion on these parts are helpful. Pupils will vary in their power to visualize, but all will be helped by the constructions.

Needless to say the formal definitions and the abstract part should follow the concrete and hand work. Learning through muscular movement, through finger tracing, through eye stimuli vivified by colors, or various dotted lines, should all precede the proof of loci.

Perhaps the more practical applications may come, from the discovery by the pupil, that graphs may be traced by obeying the rules, and that various figures such as circles, parabolas, hyperbolas, and ellipses are derived from paths of moving points, which represent loci problems. The aim is to make the subject understandable, interesting, and of practical use to the pupil. Newspapers sometimes in the Sunday supplements have dots to be joined together to make pictures. These may be suggested to the pupils.

1. William Betz, loc. cit.

b). Organization of work important. The problem of teaching loci is not a problem of approach, but rather a problem of disentangling, and making clear the various skeins bound up in the locus statement. Schultze[1] calls attention to this when he lines up the work as follows:

I. Theorem.	II. Converse.
Every point in the line is equidistant.	Every point that is equidistant lies in the line.
III. Opposite.	IV. Converse of Opposite.
Every point without is unequally distant.	Every point unequally distant is without the line.

A clear idea by the teacher of the many pieces of work to be done by the pupil in proving the locus will make it possible to show the pupil the task ahead. The writer agrees, however, that the proof should not come at the start.

Phrasing the statements in language which is readily understandable is also of value. The following is used by the writer:

Every point is a good point. Every point satisfies the given condition.	Every coin in the pocket is a penny.
No other point is a good point. No other point outside satisfies the condition.	No other coin outside is a penny.

Thus there are three gradations of statement in an effort to get clarity. The effort is to make the points objective, make them pennies; make them petals of flowers strewn along the path; make

1. Arthur Schultze, loc. cit.

b). Organization of work important. The problem of teaching loci is not a problem of approach, but rather a problem of disentangling, and making clear the various axioms bound up in the locus statement. Schultze[1] calls attention to this when he lines up the work as follows:

I. Theorem.	II. Converse.
Every point in the line is equidistant.	Every point that is equidistant lies in the line.
III. Opposite.	**IV. Converse of Opposite.**
Every point without is unequally distant.	Every point unequally distant is without the line.

A clear idea by the teacher of the many pieces of work to be done by the pupil in proving the locus will make it possible to show the pupil the task ahead. The writer agrees, however, that the proof should not come at the start.

Phrasing the statements in language which is readily understandable is also of value. The following is used by the writer:

Every point is a good point. Every point satisfies the given condition.	Every coin in the pocket is a penny.
No other point is a good point. No other point outside satisfies the condition.	No other coin outside is a penny.

Thus there are three gradations of statement in an effort to get clarity. The effort is to make the points objective, make them pennies; make them petals of flowers strewn along the path; make

1. Arthur Schultze, loc. cit.

them anything which is interesting to the pupil and understandable to him.

4. General suggestions. Following is a topical list of suggestions by the various writers:

Conception.

1 An early beginning is necessary in study of loci.
2 a. Definition in algebra.
3 b. Graphs as loci.
c. Begin early in geometry and keep it up all through.
d. Reverse order of proofs to conform with the algebraic treatment of graphs.[1]

Construction.

4 Study locus in correct order:
5 a. Conception of.
b. Discovery of.
c. Proof of.[2]

Pupil should first get the conception of a locus as the path of a moving point, thus focussing on the path rather than the point.[3]

First have pupil familiar with the language exercises and theorems through manipulation with rulers and compasses and with accompanying discussions. Final procedure is the proof.[4]

First plot the locus point by point, until a wise guess of the path can be made. Refer to moving pictures where many pictures show slight changes. In locus problems many points have only slight distance between. In each situation are certain fixed things: in moving pictures certain houses and trees are stationery; in loci problems, certain direction lines and points are stationery. The fixed things should appear first.[5]

1. George H. Selleck, Loco phobia--Its causes and cure. The Mathematics Teacher. (November, 1929). pp. 382-384. (XXII)

2. Hassler and Smith, op. cit. p. 360.

3. Hassler and Smith, loc cit.

4. Hassler and Smith, op. cit. p. 362.

5. Hassler and Smith, loc. cit.

them anything which is interesting to the pupil and understandable to him.

4. General suggestions. Following is a topical list of suggestions by the various writers:

Conception. An early beginning is necessary in study of loci.
a. Definition in algebra.
b. Graphs as loci.
c. Begin early in geometry and keep it up all through.
d. Reverse order of proofs to conform with the algebraic treatment of graphs.[1]

Construction.

Study locus in correct order:
a. Conception of.
b. Discovery of.
c. Proof of.[2]

Pupil should first get the conception of a locus as the path of a moving point, thus focussing on the path rather than the point.[3]

First have pupil familiar with the language exercises and theorems through manipulation with rulers and compasses and with accompanying discussions. Final procedure is the proof.[4]

First plot the locus point by point, until a wise guess of the path can be made. Refer to moving pictures where many pictures show slight change. In locus problems many points have only slight distance between. In each situation are certain fixed things: in moving pictures certain houses and trees are stationary; in loci problems, certain direction lines and points are stationary. The fixed things should appear first.[5]

---

1. George H. Sallock, Loco phobia--Its causes and cure. The Mathematics Teacher. (November, 1929). pp. 382-384. (XXII)

2. Hassler and Smith, op. cit. p. 380.

3. Hassler and Smith, loc cit.

4. Hassler and Smith, op. cit. p. 382.

5. Hassler and Smith, loc. cit.

Use constructions with concrete objects to aid in visualizing.

a. Path of circumference of disc eraser whose center follows a rectangular box lid.

b. Path of a pencil attached to a stick, the end of which constantly touches the circumference of a circle.

c. Path of the vertex of the carpenter's square, a point of one side remaining stationary, and the square revolving about it.[1]

Construction.

1.
2.
3.
4.

Proof.

5.
6.
7.
8.

Require definitions only when actually needed.[2]

Distinguish the various loci by means of colored chalk or dotted lines.[3]

Connect with solid geometry.[4]

Do not ask for too many things at a time. Have pupil prove only the predicate. Give the subject with modifiers.[5]

The one-way proof is generally sufficient.[6]

The proving of a locus involves four theorems.[7]

Use numerous original exercises.[8].

---

1. E. B. Woodford, Teaching Locus Concept in Plane Geometry, The Mathematics Teacher. XXVI, (February, 1933). pp. 105-106.

2. Hassler and Smith, op. cit. p. 360.

3. Arthur Schultze, op. cit. p. 225.

4. Arthur Schultze, loc cit.

5. Hassler and Smith, op. cit. p. 361-362.

6. George Selleck, op. cit. 382-384.

7. Arthur Schultze, op. cit. p. 219.

8. Arthur Schultze, loc. cit.

Construction.

1. Use constructions with concrete objects to aid in visualizing.
2. a. Path of circumference of disc-eraser whose center follows a rectangular box lid.
3. b. Path of a pencil attached to a stick, the end of which constantly touches the circumference of a circle.
4. c. Path of the vertex of the carpenter's square, a point of one side remaining stationary, and the square revolving about it.[1]

Proof.

5. Require definitions only when actually needed.[2]
6.
7. Distinguish the various loci by means of colored chalk or dotted lines.[3]
8.

Connect with solid geometry.[4]

Do not ask for too many things at a time. Have pupil prove only the predicate. Give the subject with modifiers.[5]

The one-way proof is generally sufficient.[6]

The proving of a locus involves four theorems.[7]

Use numerous original exercises.[8].

---

1. F. B. Woodford, Teaching Locus Concept in Plane Geometry, The Mathematics Teacher, XXVI, (February, 1933), pp. 105-106.

2. Hassler and Smith, op. cit. p. 360.

3. Arthur Schultze, op. cit. p. 225.

4. Arthur Schultze, loc cit.

5. Hassler and Smith, op. cit. p. 361-362.

6. George Sallack, op. cit. 382-384.

7. Arthur Schultze, op. cit. p. 219.

8. Arthur Schultze, loc. cit.

From the preceding, it will be seen that the same fundamental principles are at work for successful teaching of loci as for any other unit. There must be the correct auditory and visual image--the conception; correct practice, the construction; and the difficulty taught in isolation as exampled in Schultze's lineup of four tasks which must be separately considered. A correct conception means a favorable altitude toward learning; construction work means learning through the eye, ear and hand; while the careful separation of the difficulties call for nicety of organization. With these principles working in proper order, success should come.

From the preceding, it will be seen that the same fundamental principles are at work for successful teaching of lool as for any other unit. There must be the correct auditory and visual image--the conception; correct practice, the construction; and the difficulty taught in isolation as exemplied in Schultze's lineup of four tasks which must be separately considered. A correct conception means a favorable attitude toward learning; construction work means learning through the eye, ear and hand; while the careful separation of the difficulties call for nicety of organization. With these principles working in proper order, success should come.

PART IV.

GENERAL METHODS.

A. Organization and Continuity.

1. Importance of Grouping. That writers have sensed the importance of the teacher calling attention to grouping of propositions is evident from the quotations given below:

a). Viewpoints of writers.

"This article does oppose memorization and production of single detached proposition proofs. Should we discard these, we can spend more time in trying to give the pupil some idea of what geometry is and in treating his understanding of the relationships among the propositions...."[1]

"If the great basal theorems are selected and effectively organized into a logical system, considerable reduction can be made in the number of theorems given...."[2]

"After the selection of appropriate material, the next problem in redistribution is its effective organization and control. The essential points to be considered are sequence, grouping, generalizing, and method of development....It is important to group the material in units on the basis of a key proposition or a fundamental concept...."[3]

"Regardless of the text, it is essential that the teacher emphasize the worth and importance of the significant and group related theorems, and generalize closely the related groups in practice."[4]

1. Aaron Horn, Book Propositions in Teaching Geometry, The Mathematics Teacher, (XXV), February, 1932, pp. 76-78.

2. The Reorganization of Mathematics in Secondary Education, A Report of the National Committee on Mathematical Requirements. 1923, p. 34 under the auspices of the Mathematical Association of America, Inc. The Mathematical Association of America, Inc.

3. Gertrude E.Allen, An Experiment in Redistribution of Material for High School Geometry, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930). p. 81.

4. Gertrude E.Allen, Reorganization of Course of Study for Senior High School Mathematics, The National Council of Teachers of Mathematics, The Second Yearbook. (1927). pp. 256-298.

PART IV.

GENERAL METHODS.

A. Organization and Continuity.

1. Importance of Grouping. That writers have sensed the importance of the teacher calling attention to grouping of propositions is evident from the quotations given below:

a). Viewpoints of writers.

"This article does oppose memorization and production of single detached proposition proofs. Should we discard these, we can spend more time in trying to give the pupil some idea of what geometry is and in treating his understanding of the relationships among the propositions...."[1]

"If the great basal theorems are selected and effectively organized into a logical system, considerable reduction can be made in the number of theorems given...."[2]

"After the selection of appropriate material, the next problem in redistribution is its effective organization and control. The essential points to be considered are sequence, grouping, generalizing, and method of development....It is important to group the material in units on the basis of a key proposition or a fundamental concept...."[3]

"Regardless of the text, it is essential that the teacher emphasize the worth and importance of the significant and group related theorems, and generalize closely the related groups in practice."[4]

1. Aaron Horn, Book Propositions in Teaching Geometry, The Mathematics Teacher, (XXV), February, 1932, pp. 76-78.

2. The Reorganization of Mathematics in Secondary Education, A Report of the National Committee on Mathematical Requirements, 1923, p. 54 under the auspices of the Mathematical Association of America, Inc. The Mathematical Association of America, Inc.

3. Gertrude E.Allen, An Experiment in Redistribution of Material for High School Geometry, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), p. 81.

4. Gertrude E.Allen, Reorganization of Course of Study for Senior High School Mathematics, The National Council of Teachers of Mathematics, The Second Yearbook, (1927), pp. 256-258.

Walter H. Blaisdell[1] notes that the function of the mathematics teacher is to unify and correlate the material of the past, present and future. There is danger of the student concentrating on the particular lesson that he will not practise on the relationships between the parts.

Thus have these writers called attention to the importance of organizing around certain principles, giving the concepts certain associations. This attempt involves getting a richer and more varied setting for the central school subjects, and for grouping with the world contacts.

The Committee on Geometry with Professor Ralph Beatley[2], as chairman notes the non-existence of grouping of propositions in text books as follows; "Some text books reveal the logical structure of individual theorems by means of analyses preceding the synthetic proofs. Almost none call attention to logical chains of propositions."

It seems to the author to be of extreme importance to show the pupils how one proposition is necessary to another. The class should know the reason and should be reminded of the relationship between the propositions.

b). Advantages. Not only does the pupil retain the propositions in mind better because of their relationship when they are organized, but the work is also more interesting because the pupil can see the significance of the particular step as it

1. Daily Testing, The Mathematics Teacher, (April, 1933) XXVI, pp. 227-234.

2. Second Report of the Committee of Geometry, Presented to the National Council of Teachers of Mathematics, at its Annual meeting in Minneapolis on February 24, 1933. The Mathematics Teacher, XXVI, (October, 1933). pp. 266-271.

Walter H. Blaisdell[1] notes that the function of the mathematics teacher is to unify and correlate the material of the past, present and future. There is danger of the student concentrating on the particular lesson that he will not practise on the relationships between the parts.

Thus have these writers called attention to the importance of organizing around certain principles, giving the concepts certain associations. This attempt involves getting a richer and more varied setting for the central school subjects, and for grouping with the world contacts.

The Committee on Geometry with Professor Ralph Beatley[2], as chairman notes the non-existence of grouping of propositions in text books as follows: "Some text books reveal the logical structure of individual theorems by means of analyses preceding the synthetic proofs. Almost none call attention to logical chains of propositions."

It seems to the author to be of extreme importance to show the pupils how one proposition is necessary to another. The class should know the reason and should be reminded of the relationship between the propositions.

b). Advantages. Not only does the pupil retain the propositions in mind better because of their relationship when they are organized, but the work is also more interesting because the pupil can see the significance of the particular step as it

1. Daily Teaching, The Mathematics Teacher, (April, 1933) XXVI, pp. 227-234.

2. Second Report of the Committee of Geometry, Presented to the National Council of Teachers of Mathematics, at its Annual meeting in Minneapolis on February 24, 1933. The Mathematics Teacher, XXVI, (October, 1933), pp. 322-271.

builds into the finished whole. All learning is easier if it is properly phrased together. Music has its phrases. Pupils are encouraged to write these groupings in their note books.

2. Illustrations. It is the purpose of this unit to cite an example of grouping in relation to the area of the quadrilateral and to note an applied problem. A family of quadrilaterals is noted on pages 53 and 54 of the thesis. Further examples involve groupings through double identities.

Following this, the kindred topic of Continuity is considered. Here is noted the opportunities for the use of the principle of continuity together with graphic work showing an illustration of one of these principles at work.

a). The Area of Quadrilaterals. Below is a suggested order of steps to be taken in the consideration of areas. The initial premise, the start, must rest on definition of experiment in measuring. The task is to go from the known to the unknown, from the familiar to the unfamiliar. This is accomplished by making that which is unfamiliar familiar. The new case links up under the old. The procedure is from the rectangle to two right triangles. From two right triangles are made two general triangles, and from these latter are made either the parallelogram or the trapezoid, according as the triangles are congruent or not. The tabulation and diagrams follow:

builds into the finished whole. All learning is easier if it is properly phrased together. Music has its phrases. Pupils are encouraged to write these groupings in their note books.

2. Illustrations. It is the purpose of this unit to cite an example of grouping in relation to the area of the quadrilateral and to note an applied problem. A family of quadrilaterals is noted on pages 53 and 54 of the thesis. Further examples involve groupings through double identities.

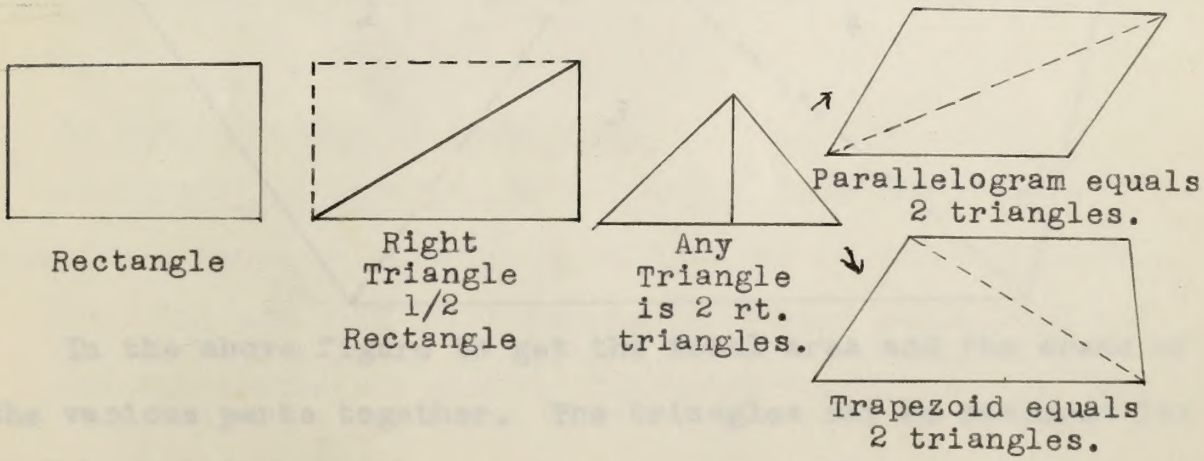
Following this, the kindred topic of Continuity is considered. Here is noted the opportunities for the use of the principle of continuity together with graphic work showing an illustration of one of these principles at work.

a). The Area of Quadrilaterals. Below is a suggested order of steps to be taken in the consideration of areas. The initial premise, the start, must rest on definition of experiment in measuring. The task is to go from the known to the unknown, from the familiar to the unfamiliar. This is accomplished by making that which is unfamiliar familiar. The new case links up under the old. The procedure is from the rectangle to two right triangles. From two right triangles are made two general triangles, and from these latter are made either the parallelogram or the trapezoid, according as the triangles are congruent or not. The tabulation and diagrams follow:

KNOWN	to	UNKNOWN
Area of rectangle		Area of right triangle equals known area, by making the right triangle one-half of the rectangle, the known.
Area of right triangle		Area of any triangle may be figured, because any triangle may be divided into two right triangles, the known.
Area of two equal triangles. Special case of the following.		Area of parallelogram may be computed because the figure consists of two equal triangles, known.
Area of two triangles, not equal. General case.		Area of trapezoid may be found by noting that the trapezoid consists of two triangles not necessarily equal, known.

A third method of representing the relationship between areas may be graphically as follows:

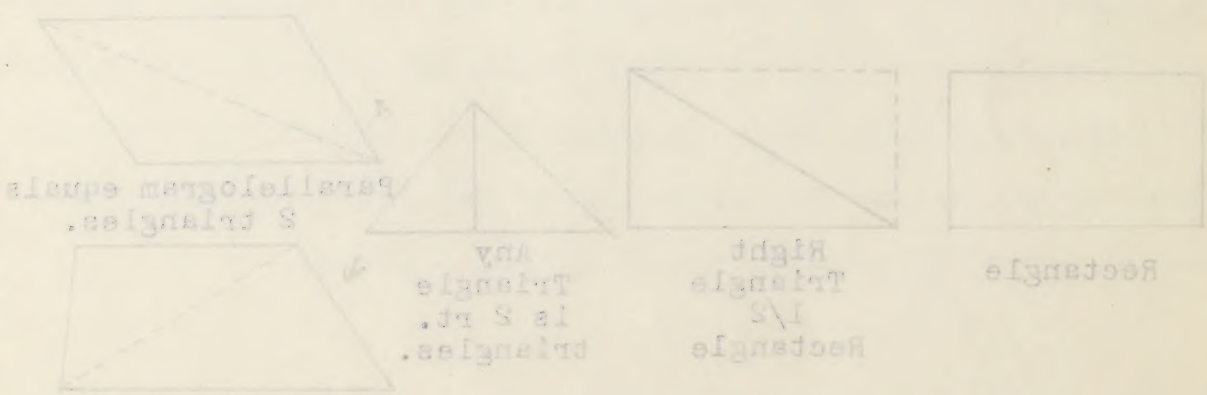
(1). Graphic representation. Below is the graphic representation.



KNOWN	to	UNKNOWN
Area of rectangle		Area of right triangle equals known area, by making the right triangle one-half of the rectangle, the known.
Area of right triangle		Area of any triangle may be figured, because any triangle may be divided into two right triangles, the known.
Area of two equal triangles. Special case of the following.		Area of parallelogram may be computed because the figure consists of two equal triangles, known.
Area of two triangles, not equal. General case.		Area of trapezoid may be found by noting that the trapezoid consists of two triangles not necessarily equal, known.

A third method of representing the relationship between areas may be graphically as follows:

(I). Graphic representation. Below is the graphic representation.



(2). <u>The applied problem</u>. Finally the applied problem will serve to retain the interest and link with practical procedure.

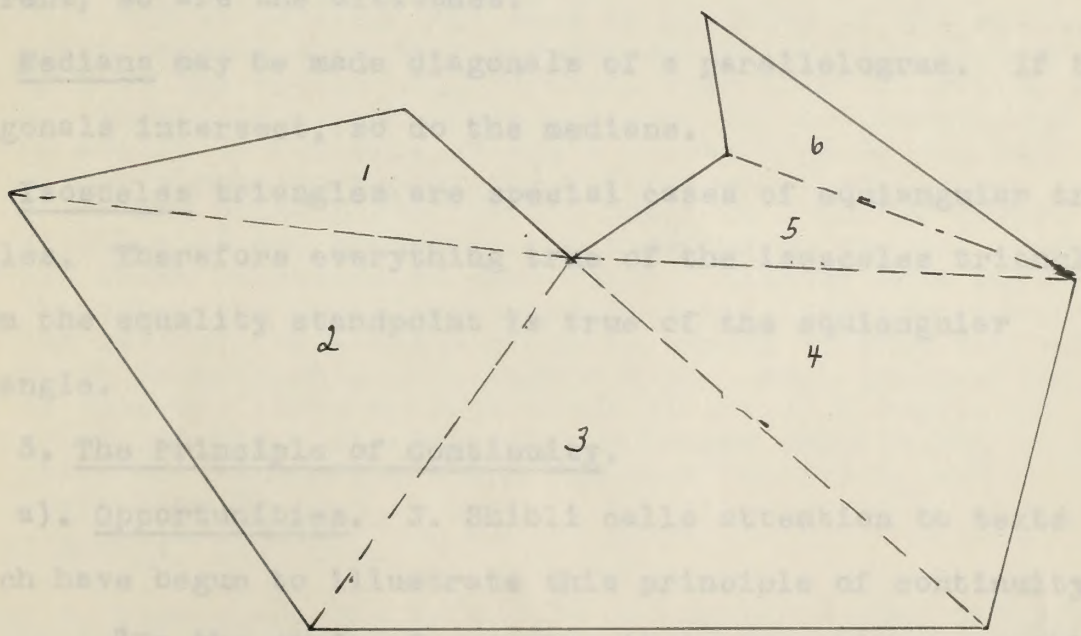
How would you find the area of the baseball field, the hay field, the tennis court, or the common?

To do this you must know the areas of certain figures in plane geometry, namely:

A. Rectangle.
B. Triangle.
C. Parallelogram.
D. Trapezoid.

The method would be to divide the various areas into the above figures.

The work preceding was to help you in finding the areas of these figures and to show you how to get from one figure to another without memorizing the area of the new figure.



In the above figure to get the total area add the areas of the various parts together. The triangles may be measured for base and altitude, thus getting the area. The trapezoid may be

(2). <u>The applied problem</u>. Finally the applied problem will serve to retain the interest and link with practical procedure.

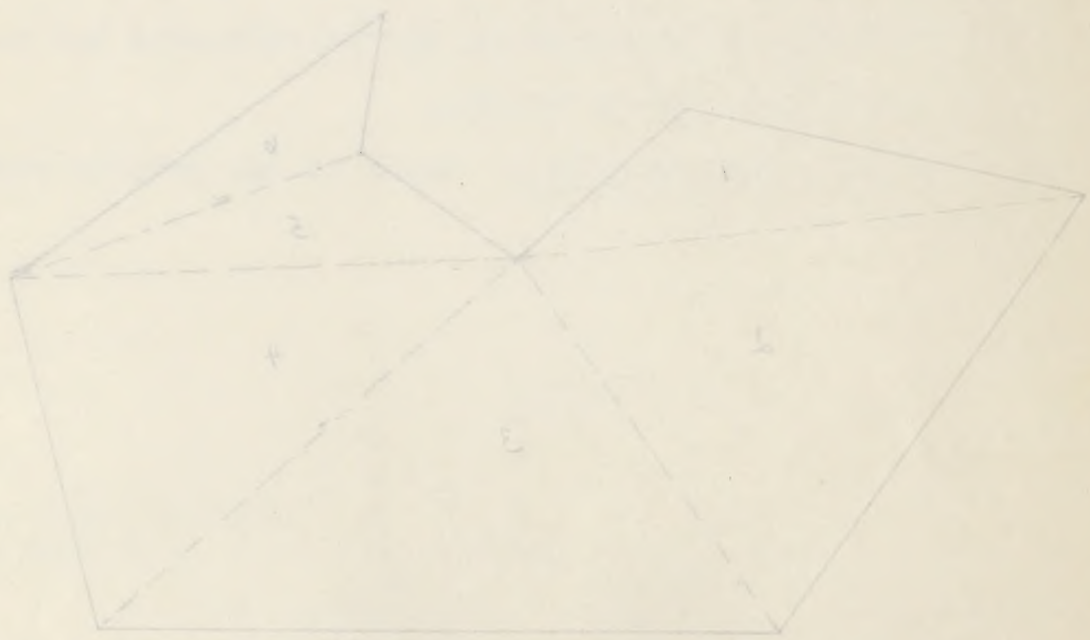
How would you find the area of the baseball field, the hay field, the tennis court, or the common?

To do this you must know the areas of certain figures in plane geometry, namely:

A. Rectangle.
B. Triangle.
C. Parallelogram.
D. Trapezoid.

The method would be to divide the various areas into the above figures.

The work preceding was to help you in finding the areas of these figures and to show you how to get from one figure to another without memorizing the area of the new figure.



In the above figure to get the total area add the areas of the various parts together. The triangles may be measured for base and altitude, thus getting the area. The trapezoid may be

measured for its bases and altitude, thus getting the area. It is further shown that any straight line area may be divided, so that these figures will appear, and by adding the various areas, the total area may be found. This is of practical advantage in field surveys where the field is an odd shape. In other words, any straight line area may now be found. It does not hurt to tell a little in regard to calculus and show that with curved figures the method is to make the triangles so small that any tiny part of a curved outline approaches a straight line.

b). Double identities.

Altitudes may be made perpendicular bisectors of another triangle. Therefore, if the perpendicular bisectors are concurrent, so are the altitudes.

Medians may be made diagonals of a parallelogram. If the diagonals intersect, so do the medians.

Isosceles triangles are special cases of equiangular triangles. Therefore everything true of the isosceles triangle from the equality standpoint is true of the equiangular triangle.

3. The Principle of Continuity.

a). Opportunities. J. Shibli calls attention to texts which have begun to illustrate this principle of continuity.[1]

> "In the study of geometry it is some times possible to consider a group of related theorems as special cases, and to formulate a general statement which includes them all. For example, after studying the various cases of

1. Recent Developments in the Teaching of Geometry. 1932. pp. 227-228.

measured for its bases and altitude, thus getting the area. It is further shown that any straight line area may be divided, so that these figures will appear, and by adding the various areas the total area may be found. This is of practical advantage in field surveys where the field is an odd shape. In other words, any straight line area may now be found. It does not hurt to tell a little in regard to calculus and show that with curved figures the method is to make the triangles so small that any tiny part of a curved outline approaches a straight line.

b). Double identities.

Altitudes may be made perpendicular bisectors of another triangle. Therefore, if the perpendicular bisectors are concurrent, so are the altitudes.

Medians may be made diagonals of a parallelogram. If the diagonals intersect, so do the medians.

Isosceles triangles are special cases of equiangular triangles. Therefore everything true of the isosceles triangle from the equality standpoint is true of the equiangular triangle.

3. The Principle of Continuity.

a). Opportunities. 1. Shibli calls attention to texts which have begun to illustrate this principle of continuity.[1]

"In the study of geometry it is some times possible to consider a group of related theorems as special cases, and to formulate a general statement which includes them all. For example, after studying the various cases of

1. Recent Developments in the Teaching of Geometry. 1932. pp. 227-228.

congruence the pupil is led to discover the general principle that any triangle is uniquely determined if three independent parts are given, except the ambiguous case where the determination is not unique, but double, since two triangles are possible. The Pythagorean Theorem may be regarded as a special case of the general principle concerning the relations of the squares on the sides of a triangle. The square on the side of a triangle is less than, equal to, or greater than the sum of the squares on the other two sides of the triangle, according as the angle opposite the first side is acute, right, or obtuse. If two sides of a triangle are held constant, but the included angle is made to vary continuously from near zero degree to a right angle and then up to 180 degrees, the square on the opposite side will vary continuously without any abrupt changes. Because of this continuous change the principle under consideration has been called the Principle of Continuity.[1] Another interesting generalization combines the group of theorems on the products of the segments of two intersecting lines that cut or touch a circle.[2] The most interesting case of generalization is that of the measure of angles by arcs. The two lines forming the angle may meet inside the circle, on the circle, or outside the circle; they may both be chords, or both secants, or both tangents, or one secant and the other tangent. Schultze suggests the development in the following statement:[3]

'If we consider all arcs as positive if the moving point travels over them counter-clockwise, and as negative if the moving point travels over them clockwise, we may summarize all theorems relating to the measurement of angles as follows: An angle is measured by one half of the algebraic sum of the intercepted arcs.'

'The principle helps the pupil to a better understanding of the proofs of the theorems on special cases and their relations to each other. Without this generalization the pupil may understand the individual proofs and yet fail to see the property common to them all and their relationship to a single unit. When instruction in geometry rises to this level of training pupils in generalization, critics will no longer deny it the power to transfer to other fields of activity, and perfection."[4]

1. J. Shibli, loc. cit.

2. F. P. Hennessey, The Principle of Continuity, The Mathematics Teacher, XXIV (January, 1931). p. 37.

3. Arthur Schultze, The Teaching of Secondary Mathematics, p. 184.

4. J. Shibli, Loc. cit.

congruence the pupil is led to discover the general principle that any triangle is uniquely determined if three independent parts are given, except the ambiguous case where the determination is not unique, but double, since two triangles are possible. The Pythagorean Theorem may be regarded as a special case of the general principle concerning the relations of the squares on the sides of a triangle. The square on the side of a triangle is less than, equal to, or greater than the sum of the squares on the other two sides of the triangle, according as the angle opposite the first side is acute, right, or obtuse. If two sides of a triangle are held constant, but the included angle is made to vary continuously from near zero degree to a right angle and then up to 180 degrees, the square on the opposite side will vary continuously without any abrupt changes. Because of this continuous change the principle under consideration has been called the Principle of Continuity.[1] Another interesting generalization combines the group of theorems on the products of the segments of two intersecting lines that cut or touch a circle.[2] The most interesting case of generalization is that of the measure of angles by arcs. The two lines forming the angle may meet inside the circle, on the circle, or outside the circle; they may both be chords, or both secants, or both tangents, or one secant and the other tangent. Schultze suggests the development in the following statement:[3]

'If we consider all arcs as positive if the moving point travels over them counter-clockwise, and as negative if the moving point travels over them clockwise, we may summarize all theorems relating to the measurement of angles as follows: An angle is measured by one half of the algebraic sum of the intercepted arcs.'

'The principle helps the pupil to a better understanding of the proofs of the theorems on special cases and their relations to each other. Without this generalization the pupil may understand the individual proofs and yet fail to see the property common to them all and their relationship to a single unit. When instruction in geometry rises to this level of training pupils in generalization, critics will no longer deny it the power to transfer to other fields of activity, and perfection."[4]

1. J. Shibli, loc. cit.

2. F. P. Hannsey, The Principle of Continuity, The Mathematics Teacher, XXIV (January, 1931). p. 37.

3. Arthur Schultze, The Teaching of Secondary Mathematics, p. 184.

4. J. Shibli, loc. cit.

(1). The Trapezoid and Triangle. F. P. Hennessey[1] cites the case of the relationship between the trapezoid and the triangle. A line parallel to the base equals one-half the sum of the bases, if it bisects the opposite sides. In the triangle there is no upper base. That is, the base is zero, but the rule still holds.

(2). The Pythagorean Theorem. E.R. Breslich[2] notes that the Pythagorean proposition is a special case of the relationship of one side to the other two, where the projection is zero, and therefore the product "Twice the projection of one side by the other upon that side" disappears, since one factor of the product is zero.

(3). The Product and Mean Proportion Theorems. Based on similar triangles which are proved equiangular. After getting triangles similar, usually by proving them equiangular, all the product propositions and mean proportional propositions, where lines are chords, tangents and secants, are simply examples of similar triangles, where the sides of the triangles are also chords, secants, and tangents. Most of the propositions are solved by means of proving triangles similar through being equiangular, and the angles are generally proved equal through equal arc measurement.

(4). The angle and circle. General case of angle as measured by one-half sum of arcs. Angle at center--arcs equal.

1. The Principles of Continuity, op. cit. pp. 32-40.

2. Demonstrative Geometry and Functional Thinking. The National Council of Teachers of Mathematics, The Third Yearbook, 1928, p. 51

(1). The Trapezoid and Triangle. F. P. Hennessey[1] cites the case of the relationship between the trapezoid and the triangle. A line parallel to the base equals one-half the sum of the bases, if it bisects the opposite sides. In the triangle there is no upper base. That is, the base is zero, but the rule still holds.

(2). The Pythagorean Theorem. H.R. Breslich[2] notes that the Pythagorean proposition is a special case of the relationship of one side to the other two, where the projection is zero, and therefore the product "Twice the projection of one side by the other upon that side" disappears, since one factor of the product is zero.

(3). The Product and Mean Proportion Theorems. Based on similar triangles which are proved equiangular. After getting triangles similar, usually by proving them equiangular, all the product propositions and mean proportional propositions, where lines are chords, tangents and secants, are simply examples of similar triangles, where the sides of the triangles are also chords, secants, and tangents. Most of the propositions are solved by means of proving triangles similar through being equiangular, and the angles are generally proved equal through equal arc measurement.

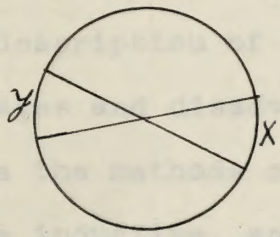
(4). The angle and circle. General case of angle as measured by one-half sum of arcs. Angle at center--arcs equal.

1. The Principles of Continuity, op. cit. pp. 32-40.

2. Demonstrative Geometry and Functional Thinking. The National Council of Teachers of Mathematics, The Third Yearbook, 1928, p. 61

Another opportunity for grouping is found in the angle with the circle, where the measure is always the sum of the intercepted arcs.

(a) Angle $\overset{m}{=}$ 1/2 Sum of arcs--At center--Special case where the arcs are equal so we can say one whole arc.

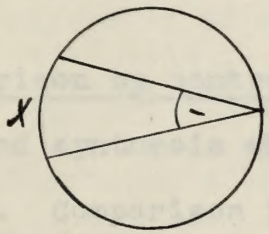


$\widehat{x} = \widehat{y}$

$\angle 1 \overset{m}{=} \frac{1}{2}(\widehat{x}+\widehat{y})$ or $\frac{1}{2}(\widehat{x}+\widehat{x})$

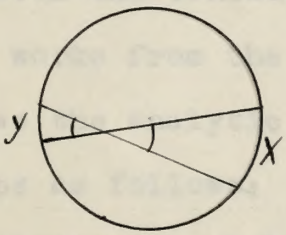
$\angle 1 = \frac{1}{2}(2\widehat{x}) = \widehat{X}$

(b). <u>Angle on circumference--one arc zero.</u> On circumference. Special case where there is no arc on one side, so one arc is zero.



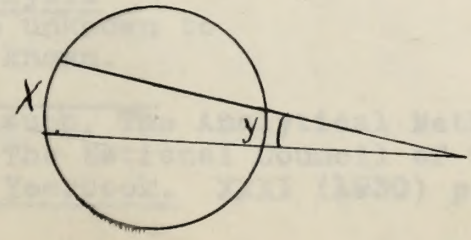
$\angle 1 \overset{m}{=} \widehat{X}$

(c). <u>Angle inside circle--general case.</u> At any other place inside of circle the rule holds as above.



$\angle 1 \overset{m}{=} \frac{1}{2}(\widehat{x}+\widehat{y})$

(d). <u>Angle outside circle--one arc negative.</u> Outside of the circle. One arc becomes negative, so the sum is the sum of a positive and negative quantity, or as we say the difference of the arcs. The rule holds if we consider one arc of negative value.



$\angle 1 \overset{m}{=} \frac{1}{2}(\widehat{x}+(-\widehat{y})$

$\angle 1 \overset{m}{=} \frac{1}{2}(\widehat{x}-\widehat{y})$

Another opportunity for grouping is found in the angle with the circle, where the measure is always the sum of the intercepted arcs.

(a) Angle = 1/2 Sum of arcs--At center--Special case where the arcs are equal so we can say one whole arc.

$\widehat{x} = \widehat{y}$

$\angle 1 = \frac{1}{2}(\widehat{x}+\widehat{y})$ or $\frac{1}{2}(\widehat{x}+\widehat{x})$

$\angle 1 = \frac{1}{2}(2\widehat{x}) = \widehat{x}$

(b). <u>Angle on circumference--one arc zero.</u> On circumference. Special case where there is no arc on one side, so one arc is zero.

$\angle 1 = \frac{1}{2}\widehat{x}$

(c). <u>Angle inside circle--general case.</u> At any other place inside of circle the rule holds as above.

$\angle 1 = \frac{1}{2}(\widehat{x}+\widehat{y})$

(d). <u>Angle outside circle--one arc negative.</u> Outside of the circle. One arc becomes negative, so the sum is the sum of a positive and negative quantity, or as we say the difference of the arcs. The rule holds if we consider one arc of negative value.

$\angle 1 = \frac{1}{2}(\widehat{x}+(-\widehat{y}))$

$\angle 1 = \frac{1}{2}(\widehat{x}-\widehat{y})$

B. Methods of Proof.

1. The methods of analysis and synthesis. Much has been written and many are the ideas in regard to the various methods of proof in geometry. It is not the purpose to enter the controversial field of this question but rather in this unit to give a brief description of the several methods and to note some of the advantages and disadvantages of them. Among those considered are the methods of analysis and synthesis, and the heuristic, the inductive, and the indirect methods. The unit ends with a statement of the method most used and best liked by the writer.

a). Comparison by contrast. First, consider the two methods of analysis and synthesis which may best be described by contrasting them. Comparison in this way shows that they differ principally in the order of steps. The analytic, assuming the proof, works from the conclusion back to the hypothesis, while the synthetic works from the hypothesis to the conclusion. Schlauch[1] shows the analytic method in a general way when he cites the steps as follows:

> "I can prove X if I can prove Y,
> And I can prove Y if I can prove A,
> But I can prove A if B is true.
> I have already proved B;
> hence I can prove X."

Another way of stating the contrast is by tabulating.

Analysis	Synthetic
From the unknown to the known.	From the known to the unknown.

1. W. S. Schlauch, The Analytical Method in the Teaching of Geometry, The National Council of Teachers of Mathematics, The Fifth Yearbook. XXXI (1930) p. 135.

B. Methods of Proof.

1. The methods of analysis and synthesis. Much has been written and many are the ideas in regard to the various methods of proof in geometry. It is not the purpose to enter the controversial field of this question but rather in this unit to give a brief description of the several methods and to note some of the advantages and disadvantages of them. Among those considered are the methods of analysis and synthesis, and the heuristic, the inductive, and the indirect methods. The unit ends with a statement of the method most used and best liked by the writer.

a). Comparison by contrast. First, consider the two methods of analysis and synthesis which may best be described by contrasting them. Comparison in this way shows that they differ principally in the order of steps. The analytic, assuming the proof, works from the conclusion back to the hypothesis, while the synthetic works from the hypothesis to the conclusion. Schlauch[1] shows the analytic method in a general way when he cites the steps as follows:

"I can prove X if I can prove Y,
And I can prove Y if I can prove A,
But I can prove A if B is true.
I have already proved B;
Hence I can prove X."

Another way of stating the contrast is by tabulating.

Analysis	Synthetic
From the unknown to the known.	From the known to the unknown.

1. W. S. Schlauch, The Analytical Method in the Teaching of Geometry, The National Council of Teachers of Mathematics, The Fifth Yearbook. XXXI (1930) p. 135.

Conclusion to Hypothesis Hypothesis to conclusion.[1]

Considerating the theorem "If a line bisects two sides of a triangle, it is parallel to the third side and equal to one-half of the third side." Schultze analyzes as follows:[2]

b). Sample theorem illustrating the analytic method.

"1. The usual method for proving that one line (ED) is one half of another (BC) is to double the smaller. Hence we produce ED by its own length to F.

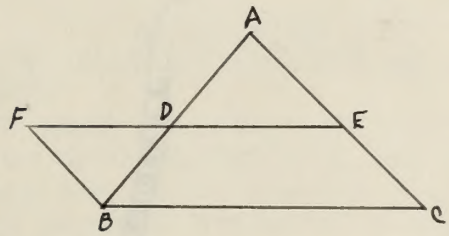
"2. Prove EF=BC, and EF∥BC. There are several methods for proving the equality of lines, and several others for proving parallelism of lines. Only one, however, proves equality and parallelism simultaneously, viz. the one based upon a parallelogram. Hence we attempt to

"3. Prove that BCEF is a parallelogram. There are again various methods of demonstrating that a figure is a parallelogram, but since nothing whatever is known in regard to BC and EF we have to use the sides BF and CE exclusively, i.e. we have

"4. To prove (a) BF∥CE and (b) BF=CE.

"(a) The fundamental method for demonstrating parallelism of lines rests upon the equality of a pair of alternate interior angles. Hence we have to prove

"5. ∠A= ∠DBF."



1. Arthur Schultze, op. cit. p. 30.

2. Arthur Schultze, op. cit. p. 33.

Conclusion to Hypothesis Hypothesis to conclusion.[1]

Considered in the theorem "If a line bisects two sides of a triangle, it is parallel to the third side and equal to one-half of the third side." Schultze analyzes as follows:[2]

b). Sample theorem illustrating the analytic method.

"1. The usual method for proving that one line (ED) is one half of another (BC) is to double the smaller. Hence we produce ED by its own length to F.

"2. Prove EF=BC, and EF∥BC. There are several methods for proving the equality of lines, and several others for proving parallelism of lines. Only one, however, proves equality and parallelism simultaneously, viz. the one based upon a parallelogram. Hence we attempt to

"3. Prove that BCEF is a parallelogram. There are again various methods of demonstrating that a figure is a parallelogram, but since nothing whatever is known in regard to BC and EF we have to use the sides BF and CE exclusively, i.e. we have to

"4. To prove (a) EF∥CE and (b) EF=CE.

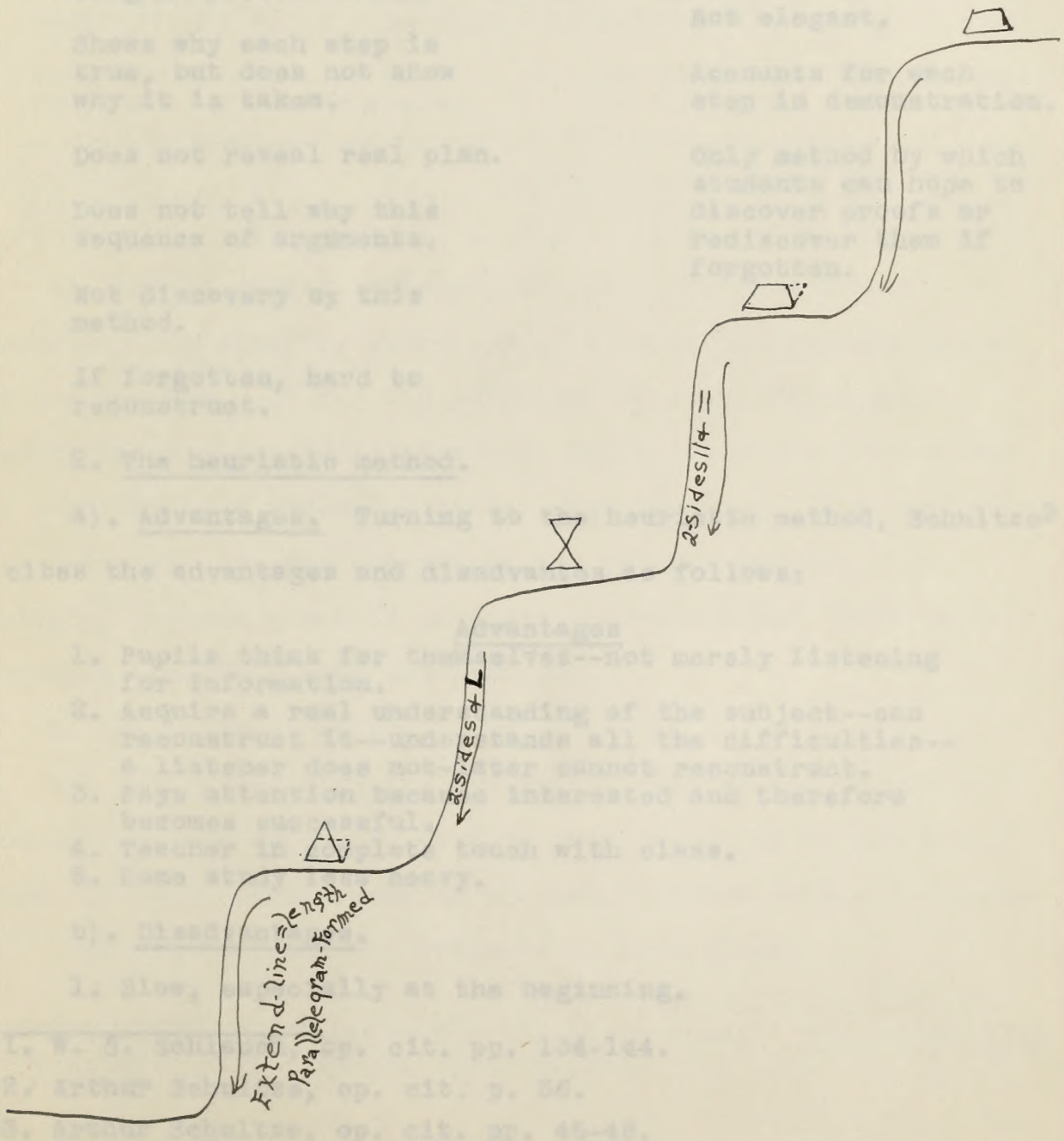
"(a) The fundamental method for demonstrating parallelism of lines rests upon the equality of a pair of alternate interior angles. Hence we have to prove

"5. ∠A=∠DEF."

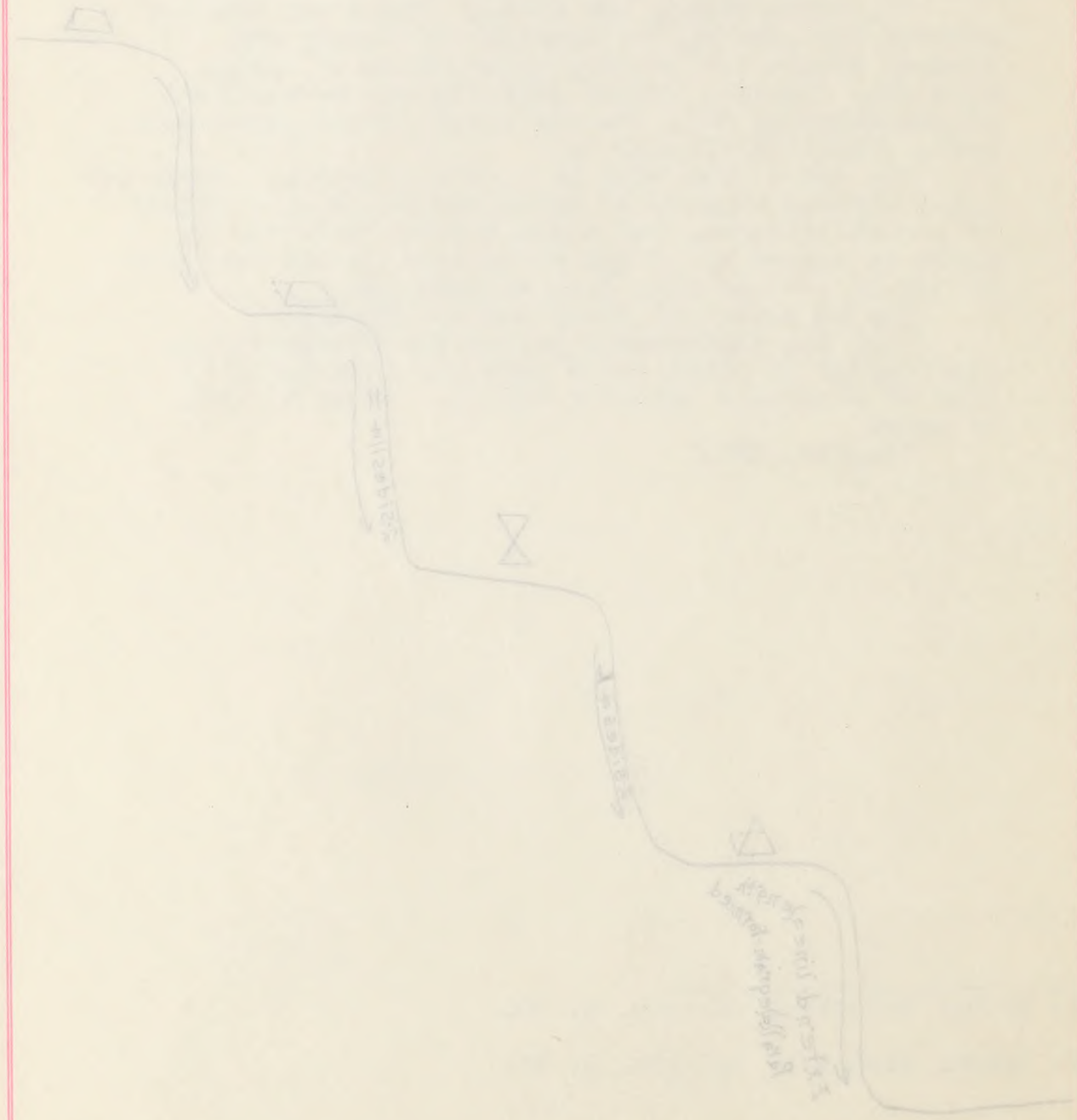
1. Arthur Schultze, op. cit. p. 50.

2. Arthur Schultze, op. cit. p. 33.

c). Graphic representation of the method of analysis. The writer has found especially effective his own method of combining the proof by analysis with a graphic representation which is usually a mountain drawing, which follows:



c). Graphic representation of the method of analysis. The writer has found especially effective his own method of combining the proof by analysis with a graphic representation which is usually a mountain drawing, which follows:



The Fifth Yearbook[1] contains an article on the method of analysis with several illustrations.

Schultze[2] describes and evaluates the two methods of synthesis and analysis as follows:

d). Description and evaluation of both methods.

Synthesis	Analysis
Method of concise and elegant presentation.	Lengthy.
	Not elegant.
Shows why each step is true, but does not show why it is taken.	Accounts for each step in demonstration.
Does not reveal real plan.	Only method by which students can hope to discover proofs or rediscover them if forgotten.
Does not tell why this sequence of arguments.	
Not discovery by this method.	
If forgotten, hard to reconstruct.	

2. The heuristic method.

a). Advantages. Turning to the heuristic method, Schultze[3] cites the advantages and disadvantes as follows:

Advantages

1. Pupils think for themselves--not merely listening for information.
2. Acquire a real understanding of the subject--can reconstruct it--understands all the difficulties--a listener does not-later cannot reconstruct.
3. Pays attention because interested and therefore becomes successful.
4. Teacher in complete touch with class.
5. Home study less heavy.

b). Disadvantages.

1. Slow, especially at the beginning.

1. W. S. Schlauch, op. cit. pp. 134-144.
2. Arthur Schultze, op. cit. p. 36.
3. Arthur Schultze, op. cit. pp. 45-46.

The Fifth Yearbook[1] contains an article on the method of analysis with several illustrations.

Schultze[2] describes and evaluates the two methods of synthesis and analysis as follows:

d). Description and evaluation of both methods.

Synthesis	Analysis
Method of concise and elegant presentation.	Lengthy.
	Not elegant.
Shows why each step is true, but does not show why it is taken.	Accounts for each step in demonstration.
Does not reveal real plan.	Only method by which students can hope to discover proofs or rediscover them if forgotten.
Does not tell why this sequence of arguments.	
Not discovery by this method.	
If forgotten, hard to reconstruct.	

2. The heuristic method.

a). Advantages. Turning to the heuristic method, Schultze[3] cites the advantages and disadvantages as follows:

Advantages

1. Pupils think for themselves--not merely listening for information.
2. Acquire a real understanding of the subject--can reconstruct it--understands all the difficulties--a listener does not--later cannot reconstruct.
3. Pays attention because interested and therefore becomes successful.
4. Teacher in complete touch with class.
5. Home study less heavy.

b). Disadvantages.

1. Slow, especially at the beginning.

1. W. S. Schlauch, op. cit. pp. 134-144.
2. Arthur Schultze, op. cit. p. 36.
3. Arthur Schultze, op. cit. pp. 45-46.

2. More difficult for the teacher, not simply text--seek devices for purpose of leading students--must modify for different students.
3. Must have skill to know how much to expect from pupils.

3. The inductive method.

a). Example. An example of the inductive method is given by William Reeve[1].

Discovery of n-2 straight angles as the sum of the interior angles in a polygon. Building a table.

Type of Polygon	No. sides	No. triangles	Sum of angles of triangle	Sum of angles of Polygon.
Triangle	3	1	1st. angle	(3-2) st. angles.
Quadrilateral	4	2	1st. angle	(4-2) st. angles.
Pentagon	5	3	1st. angle	(5-2) st. angles.

b). Seldom used. Although this is a valuable method generally, in geometry there seems to be but few places where it can be used. It has the advantage of statistical proof, where the characteristic procedure is to prove the proposition by considering a number of cases. For instance, in establishing the geometric law for the sum of the interior angles in a polygon, various kinds of polygons are considered, such as the triangle, quadrilateral and pentagon.

4. The indirect method.

a). Contrasted with the analytic. Turning now to the indirect method of proof, we find that though it is not often used, it is of great value in the propositions which do require it. Writers describe and evaluate it as follows:

1. "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher. XXVI, (March, 1933) p. 161.

2. More difficult for the teacher, not simply text--seek devices for purpose of leading students--must modify for different students.
3. Must have skill to know how much to expect from pupils.

3. The inductive method.

a). Example. An example of the inductive method is given by William Reeve[1].

Discovery of n-2 straight angles as the sum of the interior angles in a polygon. Building a table.

Type of Polygon	No. sides	No. triangles	Sum of angles of triangle	Sum of angles of polygon.
Triangle	3	1	1st. angle	(3-2) st. angles.
Quadrilateral	4	2	1st. angle	(4-2) st. angles.
Pentagon	5	3	1st. angle	(5-2) st. angles.

b). Seldom used. Although this is a valuable method generally, in geometry there seems to be but few places where it can be used. It has the advantage of statistical proof, where the characteristic procedure is to prove the proposition by considering a number of cases. For instance, in establishing the geometric law for the sum of the interior angles in a polygon, various kinds of polygons are considered, such as the triangle, quadrilateral and pentagon.

4. The indirect method.

a). Contrasted with the analytic. Turning now to the indirect method of proof, we find that though it is not often used, it is of great value in the propositions which do require it. Writers describe and evaluate it as follows:

1. "Demonstrative Geometry for the Ninth Grade" The Mathematics Teacher. XXVI. (March, 1933) p. 161.

The indirect method of reasoning is the method employed in everyday life. It may be contrasted with the analytic method in that each method starts from an assumption and draws inferences from it. In the analytic method, the assumption is that the conclusion is correct, and inferences are drawn until a known fact is reached. In the indirect method, the assumption is that the conclusion is not correct, and inferences are drawn until a fact is reached which is known to be true.[1]

b). Example. An example is the proposition that "If two parallel lines are cut by a transversal, the alternate interior angles are equal.

1. Assume the angles are not equal.
2. Draw a line making them equal.
3. This line is parallel.
4. It must coincide with the first line.

In teaching the indirect method it is important to make the procedure as definite as possible and to give the pupil ample opportunity to learn it in simple cases.

c). Place in text.

An indirect proof is more abstract than a direct proof. For this reason it is well to choose a sequence of theorems that will allow a postponement of it until the pupil is well grounded in demonstrations of the direct type.[2]

d). A valuable reference.

The Fifth Yearbook contains an article on the indirect method which is quite complete.[3]

The writer uses the graphic representation of various roads branching out from the main road as an illustration of the indirect method. In the unit on the isosceles triangle is an example of this procedure.

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics. pp. 296-299.
2. J.O.Hassler and R.R.Smith, The Teaching of Secondary Mathematics. pp. 356-357.
3. Clifford B. Upton, The Use of Indirect Proof in Geometry and in Life, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930) pp.102-133

The indirect method of reasoning is the method employed in everyday life. It may be contrasted with the analytic method in that each method starts from an assumption and draws inferences from it. In the analytic method, the assumption is that the conclusion is correct, and inferences are drawn until a known fact is reached. In the indirect method, the assumption is that the conclusion is not correct, and inferences are drawn until a fact is reached which is known to be true.[1]

b). Example. An example is the proposition that "If two parallel lines are cut by a transversal, the alternate interior angles are equal.

1. Assume the angles are not equal.

2. Draw a line making them equal.

3. This line is parallel.

4. It must coincide with the first line.

In teaching the indirect method it is important to make the procedure as definite as possible and to give the pupil ample opportunity to learn it in simple cases.

c). Place in text.

An indirect proof is more abstract than a direct proof. For this reason it is well to choose a sequence of theorems that will allow a postponement of it until the pupil is well grounded in demonstrations of the direct type.[2]

d). A valuable reference.

The Fifth Yearbook contains an article on the indirect method which is quite complete.[3]

The writer uses the graphic representation of various roads branching out from the main road as an illustration of the indirect method. In the unit on the isosceles triangle is an example of this procedure.

1. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics, pp. 295-296.

2. Leo Ussaler and R.R.Smith, The Teaching of Secondary Mathematics, pp. 356-357.

3. Clifford B. Upton, The Use of Indirect Proof in Geometry and in Life, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), pp. 105-125

5. The most valuable method.

a). Advantage of analytic over the synthetic. In the writer's experience the most outstandingly useful method is the method of analysis combined with graphic representation. It is used by him in the majority of the theorems. To go at once to the conclusion, and devise means of constructing the links between the hypothesis and the conclusion, arouses the interest of the pupil, and encourages him to join the two ends of the line, the hypothesis and conclusion, together. With synthesis, he ventures into an unknown and uncharted course with none too definite an idea of where he is going, while in the analytic method he can hold both ends of the line which gives to him a feeling of assuredness not derived in the other method. The conclusion waits for the pupil with a helping hand. It is like a friend on the other side of the stream; one can cross more easily and supply the stepping stones with greater alacrity if one knows his friend is waiting for him at a definite place, where one can see him. This idea of the helping hand is brought out in the example of the mountain graph where the conclusion holding the rope pulls the chain of climbers up from peak to the peak while the conclusion waits at the top.

5. The most valuable method.

a). Advantage of analytic over the synthetic. In the writer's experience the most outstandingly useful method is the method of analysis combined with graphic representation. It is used by him in the majority of the theorems. To go at once to the conclusion, and devise means of constructing the links between the hypothesis and the conclusion, arouses the interest of the pupil, and encourages him to join the two ends of the line, the hypothesis and conclusion, together. With synthesis, he ventures into an unknown and uncharted course with none too definite an idea of where he is going, while in the analytic method he can hold both ends of the line which gives to him a feeling of assuredness not derived in the other method. The conclusion waits for the pupil with a helping hand. It is like a friend on the other side of the stream; one can cross more easily and supply the stepping stones with greater alacrity if one knows his friend is waiting for him at a definite place, where one can see him. This idea of the helping hand is brought out in the example of the mountain graph where the conclusion holding the rope pulls the chain of climbers up from peak to the peak while the conclusions waits at the top.

C. Provision for Individual Differences.

1. Types of students. It is to be remembered that not all pupils are motivated by the same stimulus. The pupil should be given that stimulus which will motivate him into activity. In one, it is mental activity in connection with some problem to be solved; in another, creation; in still another, it involves a hand activity such as drawing, painting or constructing; or again it may be expression by composition. The main fact to keep in mind is that the pupil should be given an opportunity for expression, with an audience situation, and that the form of expression should be along the line of his natural interests and capacities. Thus growth will be achieved through natural activity.

a). Four groups of users. In this connection The Commission of 1920 states:[1]

> "...the committee believes that, speaking generally, introductory mathematics, ordinarily conceived as separate courses in algebra, geometry, and trigonometry, should be given in connection with the solving of problems and the executing of projects in fields where the pupils already have both knowledge and interest. This would make the study of mathematics more nearly approximate a laboratory course, in which individual differences could be considered....The minimum of the course might well in this way be cared for in the recitation period, reserving the outside work rather for allied projects and problems in which individual interests and capacities were prominent factors."
>
> "The committee considers that four groups of users of mathematics may be distinguished:
>
> (a). The "general' readers', who will find their

1. A Report of the Commission on The Reorganization of Secondary Education, Appointed by the National Education Association, The Problem of Mathematics in Secondary Education, Bulletin, 1920, No. 1, pp. 12-13

C. Provision for Individual Differences.

1. Types of students. It is to be remembered that not all pupils are motivated by the same stimulus. The pupil should be given that stimulus which will motivate him into activity. In one, it is mental activity in connection with some problem to be solved; in another, creation; in still another, it involves a hand activity such as drawing, painting or constructing; or again it may be expression by composition. The main fact to keep in mind is that the pupil should be given an opportunity for expression, with an audience attention, and that the form of expression should be along the line of his natural interests and capacities. Thus growth will be achieved through natural activity.

a). Four groups of users. In this connection The Commission of 1920 states:[1]

"...the committee believes that, speaking generally, introductory mathematics, ordinarily conceived as separate courses in algebra, geometry, and trigonometry, should be given in connection with the solving of problems and the execution of projects in fields where the pupils already have both knowledge and interest. This would make the study of mathematics more nearly approximate a laboratory course, in which individual differences could be considered....The minimum of the course might well in this way be cared for in the recitation period, reserving the outside work rather for allied projects and problems in which individual interests and capacities were prominent factors."

"The committee considers that four groups of users of mathematics may be distinguished:
(a). The 'general' readers', who will find their

1. A Report of the Commission on The Reorganization of Secondary Education. Appointed by the National Education Association, The Problem of Mathematics in Secondary Education. Bulletin, 1920. No. 1, pp. 12-13

"use of mathematics beyond arithmetic confined largely to the interpretative function.

(b) Those whose work in certain trades will make limited, but still specific, demand for the 'practical' use of mathematics.

(c) Those whose practical work as engineers or as students of certain sciences require considerable knowledge of mathematics.

(d) Those who specialize in the study of mathematics with a view either to research or to teaching."[1]

Several studies have been made to investigate individual differences among boys and girls. As would be expected, boys are more interested in construction exercises than girls. However, the studies seem to show that the range of differences between either boys or girls is greater than between the two sex groups.[2]

2. Differing assignments. Johnson[3] writes of three levels of achievement as follows:

a). In quantity and quality.

C student-100% requirement for minimum mastery. The aim is information, social and mechanical.

B student-Preparation for future mathematics and sciences.

A student-Near genius pupil in this group. Those able to present original solutions.

He calls these levels successively mechanistic, thinking, and high mental level. B and A are college material. He suggests that brighter students are to go deeper into a topic, rather than further along in new topics. Differentiation is to be on the basis of quality rather than quantity.

1. A Report of the Commission on The Reorganization of Secondary Education, loc. cit.

2. H. E. Benz, A Summary of Some Scientific Investigations of the Teaching of High School Mathematics, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933, p.29-30.

3. J. T. Johnson, Adapting Instructional Material to Individual Differences in Learning, The Mathematics Teacher, XXVI (April, 1933) pp. 193-199.

"use of mathematics beyond arithmetic confined largely to the interpretative function.

(b) Those whose work in certain trades will make limited, but still specific, demand for the 'practical' use of mathematics.

(c) Those whose practical work as engineers or as students of certain sciences require considerable knowledge of mathematics.

(d) Those who specialize in the study of mathematics with a view either to research or to teaching."[1]

Several studies have been made to investigate individual differences among boys and girls. As would be expected, boys are more interested in construction exercises than girls. However, the studies seem to show that the range of differences between either boys or girls is greater than between the two sex groups.[2]

2. Differing assignments. Johnson[3] writes of three levels of achievement as follows:

a). In quantity and quality.

C student-100% requirement for minimum mastery. The aim is information, social and mechanical.

B student-Preparation for future mathematics and sciences.

A student-Near genius pupil in this group. Those able to present original solutions.

He calls these levels successively mechanistic, thinking, and high mental level. B and A are college material. He suggests that brighter students are to go deeper into a topic, rather than further along in new topics. Differentiation is to be on the basis of quality rather than quantity.

---

1. A Report of the Commission on The Reorganization of Secondary Education, loc. cit.

2. H. E. Benz, A Summary of Some Scientific Investigations of the Teaching of High School Mathematics, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933, p.29-30.

3. J. T. Johnson, Adapting Instructional Material to Individual Differences in Learning, The Mathematics Teacher, XXVI (April, 1933) pp. 193-199.

In general it might be well to afford opportunities somewhat as follows:

Better Pupils	Poorer Pupils
Less drill.	More hand work.
More problem work.	More construction.
Harder problems.	Easier exercises.
Lead in suggestions.	Easier propositions.
Lead in corrections.	Easier questions.
	More drill.
	More board work on construction.
	Follow-up after the brighter have paved the way.

b). In interests. At the start of the school year information may be obtained from the individual pupils in regard to their special interests along mathematical lines. Various activities may be suggested and the pupils select the activity more interesting to them. This was noted in the unit on orientation. In case several pupils ask to make the same model or instrument; it may be made either by a group, or by the first one asking for it, or by the one who will do the best job. Sometimes it is advisable to have each one make a model and the best model is then selected for class use.

On this and the following page is mapped a tentative list of interests, together with possible activities along the line of these interests.

(1). Hand	(2). Verbal
Construction of models in manual training department, such as, transits, protractors, solid models.[1]	Presentation of history, anecdotes, dictionary of terms. Plays.

1. George Wolff, The Mathematical Collection, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933 pp. 216-263.

In general it might be well to afford opportunities somewhat as follows:

Better Pupils	Poorer Pupils
Less drill.	More hand work.
More problem work.	More construction.
Harder problems.	Easier exercises.
Lead in suggestions.	Easier propositions.
Lead in corrections.	Easier questions.
	More drill.
	More board work on construction.
	Follow-up after the brighter have paved the way.

b). In interests. At the start of the school year information may be obtained from the individual pupils in regard to their special interests along mathematical lines. Various activities may be suggested and the pupils select the activity more interesting to them. This was noted in the unit on orientation. In case several pupils ask to make the same model or instrument; it may be made either by a group, or by the first one asking for it, or by the one who will do the best job. Sometimes it is advisable to have each one make a model and the best model is then selected for class use.

On this and the following page is mapped a tentative list of interests, together with possible activities along the line of these interests.

(1). Hand	(2). Verbal
Construction of models in manual training department, such as, trensils, protractors, solid models.[1]	Presentation of history, anecdotes, dictionary of terms. Plays.

1. George Wolff, The Mathematical Collection, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933 pp. 216-283.

Field measurements.
Explanation of instruments, such as the carpenter's level.

(3). Art

Drawings--Mechanical.
Posters.
Arrangements of exhibits.
Bulletin Boards.
Designs[1].
Symmetry in Art Work.
Pictures in Mathematics room.
Graphs.
Coloring.
Pantograph use.
Construction of large figures on cardboard for drill use.
Map of local community for office use.

(5). Science.

Photography.
Physics and Geometry.
Astronomy and Geometry.
Apparatus getting.

Reports.
Magazine articles.
Biographies.
Newspaper articles.

(4). Problem

Solution of more difficult problems.
New solutions.
Recreations.
Originals.
Fallacies.
Puzzles.

The above list of activities could be placed on the bulletin board and pupils sign for the ones in which they were interested. Of course the various names of the divisions would not appear, although the grouping might be kept as it is. That is, without the caption for hand-minded appearing, the various activities under this could be grouped for convenience of the pupil and the teacher. In this way the "we feeling" gained through contribution to the class progress will create an attitude toward success and class unity.

1. A. Day Bradley, The Geometry of Repeating Design and Geometry of Design for High Schools, 1933.

Field measurements.
Explanation of instruments, such as the carpenter's level.

(3). Art

Drawings--Mechanical.
Posters.
Arrangements of exhibits.
Bulletin Boards.
Designs.
Symmetry in Art Work.
Pictures in Mathematics room.
Graphs.
Coloring.
Pantograph use.
Construction of large figures on cardboard for drill use.
Map of local community for office use.

(5). Science.

Photography.
Physics and Geometry.
Astronomy and Geometry.
Apparatus getting.

Reports.
Magazine articles.
Biographies.
Newspaper articles.

(4). Problem

Solution of more difficult problems.
New solutions.
Recreations.
Originals.
Fallacies.
Puzzles.

The above list of activities could be placed on the bulletin board and pupils sign for the ones in which they were interested. Of course the various names of the divisions would not appear, although the grouping might be kept as it is. That is, without the caption for hand-minded appearing, the various activities under this could be grouped for convenience of the pupil and the teacher. In this way the "we feeling" gained through contribution to the class progress will create an attitude toward success and class unity.

1. A. Day Bradley, The Geometry of Repeating Design and Geometry of Design for High Schools, 1932.

c). Sensory approaches. Thus through individual work, the various sensory approaches are offered as avenues of learning for the pupil. The visual approach finds its fields in board work, charts, exhibits, and objective materials such as instruments, cardboard, and room objects. Those interested in dramatics and historical anecdotes may use the auditory approach. The tactile sense is required in all hand work such as designing, drawing, construction work, and making models. The more sensory approaches which can be made, the greater the chance for successful teaching; these approaches allow for individual differences in pupils. They make the teaching stronger by repetition through the various sensory channels. They tell the story in different ways, keeping up the interest and not tiring the pupil. Children learn by handling and more than one boy, who is unable to grasp ideas easily from the printed page or the spoken work finds in the hand approach of manual training a way to success. In geometry this hand practice may come through folding, drawing, designing, collecting, measuring, making and tracing.

c). Sensory approaches. Thus through individual work, the various sensory approaches are offered as avenues of learning for the pupil. The visual approach finds its fields in board work, charts, exhibits, and objective materials such as instruments, cardboard, and room objects. Those interested in dramatics and historical anecdotes may use the auditory approach. The textile sense is required in all hand work such as designing, drawing, construction work, and making models. The more sensory approaches which can be made, the greater the chance for successful teaching; these approaches allow for individual differences in pupils. They make the teaching stronger by repetition through the various sensory channels. They tell the story in different ways, keeping up the interest and not tiring the pupil. Children learn by handling and more than one boy, who is unable to grasp ideas easily from the printed page or the spoken word finds in the hand approach of manual training a way to success. In geometry this hand practice may come through folding, drawing, designing, collecting, measuring, making and tracing.

3. Vehicles for varying kinds of expression. Having noted that pupils vary in capacities, interests and purposes, and that provision may be made for this variation in capacity through different levels of achievement, and having noted further that opportunities for various interests can be listed for choice by the pupil, one may now turn to mediums through which individual differences may find expression might be listed as follows:

The Mathematics Club.

The Mathematics Room.

Mathematics Exhibits.

Field Trips.

Manual Training Department.

Bulletin Board.

a). The Mathematics Club. In these days when so much stress is laid on the social activities of the pupil, and when it is pointed out that the happy, active pupil is the best citizen, the value of the mathematics club appears evident at once. It is the organization part of the various special interests of the geometry pupils. Here they have the fun of choosing officers, following parliamentary procedure, choosing their contributions and presenting them before an interested audience. Under this organization may well come the arranging of special mathematics features such as the decorating of the room, the arranging of exhibits, the planning for field trips, the cooperating with the manual training department and the taking charge of the bulletin board for mathematical news.

3. Vehicles for varying kinds of expression. Having noted that pupils vary in capacities, interests and purposes, and that provision may be made for this variation in capacity through different levels of achievement, and having noted further that opportunities for various interests can be listed for choice by the pupil, one may now turn to mediums through which individual differences may find expression might be listed as follows:

The Mathematics Club.

The Mathematics Room.

Mathematics Exhibits.

Field Trips.

Manual Training Department.

Bulletin Board.

a). The Mathematics Club. In these days when so much stress is laid on the social activities of the pupil, and when it is pointed out that the happy, active pupil is the best citizen, the value of the mathematics club appears evident at once. It is the organization part of the various special interests of the geometry pupils. Here they have the fun of choosing officers, following parliamentary procedure, choosing their contributions and presenting them before an interested audience. Under this organization may well come the arranging of special mathematics features such as the decorating of the room, the arranging of exhibits, the planning for field trips, the cooperating with the manual training department and the taking charge of the bulletin board for mathematical news.

This unit considers the suggestions which have been made in various articles by writers on the mathematics club. It suggests briefly the naming of the club, the choosing of an emblem, selection of officers, and then gives a number of suggestions for programs. The unit is left in topical form to aid the reader to quickly find the various suggestions. The program has been divided into the contributions which the verbal-minded pupils, the hand-minded, the problem lovers, and the art-minded may take.

(1) For the social-minded.

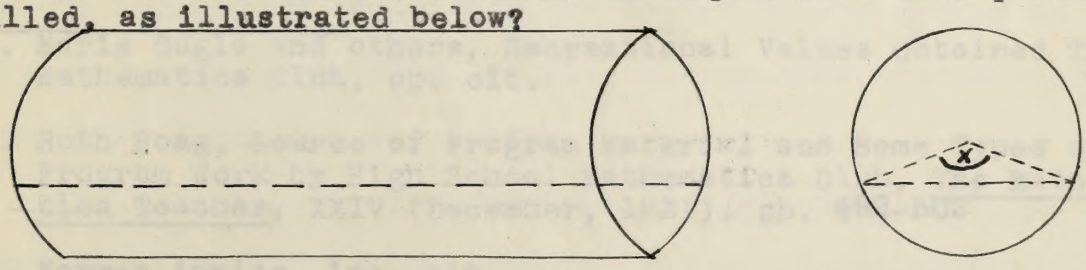
Name--After a famous mathematics teacher.
After mathematical figure.

Emblem--Geometrical figure. 1

Organization--Faculty adviser--silent member.
Officers--Chosen by the club. 2

Speakers--Science teacher who knows astronomy elements may give a talk on geometry and astronomy.
a. Finding distance to stars.
b. Finding speed of light.
c. Eclipses and geometry.
d. Latitude and longitude.

How can one find the volume of liquid in a tank partly filled, as illustrated below?



1. Marie Gugle and others, Recreational Values Obtained Through Mathematics Club. The National Coucil of Teachers of Mathematics, The First Yearbook, 1926, pp. 194-200.

2. Norman Anning, High School Mathematics Club, The Mathematics Teacher, XXVI, (February, 1933) pp. 70-76.

This unit considers the suggestions which have been made in various articles by writers on the mathematics club. It suggests briefly the meaning of the club, the choosing of an emblem, selection of officers, and then gives a number of suggestions for programs. The unit is left in topical form to aid the reader to quickly find the various suggestions. The program has been divided into the contributions which the verbal-minded pupils, the hand-minded, the problem lovers, and the art-minded may take.

(1) For the social-minded.

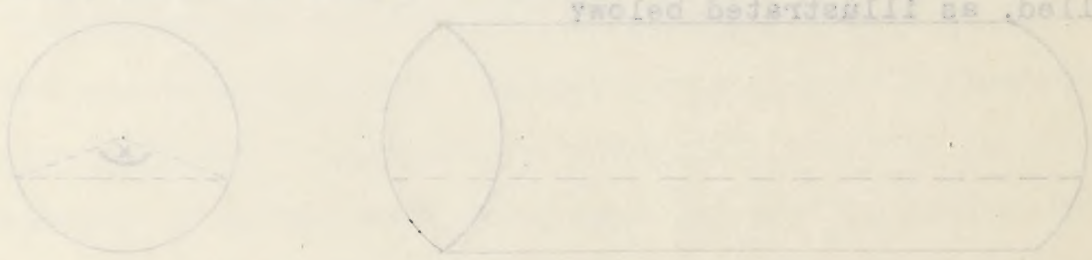
Name--After a famous mathematics teacher.
After mathematical figure.

Emblem--Geometrical figure. 1

Organization--Faculty adviser--alient member.
Officers--Chosen by the club. 2

Speakers--Science teacher who knows astronomy elements may give a talk on geometry and astronomy.
a. Finding distance to stars.
b. Finding speed of light.
c. Ellipses and geometry.
d. Latitude and longitude.

How can one find the volume of liquid in a tank partly filled, as illustrated below



---

1. Maria Gugle and others, Successional Values Obtained Through Mathematics Club. The National Council of Teachers of Mathematics, The First Yearbook, 1926, pp. 194-200.

2. Norman Anding, High School Mathematics Club, The Mathematics Teacher, XXVI, (February, 1933) pp. 70-76.

a. Find center of circle at end of tank and radius.
b. Find central angle and thus the area of sector which is $\frac{x}{360}$ of area of circle.
c. Find area of triangle. Two sides and angle included known. Formula for this area.
d. Subtract area of triangle from area of sector and the result is the area of the segment.
e. Multiply this area by the length of the tank and you have the volume of that part of the tank.[6]

(2). For the verbal-minded.

Biographical Sketch of some Great Mathematician and the story of his contributions.
Account of some scientific discovery and invention relating to mathematics.[1]
Skits, plays, dialogues.
Compiling dictionary of terms or symbols used in high school mathematics club from the texts.
Mathematical Scrap book.
History of mathematics--Stories of.
Recording what you see on a geometry trip.[2]
Club library. Books on mathematics.[3]
Astronomy Lecture. Eclipse angle.[4]

(3). For the hand-minded.

Construction of instruments to be later used in field work.
Construction of a set of models.
Demonstrations with steel square.
Field trips with plane table measure.
Measures based on congruence of triangles.
Use of transit.
Paper folding.[5]

1. Marie Gugle and others, Recreational Values Obtained Through Mathematics Club, op. cit.

2. Ruth Hoag, Source of Program Material and Some Types of Program Work by High School Mathematics Club, The Mathematics Teacher, XXIV (December, 1931). pp. 492-502

3. Norman Anning, loc. cit.

4. Mary C. Hatton, The Mathematics Club, The Mathematics Teacher, XX, (January, 1927). pp. 39-45.

5. Ruth Hoag, loc. cit.

6. Mary C. Hatton, loc. cit.

a. Find center of circle at end of tank and radius.
b. Find central angle and thus the area of sector which is $\frac{x}{360}$ of area of circle.
c. Find area of triangle. Two sides and angle included known. Formula for this area.
d. Subtract area of triangle from area of sector and the result is the area of the segment.
e. Multiply this area by the length of the tank and you have the volume of that part of the tank.[6]

(2). For the verbal-minded.

Biographical Sketch of some Great Mathematician and the story of his contributions.
Account of some scientific discovery and invention relating to mathematics.[1]
Skits, plays, dialogues.
Compiling dictionary of terms or symbols used in high school mathematics club from the texts.
Mathematical Scrap book.
History of mathematics--Stories of.
Recording what you see on a geometry trip.[2]
Club library. Books on mathematics.[3]
Astronomy Lecture. Eclipse angle.[4]

(3). For the hand-minded.

Construction of instruments to be later used in field work.
Construction of a set of models.
Demonstrations with steel square.
Field trips with plane table measure.
Measures based on congruence of triangles.
Use of transit.
Paper folding.[5]

1. Marie Gugle and others, Recreational Values Obtained Through Mathematics Club, op. cit.

2. Ruth Hoag, Source of Program Material and Some Types of Program Work by High School Mathematics Club, The Mathematics Teacher, XXIV (December, 1931). pp. 492-502

3. Norman Anning, loc. cit.

4. Mary C. Hatton, The Mathematics Club, The Mathematics Teacher, XX, (January, 1927). pp. 39-43.

5. Ruth Hoag, loc. cit.

6. Mary C. Hatton, loc. cit.

(4). For the problem lovers.
Mathematical game or fallacy.[1]
Puzzles and entertainment.[2]
Historical problems.
Slide Rule contest.
Contest.
a. Twenty-five completion sentences from the Schloring Clark Achievement Test in Plane Geometry.
b. Oral on five propositions.
c. Five originals on blackboard: other side on opposite blackboard. Each took turns in having its members give a step in the proof.[3]

(5). For the art-minded.
Posters, etc. (See the following section).[4]

b). The Mathematics Room. If the task is to be worth while, the group must feel that it is important. Helping toward this situation may be mentioned the mathematics room.

To produce a favorable attitude toward plane geometry, it is necessary to have a "we" feeling for the subject. Participation by the pupils in making various exhibits for a mathematics room will arouse this "we" feeling and will have the advantage also of providing for individual differences and affording much visual material. This will get the symbol back to the picture and object stage, for the better understanding by the pupil. Posters, exhibits, models, instruments, pictures, booklets, clippings, are all useful.

The following unit makes topical suggestions for the room decorations which give opportunity for various interests of the group. Here takes place the crowning step of the various

1. Marie Gugle and others, loc. cit.
2. Norman Anning, loc. cit.
3. Mary C. Hatton, loc. cit.
4. A. Day Bradley, loc. cit.

(4). For the problem lovers.
Mathematical game or Talley.[1]
Puzzles and entertainment.[2]
Historical problems.
Slide Rule contest.
Contest.
a. Twenty-five completion sentences from the Scholoring Clark Achievement Test in Plane Geometry.
b. Oral on five propositions.
c. Five originals on blackboard; other side on opposite blackboard. Each took turns in having its members give a step in the proof.[3]

(5). For the art-minded.
Posters, etc. (See the following section).[4]

b). The Mathematics Room. If the task is to be worth while, the group must feel that it is important. Helping toward this situation may be mentioned the mathematics room.

To produce a favorable attitude toward plane geometry, it is necessary to have a "we" feeling for the subject. Participation by the pupils in making various exhibits for a mathematics room will arouse this "we" feeling and will have the advantage also of providing for individual differences and affording much visual material. This will get the symbol back to the picture and object stage, for the better understanding by the pupil. Posters, exhibits, models, instruments, pictures, booklets, clippings, are all useful.

The following unit makes topical suggestions for the room decorations which give opportunity for various interests of the group. Here takes place the crowning step of the various

1. Marie Gugle and others, loc. cit.
2. Norman Anning, loc. cit.
3. Mary C. Hatton, loc. cit.
4. A. Day Bradley, loc. cit.

endeavors, and there is the advantage over the verbal program in its permanence. This might be said to be the publicity part of the geometry work, since other groups may visit this room and witness the visual work of the class.

(1). For art-minded. Following are the suggestions for the mathematics room:

(a). Pictures.
- Buildings.
- Bridges.
- Designs.
- Great mathematicians.[1]

(b). Posters.
- Uses of mathematics in society.
  - Geometry in the home--Picture of girl making a lamp shade with a hexagonal base.
- Mathematics in the Making.
- Mathematics in the Baking.[2]
- Suggestions for pupils in connection with classroom work.
- The A paper.
- Inciting to greater endeavor.
- Geometric forms in art and nature.[3]

(c). Graphs.
- Genealogical Tree.[4]
- Bridge.
- Stream.
- Mountain.

The following general suggestions are made:
- Make displays at eye level.[5]
- Have case for the instruments and models.
- Have exhibits at various times before the entire school.

1. Edith L. Mossman, A Mathematics Room That Speaks for Itself. School Science and Mathematics, XXXIII, (April, 1933). pp. 423-430.
2. William Betz, The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933. p. 116.
3. William Betz, Quotation from Olive A. Kee, A Mathematical Atmosphere, The National Council of Teachers of Mathematics, The Third Yearbook. 1928, pp. 268. The Eighth Yearbook.pp. 138-140.
4. J. W. A. Young, op. cit. p. 266.
5. William Betz, op. cit. pp. 138-140.

endeavors, and there is the advantage over the verbal program in its permanence. This might be said to be the publicity part of the geometry work, since other groups may visit this room and witness the visual work of the class.

(f). For art-minded. Following are the suggestions for the mathematics room:

(a). Pictures.
Buildings.
Bridges.
Designs.
Great mathematicians.[1]

(b). Posters.
Uses of mathematics in society.
Geometry in the home--Picture of girl making a lamp shade with a hexagonal base.
Mathematics in the Making.
Mathematics in the Making.[2]
Suggestions for pupils in connection with classroom work.
The A paper.
Inciting to greater endeavor.
Geometric forms in art and nature.[3]

(c). Graphs.
Genealogical Tree.[4]
Bridge.
Stream.
Mountain.

The following general suggestions are made:
Make displays at eye level.[5]
Have case for the instruments and models.
Have exhibits at various times before the entire school.

---

1. Edith L. Mossman, A Mathematics Room That Speaks for Itself. School Science and Mathematics, XXXIII, (April, 1933). pp. 423-430.

2. William Betz, The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook. 1933. p. 118.

3. William Betz, Quotation from Olive A. Kee, A Mathematical Atmosphere, The National Council of Teachers of Mathematics, The Third Yearbook. 1928. pp. 268. The Eighth Yearbook. pp. 138-140.

4. J. W. A. Young, op. cit. p. 266.

5. William Betz, op. cit. pp. 138-140.

(d). Graph board.
(e). Slogans.
Ye shall know the truth
And the Truth shall set ye free.[1]

(2). For the verbal-minded.
(a). Bulletin boards.
Newspaper clippings.
(b). Reference table.
Magazine and reference books.[2]
Source books.[3]
Booklets by pupils.[4]
Mathematical puzzles and scientific books on related subjects.[5]

(3). For the hand-minded.
(a). Geometrical instruments.
Meter rulers.
Standboards of board measure.
Compasses.
Protractors.
Pantograph.
Slated globes.
Slide rules.[6]
Carpenter's level.
Transit.
Blackboard compasses.[7]

(b). Models.
Geometric solids.[8]

1. Mary R. Cook, Stimulate the Interest in Mathematics by Creating a Mathematical Atmosphere. The Mathematics Teacher April, 1931, pp. 248-254.

2. Edith L. Mossman, op. cit. pp. 423-430.

3. William Betz, op. cit. p. 155.

4. Mary R. Cook, loc. cit.

5. Gertrude E. Allen, Reorganization of Course of Study for Senior High School Mathematics, The National Council of Teachers of Mathematics, The Second Yearbook, 1927, pp. 267-268.

6. Gertrude E. Allen, loc. cit.

7. Mary R. Cook, loc. cit.

8. Edith L. Mossman, loc. cit.

(d). Graph board.
(e). Diagrams.

Ye shall know the truth
And the Truth shall set ye free.[1]

(2). For the verbal-minded.
(a). Bulletin boards.
Newspaper clippings.
(b). Reference table.
Magazine and reference books.[2]
Source books.[3]
Booklets by pupils.[4]
Mathematical puzzles and scientific books
on related subjects.[5]

(3). For the hand-minded.
(a). Geometrical instruments.
Meter rulers.
Standboards of board measure.
Compasses.
Protractors.
Pantograph.
Slated globes.
Slide rules.[6]
Carpenter's level.
Transit.
Blackboard compasses.[7]

(b). Models.
Geometric solids.[8]

---

1. Mary R. Cook, Stimulate the Interest in Mathematics by Creating a Mathematical Atmosphere. The Mathematics Teacher April, 1931, pp. 248-254.

2. Edith L. Mossman, op. cit. pp. 423-430.

3. William Betz, op. cit. p. 185.

4. Mary R. Cook, loc. cit.

5. Gertrude E. Allen, Reorganization of Course of Study for Senior High School Mathematics, The National Council of Teachers of Mathematics, The Second Yearbook, 1927, pp. 267-268.

6. Gertrude E. Allen, loc. cit.

7. Mary R. Cook, loc. cit.

8. Edith L. Mossman, loc. cit.

The making of an attractive mathematics room will provide for the individual differences of the pupil, appealing to those who are verbal-minded, hand-minded, and art-minded. The library table being the product of the first-named, the models being the product of the hand-minded, and the posters, exhibits, graphs, etc, being made by the pupils artistically inclined.

(c). Graphs. The graphs suggested in the previous pages may be illuminated by colors and varied by polygon shapes; they are simply suggestive. Many pupils will be eager to make such a poster if the teacher gives them the idea. They may be painted, drawn in pencil or in crayon.

Lindberg flying across the Atlantic offers a challenge of "Why not fly in a straight line?" The course of a line over a tennis ball or other spherical model will demonstrate the answer.

The making of an attractive mathematics room will provide for the individual differences of the pupil, appealing to those who are verbal-minded, hand-minded, and art-minded. The library table being the product of the first-named, the models being the product of the hand-minded, and the posters, exhibits, graphs, etc. being made by the pupils artistically inclined.

(c). Graphs. The graphs suggested in the previous pages may be illuminated by colors and varied by polygon shapes; they are simply suggestive. Many pupils will be eager to make such a poster if the teacher gives them the idea. They may be painted, drawn in pencil or in crayon.

Lindberg flying across the Atlantic offers a challenge of "Why not fly in a straight line?" The course of a line over a tennis ball or other spherical model will demonstrate the answer.

D. Classroom Procedure.

1. General discussion. In large classes the writer has found no single method so powerful as the method of having the class develop the proof of the proposition which is written on the board by a student-secretary or by the teacher. Each pupil may copy the procedure at his desk which provides for hand-practice, and this is important as there is a big gap between perfect understanding of the visual and auditory impression, and the giving back of the work in writing. No other method, other than perhaps individual assignments at their desks, will keep the class so alert. A single exception of this might be in construction work in the early stages.

The old-fashioned method of having a pupil go to the board and write out a complete proposition resulted, and still results, in boredom for the others in the class. It is important that each pupil associate himself with the proposition; it must be class-discovered, class-built, class-criticized, class-owned, and this may only be accomplished by the class working together. This method provides for the leadership of those who are superior, and it also provides for those less able through the easier steps and the repeating processes.

After the proposition has been developed in one position, repeated developments with the figure in varying positions, and the construction lines varying accordingly, may provide further practice. Hands not at first raised now become active, and the teacher may inject a human touch by saying "Now the corn is popping better" as the hands increase in number as the proposition is repeated. Colored chalk adds greatly. A

D. Classroom Procedure.

1. General discussion. In large classes the writer has found no single method so powerful as the method of having the class develop the proof of the proposition which is written on the board by a student-secretary or by the teacher. Each pupil may copy the procedure at his desk which provides for hand-practice, and this is important as there is a big gap between perfect understanding of the visual and auditory impression, and the giving back of the work in writing. No other method, other than perhaps individual assignments at their desks, will keep the class so alert. A single exception of this might be in construction work in the early stages.

The old-fashioned method of having a pupil go to the board and write out a complete proposition resulted, and still results in boredom for the others in the class. It is important that each pupil associate himself with the proposition; it must be class-discovered, class-built, class-criticized, class-owned, and this may only be accomplished by the class working together. This method provides for the leadership of those who are superior, and it also provides for those less able through the easier steps and the repeating processes.

After the proposition has been developed in one position, repeated developments with the figure in varying positions, and the construction lines varying accordingly, may provide further practice. Hands not at first raised now become active, and the teacher may inject a human touch by saying "Now the corn is popping better" as the hands increase in number as the proposition is repeated. Colored chalk adds greatly. A

challenge of "No one can prove it when it is in this position" brings forth the efforts of those to whom challenges mean action.

2. Board work

a). See-it-grow method. In construction work the "see-it-grow" method of putting on a succession of figures, each showing a single step along the process of construction may be supplemented by each pupil at his eeat with the compass, copying the step. Slow progress is made in order to insure accuracy of procedure and thorough understanding.

b). Accuracy in directions. Accuracy in statement may be gained by having the pupil dictate a construction line to the teacher at the board. The pupil directs a perpendicular to be drawn to the radius. The teacher does so at some interior point. The pupil then says that he means at the end of the radius and the teacher intentionally chooses the wrong end which goes through the center of the circle. Finally the pupil directs the perpendicular to be drawn at the point of intersection of circle and radius.[1]

Geometry is a subject which because of its length of procedure and insufficiency of board space is not a fertile field for all working at the board at the same time. An exception to this is when the class is small. When construction work is to be done, certain sections of the class can be sent to the board in turns, and groups or pairs may work together on propositions.

1. J. O. Hassler, and R. R. Smith. The Teaching of Secondary Mathematics. p. 191.

challenge of "No one can prove it when it is in this position" brings forth the efforts of those to whom challenges mean action.

2. Board work

a). See-it-grow method. In construction work the "see-it-grow" method of putting on a succession of figures, each showing a single step along the process of construction may be supplemented by each pupil at his seat with the compass, copying the step. Slow progress is made in order to insure accuracy of procedure and thorough understanding.

b). Accuracy in directions. Accuracy in statement may be gained by having the pupil dictate a construction line to the teacher at the board. The pupil directs a perpendicular to be drawn to the radius. The teacher does so at some interior point. The pupil then says that he means at the end of the radius and the teacher intentionally chooses the wrong end which goes through the center of the circle. Finally the pupil directs the perpendicular to be drawn at the point of intersection of circle and radius.1

Geometry is a subject which because of its length of procedure and insufficiency of board space is not a fertile field for all working at the board at the same time. An exception to this is when the class is small. When construction work is to be done, certain sections of the class can be sent to the board in turns, and groups or pairs may work together on propositions.

1. J. O. Hassler, and R. R. Smith. The Teaching of Secondary Mathematics. p. 191.

The general procedure most used by the writer involves the following steps:

c). Class built.

Board demonstration--Class contributing.
Ear and eye approach.
Teacher in background as director, encourager, helper.
Pupil secretary at board.

Sections of the class at the board (construction work principally).
Teacher observes errors at early stage before they become crystallized. Sometimes groups or pairs of pupils are assigned a board task.

Assignment consists of that which has been learned in the class period as a minimum with optional exercises and originals in accordance with ability of pupil. (See section on individual differences). In general the practice should be correct practice. Assignment provides an extra hand-drill.

Checkup- At the beginning of the next period to catch any errors still remaining. (See section on testing).

As stated previously, the writer has quite consistently used the method of group development of a proposition using a pupil-secretary to place the work on the board. Each proposition is first treated as an original in which the various members of the class work together on determining the hypothesis and conclusion, the figure, steps and reasons for the steps, and the conclusion. At the first of the year, constant reference keeps in mind the necessity of the following steps for complete proof.

Hypothesis.
Conclusion.
Figure.
Plan.

The general procedure most used by the writer involves the following steps:

c). Class built.

Board demonstration--Class contributing.
Ear and eye approach.
Teacher in background as director, encourager, helper.
Pupil secretary at board.

Sections of the class at the board (construction work principally).
Teacher observes errors at early stage before they become crystallized. Sometimes groups or pairs of pupils are assigned a board task.

Assignment consists of that which has been learned in the class period as a minimum with optional exercises and originals in accordance with ability of pupil. (See section on individual differences). In general the practice should be correct practice. Assignment provides an extra hand-drill.

Checking. At the beginning of the next period to catch any errors still remaining. (See section on teaching).

As stated previously, the writer has quite consistently used the method of group development of a proposition using a pupil-secretary to place the work on the board. Each proposition is first treated as an original in which the various members of the class work together on determining the hypothesis and conclusion, the figure, steps and reasons for the steps, and the conclusion. At the first of the year, constant reference keeps in mind the necessity of the following steps for complete proof.

Hypothesis.
Conclusion.
Figure.
Plan.

Steps and reasons.
Conclusion.

Each pupil is taught not to memorize and is shown the reason for the necessity of each of the steps.

d). Place of the teacher. The teacher's part in this board work is to ask leading questions in case the class does not see its way out, and to pause over false steps so that time may be given for the discovery by the class of the weakness of the step. Encouragement to the timid; the hard parts to the more capable; the easier parts to the less capable so that all may feel success and contribute, is the aim. Repeating the proofs will draw in those members not successful on the first time for the proof.

Questions are asked in an effort to have the pupil see that if he asked himself these questions he would be able to solve without help.

The day of the pupil reciting gliby the memorized propositions at the board, while all the rest of the class are idle even while apparently politely listening, is gone. The board work must be dynamic, occupying the attention of all with all contributing. The proposition may be vividly outlined with the important points being given, especially in review; the plan of attack may be given; or if complete proof, each pupil will contribute his share and since no one knows when he is to be called upon to contribute the next step or to criticize the last, each one is alert and ready. Demonstration is for the class and to the class, rather than directed toward the teacher. Pupils

Steps and reasons.
Conclusion.

Each pupil is taught not to memorize and is shown the reason for the necessity of each of the steps.

d). Place of the teacher. The teacher's part in this board work is to ask leading questions in case the class does not see its way out, and to pause over false steps so that time may be given for the discovery by the class of the weakness of the step. Encouragement to the timid; the hard parts to the more capable; the easier parts to the less capable so that all may feel success and contribute, is the aim. Repeating the proofs will draw in those members not successful on the first time for the proof.

Questions are asked in an effort to have the pupil see that if he asked himself these questions he would be able to solve without help.

The day of the pupil reciting glibly the memorized propositions at the board, while all the rest of the class are idle even while apparently politely listening, is gone. The board work must be dynamic, occupying the attention of all with all contributing. The proposition may be vividly outlined with the important points being given, especially in review; the plan of attack may be given; or if complete proof, each pupil will contribute his share and since no one knows when he is to be called upon to contribute the next step or to criticize the last, each one is alert and ready. Demonstration is for the class and to the class, rather than directed toward the teacher. Pupils

watch for the weak points in the steps by the pupil.

e). Place of the demonstration by the individual pupil. After the proposition has been developed by the group, part of the period the following day may be devoted to having the class listen to a particular pupil demonstrate his written proof of the proposition at the board. The advantages here are that the group may evaluate the reasons, and the demonstration serves as another repetition with the challenge to find errors.

One writer[1] suggests the following evaluation for the demonstration.

f). Evaluation of parts of demonstration

	Points
Figure	2
Hypothesis	1
Conclusion	1
Construction	1
Proof	5

After this preliminary going over, the proposition may again be reviewed by changing the diagram and the notation, and calling on various pupils to give steps in the proof. This again keeps them on edge as they do not know when their turn is coming. Criticism of the preceding step is also asked at various times. Sometimes merely the plan of proof, or the important steps are reviewed.[2]

3. Desk Work

a). Hand Practice. An advantage of having pupils draw the figure and jot down the proof as it appears on the board is that

1. S. B. F. Rabourn, A Few Classroom Devices to Stimulate Interest in Mathematics, The Mathematics Teacher (October, 1927). pp. 328-333.

2. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics, pp. 247-249

watch for the weak points in the steps by the pupil.

e). Place of the demonstration by the individual pupil.

After the proposition has been developed by the group, part of the period the following day may be devoted to having the class listen to a particular pupil demonstrate his written proof of the proposition at the board. The advantages here are that the group may evaluate the reasons, and the demonstration serves as another repetition with the challenge to find errors.

One writer[1] suggests the following evaluation for the demonstration.

f). Evaluation of parts of demonstration

	Points
Figure	2
Hypothesis	1
Conclusion	1
Construction	1
Proof	5

After this preliminary going over, the proposition may again be reviewed by changing the diagram and the notation, and calling on various pupils to give steps in the proof. This again keeps them on edge as they do not know when their turn is coming. Criticism of the preceding step is also asked at various times. Sometimes merely the plan of proof, or the important steps are reviewed.[2]

3. Desk Work

a). Hand Practice. An advantage of having pupils draw the figure and jot down the proof as it appears on the board is that

1. C. E. F. Rabourn, A Few Classroom Devices to Stimulate Interest in Mathematics, The Mathematics Teacher (October, 1927). pp. 328-333.

2. Ernest R. Breslich, Problems in Teaching Secondary School Mathematics, pp. 247-249

it gives everyone hand practice. The teacher may pass from desk to desk assisting.

b). Drill sheets. Drill sheets are also favored by some teachers on summaries of equalities and parallels.

c). Direction sheets. Where a text is not used, sheets of directions insure practice in following directions and involve construction work. Work sheets involving new terms give a felt need for definitions.

d). Conference groups. Later, conference groups evaluate the work, and final discussion and evaluation come under the presiding of the teacher.

e). Note books, or work books. Standard notebooks or workbooks in which each pupil writes important introductory exercises, while the teacher directs the work orally, or from the blackboard are also used. At first, each step is the result of careful questions and oral responses or conclusions reached by the collective cooperation of the class. As the work progresses additional problems and exercises provide for the various ability levels. Each pupil is then expected to work at his own best rate.[1]

4. Grouping.

a). Strong and weak pupil. Good and poor student work at board together. Then two weaker students work together, after they have worked with the better students.[2]

1. William Betz, The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook, 1933. p. 144.

2. Gertrude V. Pratt, Popularizing Plane and Solid Geometry. The Mathematics Teacher, XXI (November, 1928) p. 413.

it gives everyone hand practice. The teacher may pass from desk to desk assisting.

b). Drill sheets. Drill sheets are also favored by some teachers on summaries of equalities and parallels.

c). Direction sheets. Where a text is not used, sheets of directions insure practice in following directions and involve construction work. Work sheets involving new terms give a felt need for definitions.

d). Conference groups. Later, conference groups evaluate the work, and final discussion and evaluation come under the presiding of the teacher.

e). Note books, or work books. Standard notebooks or work-books in which each pupil writes important introductory exer-cises, while the teacher directs the work orally, or from the blackboard are also used. At first, each step is the result of careful questions and oral responses or conclusions reached by the collective cooperation of the class. As the work progresses additional problems and exercises provide for the various ability levels. Each pupil is then expected to work at his own best rate.[1]

4. Grouping.

a). Strong and weak pupil. Good and poor student work at board together. Then two weaker students work together, after they have worked with the better students.[2]

1. William Betz, The Teaching of Intuitive Geometry, The Nation-al Council of Teachers of Mathematics, The Eighth Yearbook, 1933. p. 144.

2. Gertrude V. Pratt, Popularizing Plane and Solid Geometry. The Mathematics Teacher, XXI (November, 1928) p. 413.

b). Two sides. The writer has had the class select two leaders, who in turn choose sides and a geometry match, something like the old-fashioned spelling bee, may add fun. Definitions, summaries of equalities, or important methods, may be the questions for sides. When a member of one side fails, the other side has a chance, and if successful, draws a member from the losing side.

c). Row. Another method of grouping is to have a row of pupils responsible for certain originals. This encourages the teaching by the brighter pupils so that the poorer may have success.

All these methods of groupings are to give courage, success, cooperativeness, and discourage dishonesty.

5. Pupil helper.

a). Checking recitations. Not only may pupils help each other but they may be helpful to the teacher. Checking recitations, which does not involve evaluation, may be done by some pupil through the use of a tally sheet.

b). Written test on board. Placing a written lesson on the board just prior to the recitation may also be done by some pupil, or the putting on of figures for class demonstration may also save time.[1]

6. Tests.

a). Place in class period. One writer[2] says, "As to the recitation, I think written work should occupy the greater part

1. J. O. Hassler, loc. cit.
2. J. M. Jacobs, Reaching the Individual in Mathematics Instruction, School Science and Mathematics, (May, 1931) XXXI. pp. 575-585.

b). Two sides. The writer has had the class select two leaders, who in turn choose sides and a geometry match, something like the old-fashioned spelling bee, may add fun. Definitions, summaries of equalities, or important methods, may be the questions for sides. When a member of one side fails, the other side has a chance, and if successful, draws a member from the losing side.

c). Row. Another method of grouping is to have a row of pupils responsible for certain originals. This encourages the teaching by the brighter pupils so that the poorer may have success.

All these methods of grouping are to give courage, success, cooperativeness, and discourage dishonesty.

5. Pupil helper.

a). Checking recitations. Not only may pupils help each other but they may be helpful to the teacher. Checking recitations, which does not involve evaluation, may be done by some pupil through the use of a tally sheet.

b). Written test on board. Placing a written lesson on the board just prior to the recitation may also be done by some pupil, or the putting on of figures for class demonstration may also save time.[1]

6. Tests.

a). Place in class period. One writer[2] says, "As to the recitation, I think written work should occupy the greater part

1. J. O. Hassler, loc. cit.

2. J. M. Jacobs, Reasoning the Individual in Mathematics Instruction, School Science and Mathematics, (May, 1931) XXXI, pp. 578-583.

of the period. We can dispense with oral demonstrations. One person need not give a whole proposition. Break in and ask others to do it."[1]

b). Frequent need. Blaisdell[2] calls attention to the need of frequent testing to unify and correlate the cumulative subject of mathematics which otherwise would go neglected on review tests because of the pupil's concentration on the current topic.

c). Test on errors. Testing should be on the things needing testing. Common errors should form the subject of tests. Various devices may be used for the discovery and recording of these. One form might be a tally sheet at the top of which would appear the various errors and at the side would be the names of the pupils. A large enough square would be made by the vertical and horizontal lines so that check marks could be placed in the proper square opposite the pupil's name and under the correct error column. This would be a key sheet for the teacher.

(1). How record. In addition each pupil could keep his own sheet of errors in a note book. Code letters have been suggested for the errors involved in order to save time in writing when correcting papers. Each pupil would have a key to the code.[3]

1. J. M. Jacobs, loc. cit.

2. Walter H. Blaisdell, Daily Testing, The Mathematics Teacher XXVI (April, 1933) pp. 227-234.

3. G. Hapgood Parks, A Mathematics Teacher Seeks Guidance in Guidance, The Mathematics Teacher, (February, 1933) XXVI. pp. 89-98.

of the period. We can dispense with oral demonstrations. One person need not give a whole proposition. Break in and ask others to do it."[1]

b). Frequent need. Blisdell[2] calls attention to the need of frequent testing to unify and correlate the cumulative subject of mathematics which otherwise would go neglected on review tests because of the pupil's concentration on the current topic.

c). Test on errors. Testing should be on the things needing testing. Common errors should form the subject of tests. Various devices may be used for the discovery and recording of these. One form might be a tally sheet at the top of which would appear the various errors and at the side would be the names of the pupils. A large enough square would be made by the vertical and horizontal lines so that check marks could be placed in the proper square opposite the pupil's name and under the correct error column. This would be a key sheet for the teacher.

(1). How record. In addition each pupil could keep his own sheet of errors in a note book. Code letters have been suggested for the errors involved in order to save time in writing when correcting papers. Each pupil would have a key to the code.[3]

---

1. J. M. Jacobs, loc. cit.

2. Walter H. Blisdell, Daily Testing, The Mathematics Teacher XXVI (April, 1933) pp. 227-234.

3. G. Haywood Parks, A Mathematics Teacher Seeks Guidance in Guidance, The Mathematics Teacher, (February, 1933) XXVI. pp. 89-94.

(2). Common errors. Among the errors which are common might be noted:

a. Confusion of terms.
b. Failure to visualize figures in different positions.
c. Failure to think of figures other than conventional type.
d. Inability to use formulas.
e. Basing conclusions on appearances.
f. Using specific instead of general cases.[1]

d). Length. The five-minute test at the beginning of the period has been found useful to call the pupils to attention and to provide an incentive for daily preparation of the work.

e). Subject. This may consist of the various parts of the propositions, sometimes the plan of work or the important steps, sometimes the construction figures, sometimes completion tests on equality or similarity. In the short test, ease of correction and rapidity in administering are important phases. For this reason the less writing required of pupils the better. These short tests are factual in nature.

f). Value. In spite of the artificiality of the motive, the final examination seems to remain powerful in having the class master the necessary facts. In hot weather, near the fag end of the year, it provides a goal for final endeavors.

7. Assignment.

a). Suited to capacity of pupil. Various writers call attention to the shortness of the gaps which must prevail in the teaching of plane geometry. Text books have done considerable in meeting the requirements of this situation, but the

1. Alma M. Wolf, The Teaching of Secondary Mathematics, Boston University, M.A. Thesis, 1932. p. 44. Quotation from Diagnostic Tests in the Teaching of Plane Geometry, Carrie B. Ramsey, Colorado State Teachers' College, 1929.

(2). Common errors. Among the errors which are common might be noted:

a. Confusion of terms.
b. Failure to visualize figures in different positions.
c. Failure to think of figures other than conventional type.
d. Inability to use formulae.
e. Basing conclusions on appearances.
f. Using specific instead of general cases.[1]

d). Length. The five-minute test at the beginning of the period has been found useful to call the pupils to attention and to provide an incentive for daily preparation of the work.

e). Subject. This may consist of the various parts of the propositions, sometimes the plan of work or the important steps, sometimes the construction figures, sometimes completion tests on equality or similarity. In the short test, ease of correction and rapidity in administering are important phases. For this reason the less writing required of pupils the better. These short tests are factual in nature.

f). Value. In spite of the artificiality of the motive, the final examination seems to remain powerful in having the class master the necessary facts. In hot weather, near the fag end of the year, it provides a goal for final endeavors.

7. Assignment.

a). Suited to capacity of pupil. Various writers call attention to the shortness of the gaps which must prevail in the teaching of plane geometry. Text books have done considerable in meeting the requirements of this situation, but the

1. Alma M. Wolf, The Teaching of Secondary Mathematics, Boston University, M.A. Thesis, 1932. p. 44. Quotation from Diagnostic Tests in the Teaching of Plane Geometry, Carrie B. Ramsey, Colorado State Teachers' College, 1929.

teacher of geometry can make these gaps even shorter by calling attention to what the text book has done. For instance, in referring to exercises after the proposition, especially early in the text, the teacher may note that the exercises involve the proposition just studied, and that they vary very slightly.

b). Preparation for economical study. For example, in regard to the exercises which center about the proof of congruency when two sides and the included angle are equal, the two sides may be given directly and the angle may be equal either because it is a vertical angle, or because the word "perpendicular" occurs. When a line is the missing quantity, the word "biset" or "isosceles" may provide it. In any event, if the pupil is taught to see the given quantities and how to search for the missing quantities, he becomes encouraged in his task. The task cannot be made too clear for the pupil. He should watch for the variations, such as different sets of triangles chosen, methods of lines and angles being made equal, and remember that the final aim is to make the figure agree with the conditions for congruence, such as two sides and the included angle.

8. Failures in geometry.

a). A testing program to guard against failure. After a number of years of experience in which the writer gave a test on the entire proposition after the class studied it, which resulted in failure of many pupils and discouragement on the part of the teacher and which included a mental set of the teacher toward the ability of the pupil failing, for often the

teacher of geometry can make these gaps even shorter by calling attention to what the text book has done. For instance, in referring to exercises after the proposition, especially early in the text, the teacher may note that the exercises involve the proposition just studied, and that they vary very slightly.

b). Preparation for economical study. For example, in regard to the exercises which center about the proof of congruency when two sides and the included angle are equal, the two sides may be given directly and the angle may be equal either because it is a vertical angle, or because the word "perpendicular" occurs. When a line is the missing quantity, the word "bisect" or "isosceles" may provide it. In any event, if the pupil is taught to see the given quantities and how to search for the missing quantities, he becomes encouraged in his task. The task cannot be made too clear for the pupil. He should watch for the variations, such as different sets of triangles chosen, methods of lines and angles being made equal, and remember that the final aim is to make the figure agree with the conditions for congruence, such as two sides and the included angle.

8. Failures in geometry.

a). A testing program to guard against failure. After a number of years of experience in which the writer gave a test on the entire proposition after the class studied it, which resulted in failure of many pupils and discouragement on the part of the teacher and which included a mental set of the teacher toward the ability of the pupil failing, for often the

answers seemed to the teacher absurd, a complete change in the method of testing was made.

(1). The start for short tests with careful preparation. For three years the author has given tests with very short gaps and has tried to carefully prepare the pupils for successful work. At the present time he has selected a testing program which involves a constantly increasing number of questions on the proposition which gradually work up to the presentation of the entire proposition, which latter is deferred until the middle or end of the third month in teaching.

(2). Postponement of complete presentation of the theorem for three months. It has been found that this method saves the patience of the corrector of the papers and is much more effective in giving power to the pupils in writing out propositions in full with real understanding, as will be shown later by the results of the first complete period test.

(3). Examples of the short test. These short tests, from three to ten minutes in length, start with simple construction work and with no formal demonstration. The first question on demonstration will consist simply of marking off the hypothesis and conclusion by a vertical line; this will follow the unit on the topic of picking out the hypothesis previously written. A number of theorems are given and the class will insert the vertical line in the theorem at the proper place. Following this test may come a memory test of naming the seven steps in the presentation of a theorem as follows:

answers seemed to the teacher absurd, a complete change in the method of testing was made.

(1). The start for short tests with careful preparation. For three years the author has given tests with very short pages and has tried to carefully prepare the pupils for successful work. At the present time he has selected a testing program which involves a constantly increasing number of questions on the proposition which gradually work up to the presentation of the entire proposition, which latter is deferred until the middle or end of the third month in teaching.

(2). Postponement of complete presentation of the theorem for three months. It has been found that this method saves the patience of the corrector of the papers and is much more effective in giving power to the pupils in writing out propositions in full with real understanding, as will be shown later by the results of the first complete period test.

(3). Examples of the short test. These short tests, from three to ten minutes in length, start with simple construction work and with no formal demonstration. The first question on demonstration will consist simply of marking off the hypothesis and conclusion by a vertical line; this will follow the unit on the topic of picking out the hypothesis previously written. A number of theorems are given and the class will insert the vertical line in the theorem at the proper place. Following this test may come a memory test of naming the seven steps in the presentation of a theorem as follows:

1. Selecting the hypothesis.
2. Selecting the conclusion.
3. Drawing the figure.
4. Making the plan.
5. Construction the auxiliary line.
6. Steps and reasons.
7. The conclusion.

This test is important for the reason that the pupil should have these seven steps so thoroughly in mind that they can be brought back automatically. This makes it possible for him later, when presenting an entire proposition, to focus on the actual work, without worry in regard to whether he has considered the proposition in all the necessary parts.

A third type of test may next come which might consist of one to three questions on any of the steps mentioned above in connection with a particular theorem. The number of questions may constantly increase until the pupil has answered questions which involve in an isolated, but suggestive way, the entire proposition. Completion tests may be dictated in connection with these which have the advantage of ease of correction and ease of writing. However, they have the disadvantage of having only the auditory approach unless the school is well supplied with clerical help for mimeograph work, and occasionally there will be a pupil or two who will have difficulty in understanding them.

In all cases, the written test is preceded by careful preparation. If it is a construction test, the pupil practises at the board, at his seat, and in his home assignment. If it is a test which may be answered by the written sentence, the pupil is drilled on the questions, and a day or two is allowed for the work to seep in before the test is given. The short test is

1. Selecting the hypothesis.
2. Selecting the conclusion.
3. Drawing the figure.
4. Making the plan.
5. Construction the auxiliary line.
6. Steps and reasons.
7. The conclusion.

This test is important for the reason that the pupil should have these seven steps so thoroughly in mind that they can be brought back automatically. This makes it possible for him later, when presenting an entire proposition, to focus on the actual work, without worry in regard to whether he has considered the proposition in all the necessary parts.

A third type of test may next come which might consist of one to three questions on any of the steps mentioned above in connection with a particular theorem. The number of questions may constantly increase until the pupil has answered questions which involve in an isolated, but suggestive way, the entire proposition. Completion tests may be dictated in connection with these which have the advantage of ease of correction and ease of writing. However, they have the disadvantage of having only the auditory approach unless the school is well supplied with clerical help for mimeograph work, and occasionally there will be a pupil or two who will have difficulty in understanding them.

In all cases, the written test is preceded by careful preparation. If it is a construction test, the pupil practices at the board, at his seat, and in his home assignment. If it is a test which may be answered by the written sentence, the pupil is drilled on the questions, and a day or two is allowed for the work to seep in before the test is given. The short test is

given without warning; the longer tests are given with warning. The aim in all cases is to have the pupil successful in these early tests; the ones who fail in the short tests are those who have not prepared their lesson for the day; these frequent tests incite them to daily preparation, and seldom does a pupil fail twice consecutively.

After these short term tests, a group of propositions are studied over a period of three or four days. These are class-built as described in the unit on class procedure. Repetitions are given and the weaker members of the class are encouraged to make their contributions. Notice is given that a written test is to be given on these propositions. The written test itself consists the first time of a single proposition or sometimes a choice of two out of three. The aim again is to make the pupil successful.

(4). The sliding scale test. After two or three of such tests, a sliding scale test is given. For instance, the writer has just given at the end of November, his first sliding scale test. It was explained to the class that success in one proposition would give a passing mark and was the minimum assignment; in the marking system this would mean a C; completion of two of the theorems would net a B which is college entrance credit; while completion of the three would mean an A rank, superior. Of the thirty taking the test, 8 successfully completed the three; 12 additional pupils completed two, while the remainder of the class succeeded in doing a single theorem.

given without warning; the longer tests are given with warning. The aim in all cases is to have the pupil successful in these early tests; the ones who fail in the short tests are those who have not prepared their lesson for the day; these frequent tests incite them to daily preparation, and seldom does a pupil fail twice consecutively.

After these short term tests, a group of propositions are studied over a period of three or four days. These are class-built as described in the unit on class procedure. Repetitions are given and the weaker members of the class are encouraged to make their contributions. Notice is given that a written test is to be given on these propositions. The written test itself consists the first time of a single proposition or sometimes a choice of two out of three. The aim again is to make the pupil successful.

(4). <u>The sliding scale test.</u> After two or three of such tests, a sliding scale test is given. For instance, the writer has just given at the end of November, his first sliding scale test. It was explained to the class that success in one proposition would give a passing mark and was the minimum assignment; in the marking system this would mean a D; completion of two of the theorems would net a B which is college entrance credit; while completion of the three would mean an A rank, superior. Of the thirty taking the test, 8 successfully completed the three; 12 additional pupils completed two, while the remainder of the class succeeded in doing a single theorem.

b). Types of pupils. It might be well to explain here that the class members are drawn from various high school courses, not more than half are in the scientific course. A number of the pupils have taken either a single course in algebra or no mathematics at all in high school.

(1). Geometry as a cultural subject. Geometry is thought of as a general cultural subject, somewhat in the same line as junior business training, history, civics, general science or biology. It is the one mathematics course that is thought of in this way.

(2). Reasons for taking geometry. As mentioned in the previous section there is no selection of geometry pupils. In some cases pupils are advised to take the course for various reasons as follows:

1. Higher institutions require it.
2. Mechanical drawing and manual training work is connected with it.
3. A pupil has a naturally debative or argumentative type of mind, and therefore has a natural interest in the problem work and in the demonstration.
4. The pupil is seeking a subject general in character to complete his program; if a boy, the choice is quite likely to be geometry.

About sixty per cent of the pupils in school take this subject during their high school course; most of these take it during the sophomore year, but a few include it in their program in the junior or senior year.

c). Percentage of failures. As outlined in the previous topic on testing, careful plans are made to insure the success of the pupil. Because of this, failures have been reduced

b). Types of pupils. It might be well to explain here that the class members are drawn from various high school courses, not more than half are in the scientific course. A number of the pupils have taken either a single course in algebra or no mathematics at all in high school.

(1). Geometry as a cultural subject. Geometry is thought of as a general cultural subject, somewhat in the same line as junior business training, history, civics, general science or biology. It is the one mathematics course that is thought of in this way.

(2). Reasons for taking geometry. As mentioned in the previous section there is no selection of geometry pupils. In some cases pupils are advised to take the course for various reasons as follows:

1. Higher institutions require it.
2. Mechanical drawing and manual training work is connected with it.
3. A pupil has a naturally debative or argumentative type of mind, and therefore has a natural interest in the problem work and in the demonstration.
4. The pupil is seeking a subject general in character to complete his program; if a boy, the choice is quite likely to be geometry.

About sixty per cent of the pupils in school take this subject during their high school course; most of these take it during the sophomore year, but a few include it in their program in the junior or senior year.

c). Percentage of failures. As outlined in the previous topic on teaching, careful plans are made to insure the success of the pupil. Because of this, failures have been reduced

considerably in geometry. Ordinarily they will average perhaps not more than 5% of the class. Possibly if one considers the one or two who wander into the course and wander out again the first week in school, the failures might mount to 10%, but at no time will they rise above this.

As inferred above, and throughout the entire thesis, in the opinion of the writer, it is possible to make geometry both interesting and beneficial to almost all of the pupils who enter the class. A single exception might be those of particularly low capacity who have no interests which bear on geometry.

considerably in geometry. Ordinarily they will average perhaps not more than 5% of the class. Possibly if one considers the one or two who wander into the course and wander out again the first week in school, the failures might mount to 10%, but at no time will they rise above this.

As inferred above, and throughout the entire thesis, in the opinion of the writer, it is possible to make geometry both interesting and beneficial to almost all of the pupils who enter the class. A single exception might be those of particularly low capacity who have no interests which bear on geometry.

E. The Drill.

1. Drill as part of the learning process.

It is noted in the conclusion that geometry is primarily a problem subject, but with the aim of appreciation being particularly strong in the beginning of geometry and running through the entire work to a considerable extent.

The mastery of the fundamental knowledge offers a smaller field, but none the less important. This mastery involves the technique of drill procedure, and though the total proportion of time so spent is small when compared to the time spent on the problem situation, or the time spent in developing appreciation, yet drill must not be neglected, nor must it be applied in a haphazard manner. Only by keeping the tools, the fundamental facts, shiny, can they be used. When and where to drill is a vital matter. Too much drill or drilling on the wrong work does as much damage as not drilling at all.

The following unit on drill attempts a suggestive procedure by which the pupil can see the value of drill, work on which the drill procedure may be properly emplyed, the elements of a successful drill procedure, the goal of the drill, and suggested class procedure on drill.

> "How does learning in fact take place? (1) Repetition is a factor in learning known to all. (2) An inclusive "set" which shall predispose the attention, focus available inner resources, and secure repetition is a necessary condition less[1]

1. The Problem of Mathematics in Secondary Education, Bulletin, 1920, No. 1. p. 12.

E. The Drill.

1. Drill as part of the learning process.

It is noted in the conclusion that geometry is primarily a problem subject, but with the aim of appreciation being particularly strong in the beginning of geometry and running through the entire work to a considerable extent.

The mastery of the fundamental knowledge offers a smaller field, but none the less important. This mastery involves the technique of drill procedure, and though the total proportion of time so spent is small when compared to the time spent on the problem situation, or the time spent in developing appreciation, yet drill must not be neglected, nor must it be applied in a haphazard manner. Only by keeping the tools, the fundamental facts, shiny, can they be used. When and where to drill is a vital matter. Too much drill or drilling on the wrong work does as much damage as not drilling at all.

The following unit on drill attempts a suggestive procedure by which the pupil can see the value of drill, work on which the drill procedure may be properly employed, the elements of a successful drill procedure, the goal of the drill, and suggested class procedure on drill.

> "How does learning in fact take place? (1) Repetition is a factor in learning known to all. (2) An inclusive "set" which shall predispose the attention, focus available inner resources, and secure repetition is a necessary condition less[1]

---

1. The Problem of Mathematics in Secondary Education, Bulletin, 1920, No. 1, p. 12.

"commonly considered. (3) The effect of accompanying satisfaction to foster habit formation is a third factor to be noted. These three facts are necessary, then, to adequate consideration of the problem of method. It accords with these considerations and with undisputed observation that, other things being equal, any item is more readily learned if its bearing and need are definitely recognized. The felt need predisposes attention, calls into play accessory mental resources, and in proportion to its strength secures the necessary repetition. As the need is met, satisfaction ensues. All factors thus cooperate to fix in place the new item of knowledge. The element of felt need thus secures not only the learning of the new item, but it has at the same time called into play the allied intellectual resources so that new and old are welded together in effective organization with reference to the need which originally motivated the process."[1]

2. Necessity of drill.

The class may be addressed somewhat as follows: Your classmate could solve this problem because she had at hand the facts which could be applied. Just as in manual training, you must keep the tools in order and shiny, so in plane geometry you must keep certain facts in order and be able to bring them to mind automatically for the purpose of solving the problem. In any factory containing machine parts, the separate shelves contain the various parts. Unorganized knowledge and knowledge not readily available is worth little.

If you smile at Mr. Equal Angle, and say, "Good morning", he will smile back at you. You recognize him as being Mr. Equal Angle because he answers the conditions, for instance, of being one of two corresponding angles of parallel lines.

"Mr. Parallelogram", I see you have on your coat of

1. The Problem of Mathematics in Secondary Education, p. 12. op. cit.

"commonly considered. (3) The effect of accompanying satisfaction to foster habit formation is a third factor to be noted. These three facts are necessary, then, to adequate consideration of the problem of method. It accords with these considerations and with undisputed observation that, other things being equal, any item is more readily learned if its bearing and need are definitely recognized. The felt need predisposes attention, calls into play accessory mental resources, and in proportion to its strength secures the necessary repetition. As the need is met, satisfaction ensues. All factors thus cooperate to fix in place the new item of knowledge. The element of felt need thus secures not only the learning of the new item, but it has at the same time called into play the allied intellectual resources so that new and old are welded together in effective organization with reference to the need which originally motivated the process."[1]

2. Necessity of drill.

The class may be addressed somewhat as follows: Your classmate could solve this problem because she had at hand the facts which could be applied. Just as in manual training, you must keep the tools in order and shiny, so in plane geometry you must keep certain facts in order and be able to bring them to mind automatically for the purpose of solving the problem. In any factory containing machine parts, the separate shelves contain the various parts. Unorganized knowledge and knowledge not readily available is worth little.

If you smile at Mr. Equal Angle, and say, "Good morning", he will smile back at you. You recognize him as being Mr. Equal Angle because he answers the conditions, for instance, of being one of two corresponding angles of parallel lines.

"Mr. Parallelogram", I see you have on your coat of

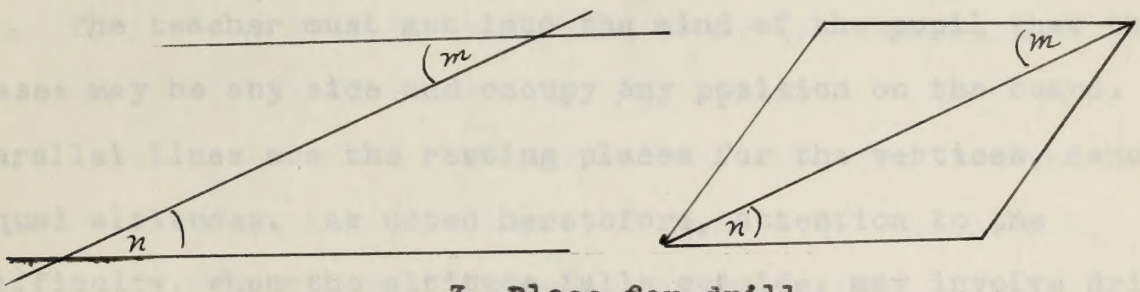
1. The Problem of Mathematics in Secondary Education, p. 12. op. cit.

"opposite side are parallel" and that you are wearing your trousers of "opposite sides equal" and your shoes of "adjacent angles are supplementary".

Joseph A. Nyeberg writes:[1] "How can the class be taught to select the correct angles that must be proved in order to get certain lines parallel?

"If several pupils cannot select the angles correctly, how should the trouble be remedied? Can the teacher expect the trouble will gradually be overcome as the recitation proceeds, or should the day's work be set aside and the subject of angles relating to two lines and the transveral be retaught at once."

Ernest R. Breslich notes:[2] "A pupil may know the facts of geometry and be unable to recognize them in situations that are new and complicated. For example he may readily note that the alternate-interior angles formed by two parallel lines and a transveral are equal when it appears in the first of the two figures below but fail to recognize it in the second."



3. Place for drill.

"It is also a good drill to enumerate the properties of the figure which are known in consequence of data; what lines, angles, triangles are equal; what lines are parallel, perpendicular; what figures are of equal area, etc. A list of the chief things to look for might be helpful in the pupil's hands."[3]

1. Verbatim Report of a Recitation in Geometry, School Science and Mathematics, XXXIII, (February, 1933) p. 68.

2. Problems in Teaching Secondary School Mathematics, pp. 289-290

3. J. W. A. Young, The Teaching of Mathematics in the Elementary and Secondary School. p. 267.

"opposite side are parallel" and that you are wearing your trousers of "opposite sides equal" and your shoes of "adjacent angles are supplementary".

Joseph A. Myeberg writes:1 "How can the class be taught to select the correct angles that must be proved in order to get certain lines parallel?

"If several pupils cannot select the angles correctly, how should the trouble be remedied? Can the teacher expect the trouble will gradually be overcome as the realization proceeds, or should the day's work be set aside and the subject of angles relating to two lines and the transversal be taught at once."

Ernest R. Breslich notes:2 "A pupil may know the facts of geometry and be unable to recognize them in situations that are new and complicated. For example he may readily note that the alternate-interior angles formed by two parallel lines and a transversal are equal when it appears in the first of the two figures below but fail to recognize it in the second."

3. Place for drill.

"It is also a good drill to enumerate the properties of the figure which are known in consequence of data; what lines, angles, triangles are equal; what lines are parallel, perpendicular; what figures are of equal area, etc. A list of the chief things to look for might be helpful in the pupil's hands."3

1. Verbatim Report of a Recitation in Geometry, School Science and Mathematics, XXXIII, (February, 1933) p. 68.
2. Problems in Teaching Secondary School Mathematics, pp. 289-290
3. J. W. A. Young, The Teaching of Mathematics in the Elementary and Secondary School. pp.267.

a). Drill on equivalent triangles. Since the area of the triangle is the "core" area of the polygon, as all other areas are built on it, first drill on the triangle as an equivalent figure when bases and altitudes are equal. This drill will be first on triangles having vertices lying in a line parallel to the base. The triangles are equal if they have the same or equal bases.

A variation is the drill on showing that triangles with equal bases very as their altitudes and the converse of this. All this work may be made visual by drawing many triangles, first in simple situations, then in more difficult.

The teacher must get into the mind of the pupil that the bases may be any side and occupy any position on the board. Parallel lines are the resting places for the vertices, denoting equal altitudes. As noted heretofore, attention to the difficulty, when the altitude falls outside, may involve drill work.

(a). Drill on equivalent triangles. Since the area of the triangle is the "core" area of the polygon, as all other areas are built on it, first drill on the triangle as an equivalent figure when bases and altitudes are equal. This drill will be first on triangles having vertices lying in a line parallel to the base. The triangles are equal if they have the same or equal bases.

A variation is the drill on showing that triangles with equal bases vary as their altitudes and the converse of this. All this work may be made visual by drawing many triangles, first in simple situations, then in more difficult.

The teacher must get into the mind of the pupil that the bases may be any side and occupy any position on the board. Parallel lines are the resting places for the vertices, denoting equal altitudes. As noted beneficiary, attention to the difficulty, when the altitude falls outside, may involve drill work.

Ernest R. Breslich notes the following places where drill can be used:[1]

"Two line segments equal.
Two angles equal.
Lines parallel.
Lines perpendicular.
Triangles similar.
Triangles congruent.
A quadrilateral as a parallelogram".

4. A Successful Start.

To make the drill work effective at the start, J. W. A. Young suggests:[2]

"a. Pupil sets goal: selects task: finds own weaknesses.
b. Pupil understands reason for practice.
c. Pupil recognizes and selects own errors for drill."

The pre-inventory test will satisfy "a" and "c".

This may consist of a written test which is made as simple, as far as writing is concerned, as possible. Mimeographed sheets handed out will have different units.

Angles are equal when:--and then there will be the necessary number of blank lines all numbered. Pupils are encouraged to use geometric symbolism in writing their statements, using symbols for angles, triangles, etc.

Henry L. Morrison writes:[3]

"Mastery formula:
Pre-test
Teach
Teach the result
Adapt procedure
Teach and test again
All going to the center of mastery.

1. Ernest R. Breslich, op. cit.
2. J. W. A. Young, op. cit.
3. The Practice of Teaching In The Secondary School, pp. 79-99.

Ernest R. Breslich notes the following places where drill can be used:[1]

"Two line segments equal.
Two angles equal.
Lines parallel.
Lines perpendicular.
Triangles similar.
Triangles congruent.
A quadrilateral as a parallelogram".

4. A Successful Start.

To make the drill work effective at the start, J. W. A. Young suggests:[2]

"a. Pupil sets goal: selects task: finds own weaknesses.
b. Pupil understands reason for practice.
c. Pupil recognizes and selects own errors for drill."

The pre-inventory test will satisfy "a" and "c".

This may consist of a written test which is made as simple as far as writing is concerned, as possible. Mimeographed sheets handed out will have different units.

Angles are equal when:--and then there will be the necessary number of blank lines all numbered. Pupils are encouraged to use geometric symbolism in writing their statements, using symbols for angles, triangles, etc.

Henry L. Morrison writes:[3]

"Mastery Formula:
Pre-test
Teach
Teach the result
Adapt procedure
Teach and test again
All going to the center of mastery.

---

1. Ernest R. Breslich, op. cit.
2. J. W. A. Young, op. cit.
3. The Practice of Teaching in The Secondary School, pp. 78-99.

"Advantages of pre-test:

1. Orients the teacher for intelligent approach.
2. Gives guide post in progress.
3. Makes redirected teacher.
4. Finds trouble.
5. Teaches to mastery.
6. Shows types of pupils who fail.
7. Shows reasons for failure.

"Pre-test orients the teacher: tends to establish in the minds of the pupils a connection between prospective learning and present attainments.

"Supplementary work should be for the superior: if released from class work, personalities are unfavorably affected in that they were given an utterly false sense of their intellectual attainments."[1]

5. Significant practice.

Various writers make the following suggestions for successful drill work:

"It is a good plan for students to keep notebooks and in review of a unit to outline and summarize. For example, they can summarize (1) ways of proving triangles congruent; (2) ways of proving lines equal; (3) ways of proving angles equal; (4) ways of proving a quadrilateral; a parallelogram; (5) ways of proving lines parallel; (6) ways of proving triangles similar; (7) facts they have proved about parallel lines; (8) facts they have proved about regular polygons; (9) facts they have proved about two intersecting secants of a circle."[2]

"Drill must be: Individual.
Specific.
Standardized.
For the new skills.
Have scoring technique where pupil can watch his daily growth.
In natural setting.
In diminishing amounts and at increasing intervals.
Have more interesting aspects taught first".[3]

1. Henry L. Morrison, op. cit. pp. 79-99.

2. Gertrude Allen, "An Experiment in Redistribution of Material for High School Geometry", The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), p. 82.

3. Raleigh Schloring, "Suggestions for the Solution of the Important Problem That Has Arisen in the Last Quarter of the Century", The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), p. 105.

"Advantages of pre-test:
1. Orients the teacher for intelligent approach.
2. Gives guide post in progress.
3. Makes redirected teacher.
4. Finds trouble.
5. Teaches to mastery.
6. Shows types of pupils who fail.
7. Shows reasons for failure.
"Pre-test orients the teacher: tends to establish in the minds of the pupils a connection between prospective learning and present attainments.
"Supplementary work should be for the superior: it released from class work personalities are unfavorably affected in that they were given an utterly false sense of their intellectual attainments."[1]

5. Significant practice.

Various writers make the following suggestions for successful drill work:
"It is a good plan for students to keep notebooks and in review of a unit to outline and summarize. For example, they can summarize (1) ways of proving triangles congruent; (2) ways of proving lines equal; (3) ways of proving angles equal; (4) ways of proving a quadrilateral a parallelogram; (5) ways of proving lines parallel; (6) ways of proving triangles similar; (7) facts they have proved about parallel lines; (8) facts they have proved about regular polygons; (9) facts they have proved about two intersecting secants of a circle."[2]

"Drill must be: Individual.
Specific.
Standardized.
For the new skills.
Have scoring technique where pupil can watch his daily growth.
In natural setting.
In diminishing amounts and at increasing intervals.
Have more interesting aspects taught first".[3]

---

1. Henry L. Morrison, op. cit. pp. 79-99.

2. Gertrude Allen, "An Experiment in Redistribution of Material for High School Geometry", The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), p. 82.

3. Raleigh Schorling, "Suggestions for the Solution of the Important Problem That Has Arisen in the Last Quarter of the Century", The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), p. 105.

Guy M. Wilson makes the following suggestions:[1]

Chart kept of errors made by various pupils; they have a copy; teacher has master chart of all pupils. This makes the drill individual and specific; eliminates that already known; makes simple and understandable; limits in amount; keeps meaning first and learning first.

Systematizing: This comes from the pre-test itself and from the note books. Teacher's key chart and pupil's individual need chart will point to the weak points necessary to learn more thoroughly by drill by self prior to oral and written test.

Repetition: Interest retained by:

1. Contest race by groups. Has all the power of socialization; audience situation; group power over the individual.
2. Games.
3. Display of best papers.
4. Release from further drill for other work.
5. Repeat necessity to keep tools bright: can do the work if in order: can prove the propositions.
6. Necessity of having margin of safety: must make responses so automatic they come at will and at call.
7. Oral drills: must succeed before group.
8. Written inventory: must get good mark so as to go on.
9. Cite story of swimmer coming to surface: Cannot see him until above water: so with repetition: one more may be effective enough to register: if almost learned, come the rest of the way.
10. Sees definite task: goal release when reached.
11. Chart of daily growth.
12. Repetition itself satisfying: familiarity: getting easier: satisfaction from success at achievement.
13. Graph for bright child.

6. Typical class procedure.

The author has utilized the class period for drill in the following manner:

The oral drill is carried out while superior students may

1. Boston University Summer School, Boston, Mass. 1933.

Guy M. Wilson makes the following suggestions:[1]

Chart kept of errors made by various pupils; they have a copy; teacher has master chart of all pupils. This makes the drill individual and specific; eliminates that already known; makes simple and understandable; limits in amount; keeps meaning first and learning first.

Systematizing: This comes from the pre-test itself and from the note books. Teacher's key chart and pupil's individual need chart will point to the weak points necessary to learn more thoroughly by drill by self prior to oral and written test.

Repetition: Interest retained by:

1. Contest race by groups. Has all the power of socialization; audience situation; group power over the individual.
2. Games.
3. Display of best papers.
4. Releases from further drill for other work.
5. Repeat necessity to keep tools bright: can do the work if in order: can prove the propositions.
6. Necessity of having margin of safety; must make responses so automatic they come at will and at call.
7. Oral drills; must succeed before group.
8. Written inventory: must get good mark so as to go on.
9. Cite story of swimmer coming to surface: Cannot see him until above water: so with repetition: one more may be effective enough to register: if almost learned. come the rest of the way.
10. Sees definite task: goal releases when reached.
11. Chart of daily growth.
12. Repetition itself satisfying: familiarity: getting easier: satisfaction from success at achievement.
13. Graph for bright child.

6 Typical class procedure.

The author has utilized the class period for drill in the following manner:

The oral drill is carried out while superior students may

1. Boston University Summer School, Boston, Mass. 1932.

be preparing a special problem at the board or working at their seats. It is rapid, quick-fire, and of short duration, usually at the beginning of the period. If a written test is given, it is usually from five to ten minutes duration and entails a minimum of writing by using mimeographed forms and geometric symbolism as much as possible. It is written in such form that it is easy to correct and put on a chart. Each pupil also keeps a chart.

Drill material in geometry is rather cumulative--that is, it involves all of the material as the year wears on. It is distributive, lessening in emphasis on the old material as the new is taken up, but at intervals re-tests are given for all the drill material to be mastered. A game may be played in which the class chooses leaders who pick sides and they stand much as in the old-fashioned spelling bee with either leaders or teachers asking questions and going from side to side as they fail. Notice ahead, of course, results in attempts at perfect scores. The weak do not sit down but go to opposite side.

Application on drill procedure involves the following items: a. In increasing complexity of situations; in case the pupil has difficulty in seeing it, erase the added part and reveal to him the original drill material.

b. Link up: if you know this, you can solve this. Relate to practical problems interesting to him; if manual training, those problems, etc.

c. Recreations.

d. Interrelations with occupations.

e. Interrelations with science.

be preparing a special problem at the board or working at their seats. It is rapid, quick-fire, and of short duration, usually at the beginning of the period. If a written test is given, it is usually from five to ten minutes duration and entails a minimum of writing by using mimeographed forms and geometric symbolism as much as possible. It is written in such form that it is easy to correct and put on a chart. Each pupil also keeps a chart.

Drill material in geometry is rather cumulative--that is, it involves all of the material as the year wears on. It is distributive, lessening in emphasis on the old material as the new is taken up, but at intervals re-tests are given for all the drill material to be mastered. A game may be played in which the class chooses leaders who pick sides and they stand much as in the old-fashioned spelling bee with either leaders or teachers asking questions and going from side to side as they fail. Notice ahead, of course, results in attempts at perfect scores. The weak do not sit down but go to opposite side.

Application on drill procedure involves the following items: a. In increasing complexity of situations; in case the pupil has difficulty in seeing it, erase the added part and reveal to him the original drill material.

b. Link up: if you know this, you can solve this. Relate to practical problems interesting to him; if manual training, those problems, etc.

c. Recreations.

d. Interrelations with occupations.

e. Interrelations with science.

7. Topical summary.

Drill material--100% mastery:

See use and application.

Relate to present interests.

Make a game.

Understand task.

Eliminate that already known.

Systematize the drill: know the goal.

All successful.

Clear meaning.

Teaching scheme.

Always keep behind a. Meaning.

b. What the child has learned.

Meaning first: learning first: correct image: then drill.

Simple.

Inventory at start and end.

Provision for variety of procedure.

Amount limited.

Automatic response.

Repetition key point.

Repetition effective in proportion to interest.

Observation of time limits.

All responses to be correct.

Gradually lengthening intervening periods.

Graph meaningful to bright child[1].

Guy M. Wilson, loc. cit.

7. Topical summary.

Drill material--100% mastery:

See use and application.

Relate to present interests.

Make a game.

Understand task.

Eliminate that already known.

Systematize the drill: know the goal.

All successful.

Clear meaning.

Teaching scheme.

Always keep behind a. Meaning.

b. What the child has learned.

Meaning first: learning first: correct image:

then drill.

Simple.

Inventory at start and end.

Provision for variety of procedure.

Amount limited.

Automatic response.

Repetition key point.

Repetition effective in proportion to interest.

Observation of time limits.

All responses to be correct.

Gradually lengthening intervening periods.

Graph meaningful to bright child[1].

[1] Guy M. Wilson, loc. cit.

8. Comment.

It is possible that this section on drill may be overemphasized. The Morrison Mastery Formula does not apply very fully to plane geometry. Nevertheless, there is opportunity for repetition. While the subject is primarily an appreciation subject, yet there needs to be some repetition for drill purposes. It is to be emphasized that although the purpose is not mastery, yet there needs to be some mastery in order to go forward.

B. Comment.

It is possible that this section on drill may be over-emphasized. The Morrison Mastery Formula does not apply very fully to plane geometry. Nevertheless, there is opportunity for repetition. While the subject is primarily an appreciation subject, yet there needs to be some repetition for drill purposes. It is to be emphasized that although the purpose is not mastery, yet there needs to be some mastery in order to go forward.

F. Miscellaneous Devices for Motivation.

1. Clarity.

a). Introduction. While the author, for convenience, has grouped the miscellaneous devices under two topics of clarity and attitude, yet the reader will realize that there are no sharp distinctions between these two aims. Anything that is made clear has a tendency to produce a favorable attitude, and conversely anything which produces a favorable attitude must be made clear.

However, there are some devices which have a stronger leaning toward clarity as a characteristic aim, while others lean more toward producing a favorable attitude. For this reason, it may be well to consider the devices under the two groupings, rather than under one.

b). The figure.

(1). The line. If the subject is put on the board so that the pupil can easily see it, he will be more apt to be attentive. Much of the power of the moving pictures comes from its vividness because of its large size. The figure in geometry should be large and placed on the portion of the board best seen by the class as a whole. Lines showing up to the best advantage should be drawn; these may be ordinary lines of white chalk pressed hard enough against the board to get away from the gray color which appears when the line is lightly drawn, and which is not as well seen. Lines made with the chalk held broadways to the board to produce heavy lines are also useful. When using colors, contrasting colors are always easily seen; bright colors, such

F. Miscellaneous Devices for Motivation.

1. Clarity.

a). Introduction. While the author, for convenience, has grouped the miscellaneous devices under two topics of clarity and attitude, yet the reader will realize that there are no sharp distinctions between these two aims. Anything that is made clear has a tendency to produce a favorable attitude, and conversely anything which produces a favorable attitude must be made clear.

However, there are some devices which have a stronger leaning toward clarity as a characteristic aim, while others lean more toward producing a favorable attitude. For this reason, it may be well to consider the devices under the two groupings, rather than under one.

b). The Figure.

(1). The line. If the subject is put on the board so that the pupil can easily see it, he will be more apt to be attentive. Much of the power of the moving pictures comes from its vivid-ness because of its large size. The figure in geometry should be large and placed on the portion of the board best seen by the class as a whole. Lines showing up to the best advantage should be drawn; these may be ordinary lines of white chalk pressed hard enough against the board to get away from the gray color which appears when the line is lightly drawn, and which is not as well seen. Lines made with the chalk held broadways to the board to produce heavy lines are also useful. When using colors, contrasting colors are always easily seen; bright colors, such

as yellow and orange are good, while blue and red show up rather poorly on the blackboard except in contrast, as in design drawing. Dotted lines are used for auxiliary construction lines.

Colored chalk may also be used in the beginning to call attention to certain parts of figures, such as pointing out an angle in a triangle, the vertex, the base angle, or the extremeties of a line segment.

(2). Notation. Of late, text books have been tending toward simple notation. Instead of reading the three letters of a triangle, it is often customary to use roman numerals to designate the triangles, the numerals being placed inside the figure. A single letter is used for a line. When only one angle is at a vertex, a single capital letter is used to denote the angle. Corresponding lines in two triangles are designated by a short line through the side of each triangle to denote the first set of corresponding sides; two short lines to denote the second set, and three such lines to denote the third set. Braces used singly, doubly, and in triplicate are also used to denote corresponding sides the same as the one, two and three short lines are used. Corresponding angles are shown by single, double and triple arcs for each of the three sets. Not only does simplicity in notation save time, but it makes the figure clearer and so aids to easier understanding and thus makes for success by the pupil. Just as the voice too low to be heard is not interesting, so the figure too dim and clouded to be seen is not attractive to the pupil.

as yellow and orange are good, while blue and red show up rather poorly on the blackboard except in contrast, as in design drawing. Dotted lines are used for auxiliary construction lines.

Colored chalk may also be used in the beginning to call attention to certain parts of figures, such as pointing out an angle in a triangle, the vertex, the base angle, or the extremities of a line segment.

(2). Notation. Of late, text books have been tending toward simple notation. Instead of reading the three letters of a triangle, it is often customary to use roman numerals to designate the triangles, the numerals being placed inside the figure. A single letter is used for a line. When only one angle is at a vertex, a single capital letter is used to denote the angle. Corresponding lines in two triangles are designated by a short line through the side of each triangle to denote the first set of corresponding sides; two short lines to denote the second set, and three such lines to denote the third set. Braces used singly, doubly, and in triplicate are also used to denote corresponding sides the same as the one, two and three short lines are used. Corresponding angles are shown by single, double and triple arcs for each of the three sets. Not only does simplicity in notation save time, but it makes the figure clearer and so aids to easier understanding and thus makes for success by the pupil. Just as the voice too low to be heard is not interesting, so the figure too dim and clouded to be seen is not attractive to the pupil.

(3). **Cardboard.** To facilitate drill work and review work in general, and also board demonstration work, large figures may be drawn on cardboard and serve in much the same manner as maps do in the history class. They may be suspended from a string stretched across the top of the blackboard. In the drill work, the pointer may be used to indicate the parts being talked about and this is also useful in the review. When class time is limited the use of these figures on cardboard may save time when a demonstration is being worked on the blackboard; they will be better than hastily drawn figures on the board. They will have the advantage over the figure in the book in that you can point to the various parts of the board figure. Being drawn leisurely, they may be accurate and attractive and may be in suitable colors. The making of these cardboards will serve as an opportunity for some one in the geometry class not too much interested in the actual demonstration but who wishes to earn credit by contributing along the line of his interest and capacity. This activity will have the double advantage of affording a hand-activity opportunity and an audience situation for the finished product.

(4). Distorted figure. Having noted that it is necessary to have a figure easily seen, and easily read, and that cardboard figures help in the geometry situation, one may next consider the difficulties caused by distorted figures. Since the writer of the geometry book cannot make his lines move, and since it is expensive to make the figure in a vast succession of positions to denote motion as moving pictures are made, he is faced with the necessity of distorting the line in an effort to prove his theorem. This situation occurs largely in the indirect proof propositions. The following example of such a

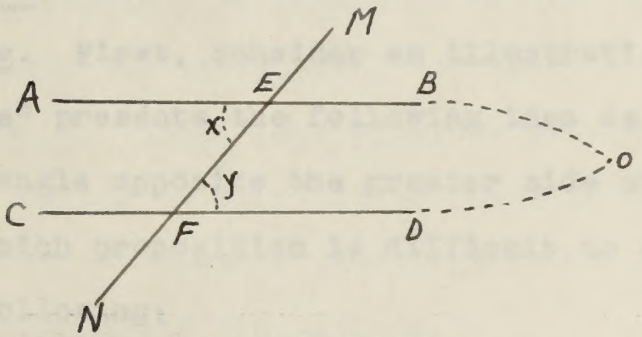
(3). Cardboard. To facilitate drill work and review work in general, and also board demonstration work, large figures may be drawn on cardboard and serve in much the same manner as maps do in the history class. They may be suspended from a string stretched across the top of the blackboard. In the drill work, the pointer may be used to indicate the parts being talked about and this is also useful in the review. When class time is limited the use of these figures on cardboard may save time when a demonstration is being worked on the blackboard; they will be better than hastily drawn figures on the board. They will have the advantage over the figure in the book in that you can point to the various parts of the board figure. Being drawn leisurely, they may be accurate and attractive and may be in suitable colors. The making of these cardboards will serve as an opportunity for some one in the geometry class not too much interested in the actual demonstration but who wishes to earn credit by contributing along the line of his interest and capacity. This activity will have the double advantage of affording a hand-activity opportunity and an audience situation for the finished product.

(4). Distorted figure. Having noted that it is necessary to have a figure easily seen, and easily read, and that cardboard figures help in the geometry situation, one may next consider the difficulties caused by distorted figures. Since the writer of the geometry book cannot make his lines move, and since it is expensive to make the figure in a vast succession of positions to denote motion as moving pictures are made, he is faced with the necessity of distorting the line in an effort to prove his theorem. This situation occurs largely in the indirect proof propositions. The following example of such a

situation is given, and an alternate plan is suggested by Clifford B. Upton[1].

. Distorted figure.

"Difficulties Caused by Distorted Figures. Let us now discuss certain practical aspects of the teaching of the indirect method of reasoning. There is no question that this type of reasoning is difficult for pupils and also for teachers. Undoubtedly, some of the difficulty that pupils have with indirect proof is due to the fact that the figures for certain propositions are often distorted and misleading. The following proposition with its accompanying figure illustrates this point.
Proposition: When two lines in the same plane are cut by a transversal, if the alternate interior angles are equal, the two lines are parallel.



"It is given that lines AB and CD are cut by a transversal MN so that angle x equals angle y. We are to prove that lines AB and CD are parallel. The proof of this proposition is commonly given by the indirect method, according to which we first assume lines AB and CD to meet on the right of MN as at O, thus forming a triangle EOF. It would then follow that angle x is greater than angle y, which contradicts the given data. Similarly, we assume AB and CD to meet on the left of MN. Since both these assumptions lead to absurdities, it follows that AB and CD are parallel.

"The difficulty which the pupil experiences with this proposition is that the figure above suggests that the lines AB and CD can each bend so as to meet at O, but the pupil knows that this is impossible since AB and CD are straight lines which could not

1. The Use of Indirect Proof in Geometry and in Life, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), pp. 119-120.

situation is given, and an alternate plan is suggested by Clifford B. Upton[1].

. Distorted figure.

"Difficulties Caused by Distorted Figures. Let us now discuss certain practical aspects of the teaching of the indirect method of reasoning. There is no question that this type of reasoning is difficult for pupils and also for teachers. Undoubtedly, some of the difficulty that pupils have with indirect proof is due to the fact that the figures for certain propositions are often distorted and misleading. The following proposition with its accompanying figure illustrates this point.

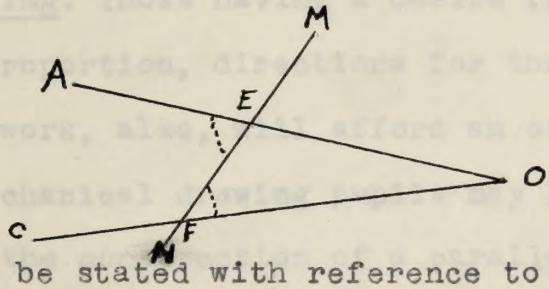
Proposition: When two lines in the same plane are cut by a transversal, if the alternate interior angles are equal, the two lines are parallel.

"It is given that lines AB and CD are cut by a transversal MN so that angle x equals angle y. We are to prove that lines AB and CD are parallel. The proof of this proposition is commonly given by the indirect method, according to which we first assume lines AB and CD to meet on the right of MN as at O, thus forming a triangle EOF. It would then follow that angle x is greater than angle y, which contradicts the given data. Similarly, we assume AB and CD to meet on the left of MN. Since both these assumptions lead to absurdities, it follows that AB and CD are parallel.

"The difficulty which the pupil experiences with this proposition is that the figure above suggests that the lines AB and CD can each bend so as to meet at O, but the pupil knows that this is impossible since AB and CD are straight lines which could not

1. The Use of Indirect Proof in Geometry and in Life, The National Council of Teachers of Mathematics, The Fifth Yearbook, (1930), pp. 119-120.

"possibly bend as indicated. This difficulty could be avoided by omitting the dotted lines BO and DO in the above figure and by supplementing this figure by a second one like that given below. This given fact



could then be stated with reference to the first figure, while the assumption that the lines meet at O could be referred to the second figure in which it is clearly seen that EOF is a triangle. A third figure, which would show the lines meeting on the left of MN, might also be included."[1]

c). Hand work.

(1). Folding. First, consider an illustration in folding. Arthur Schultze[2] presents the following idea as a solution to the proof that an angle opposite the greater side of a triangle is the greater, which proposition is difficult to analyze. He suggests the following:

"Cut out a piece of paper and mark, on both sides of the paper, the sides whose lengths we assume to be different. Mark also differently, on both sides of the paper, the angles whose relative size we wish to discover, and ask the following questions:

"Which is the only theorem which tells that one angle is greater than another? Ans. The exterior angle of a triangle, etc.

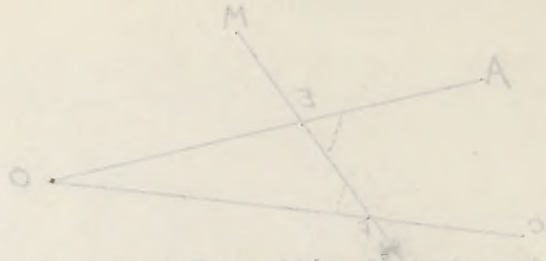
"Who can fold this paper so that ∠B becomes the exterior angle of a triangle, while ∠A becomes a remote interior?

"Let all students cut out pieces of paper and try, and soon some will find the answer that is indicated in the annexed diagram. Students see now the truth of the proposition, and it remains to translate this idea into geometric terminology. We unfold the paper, find that CD is the bisector of ∠C, find the position of B', the equality of △ BDC and B'DC, etc."

1. Clifford B. Upton, op. cit. pp. 119-120.

2. The Teaching of Mathematics in Secondary Schools, pp. 160-161.

"possibly bend as indicated. This difficulty could be avoided by omitting the dotted lines BO and DO in the above figure and by supplementing this figure by a second one like that given below. This given fact



could then be stated with reference to the first figure, while the assumption that the lines meet at O could be referred to the second figure in which it is clearly seen that EOF is a triangle. A third figure, which would show the lines meeting on the left of MN, might also be included."[1]

c). Hand work.

(1). Folding. First, consider an illustration in folding. Arthur Schultze[2] presents the following idea as a solution to the proof that an angle opposite the greater side of a triangle is the greater, which proposition is difficult to analyze. He suggests the following:

"Cut out a piece of paper and mark, on both sides of the paper, the sides whose lengths we assume to be different. Mark also differently, on both sides of the paper, the angles whose relative size we wish to discover, and ask the following questions:

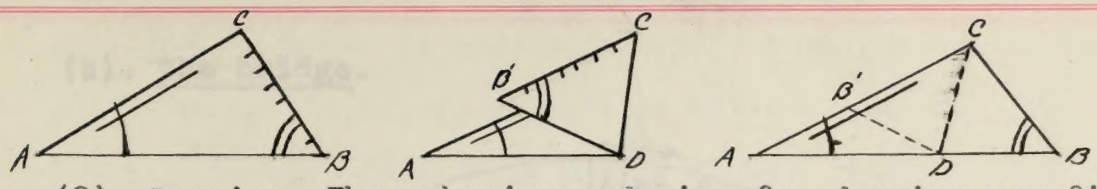
"Which is the only theorem which tells that one angle is greater than another? Ans. The exterior angle of a triangle, etc.

"Who can fold this paper so that ∠B becomes the exterior angle of a triangle, while ∠A becomes a remote interior?

"Let all students cut out pieces of paper and try, and soon some will find the answer that is indicated in the annexed diagram. Students see now the truth of the proposition, and it remains to translate this idea into geometric terminology. We unfold the paper, find that CD is the bisector of ∠C, find the position of B', the equality of △BDC and B'DC, etc."

1. Clifford B. Upton, op. cit. pp. 119-120.

2. The Teaching of Mathematics in Secondary Schools, pp. 180-181.

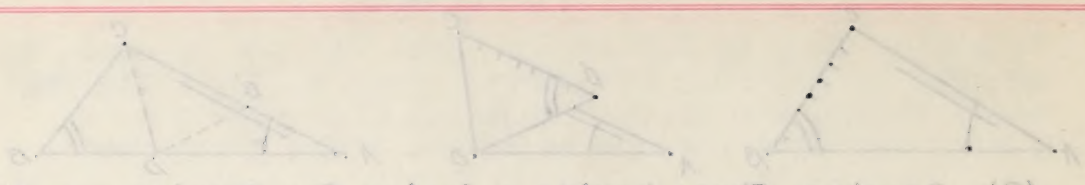


(2). Drawing. Those having a desire for drawing may find in the unit on proportion, directions for the enlarging of a picture. Graph work, also, will afford an outlet for their interests. The mechanical drawing pupils may link physics with geometry through the construction of a parallelogram with two adjacent sides given, as an illustration of the resolution of forces with the picture hung on the wall as an application. The inclined plane is another opportunity for them. Familiarity with drawing instruments as suggested in the list given by Louise B. Eddy will be interesting to them.

(3). Measuring. Those interested in problem work may find hand practice linking up with the problems in making and using a large protractor to measure inaccessible heights and distances.

d). Presenting ideas graphically.

(1). General discussion. Previously in the thesis has appeared illustrations of the graphic method of presenting the steps in proportions. The newspapers recognize the power of cartoons and comic strips to tell their stories of politics, science or history. To give these various geometric steps familiar names will add to their interest. Colored chalk will add to their attractiveness. Here also is an opportunity for the pupil interested in drawing. His ingenuity can be called upon for the proper drawing after the rough ideas have been suggested by the teacher, or he may himself have an original idea for visual presentation. The class makes an excellent audience. The pupil achieves and the class benefits.



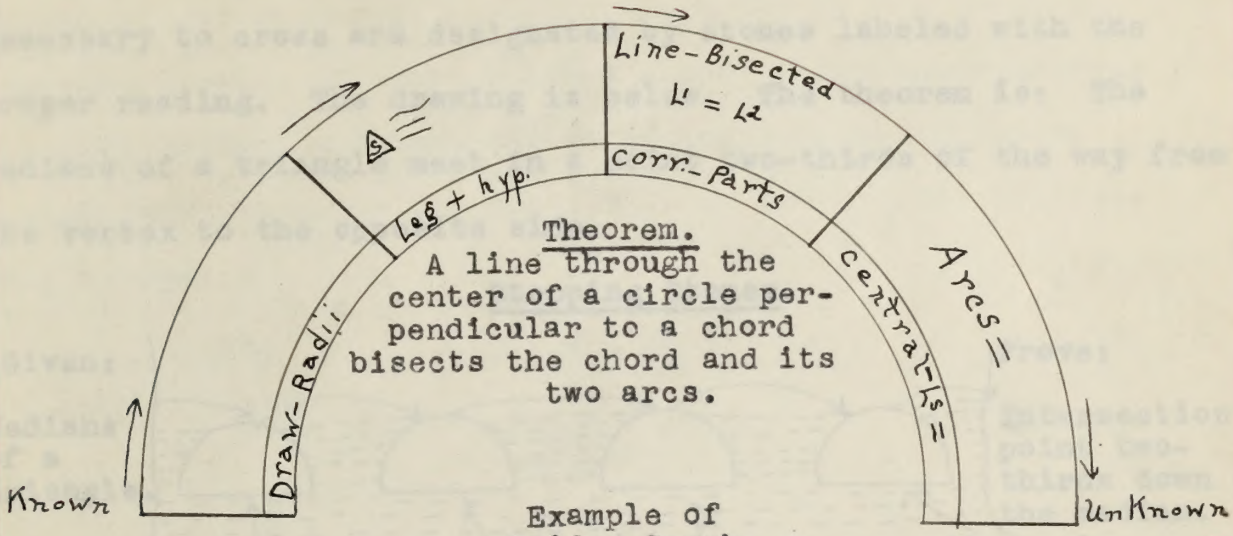
(2). Drawing. Those having a desire for drawing may find in the unit on proportion, directions for the enlarging of a picture. Graph work, also, will afford an outlet for their interests. The mechanical drawing pupils may link physics with geometry through the construction of a parallelogram with two adjacent sides given, as an illustration of the resolution of forces with the picture hung on the wall as an application. The inclined plane is another opportunity for them. Familiarity with drawing instruments as suggested in the list given by Louise B. Eddy will be interesting to them.

(3). Measuring. Those interested in problem work may find hand practice linking up with the problems in making and using a large protractor to measure inaccessible heights and distances.

d). Presenting ideas graphically.

(1). General discussion. Previously in the thesis has appeared illustrations of the graphic method of presenting the steps in proportions. The newspapers recognize the power of cartoons and comic strips to tell their stories of politics, science or history. To give these various geometric steps familiar names will add to their interest. Colored chalk will add to their attractiveness. Here also is an opportunity for the pupil interested in drawing. His ingenuity can be called upon for the proper drawing after the rough ideas have been suggested by the teacher, or he may himself have an original idea for visual presentation. The class makes an excellent audience. The pupil achieves and the class benefits.

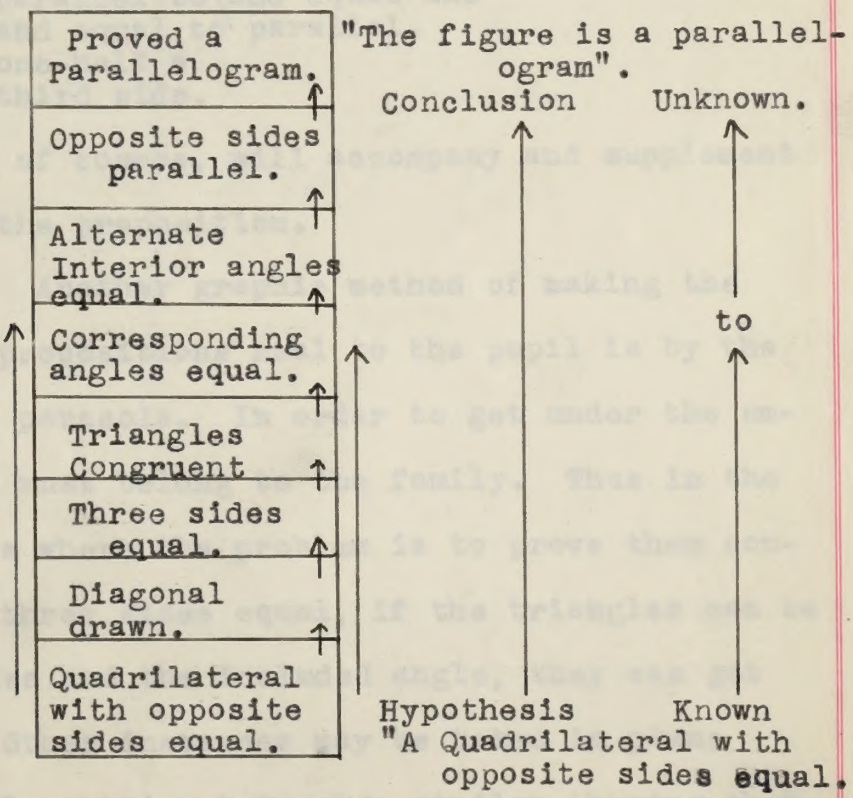
(a). The bridge.



Theorem.
A line through the center of a circle perpendicular to a chord bisects the chord and its two arcs.

Example of Bridge Graph

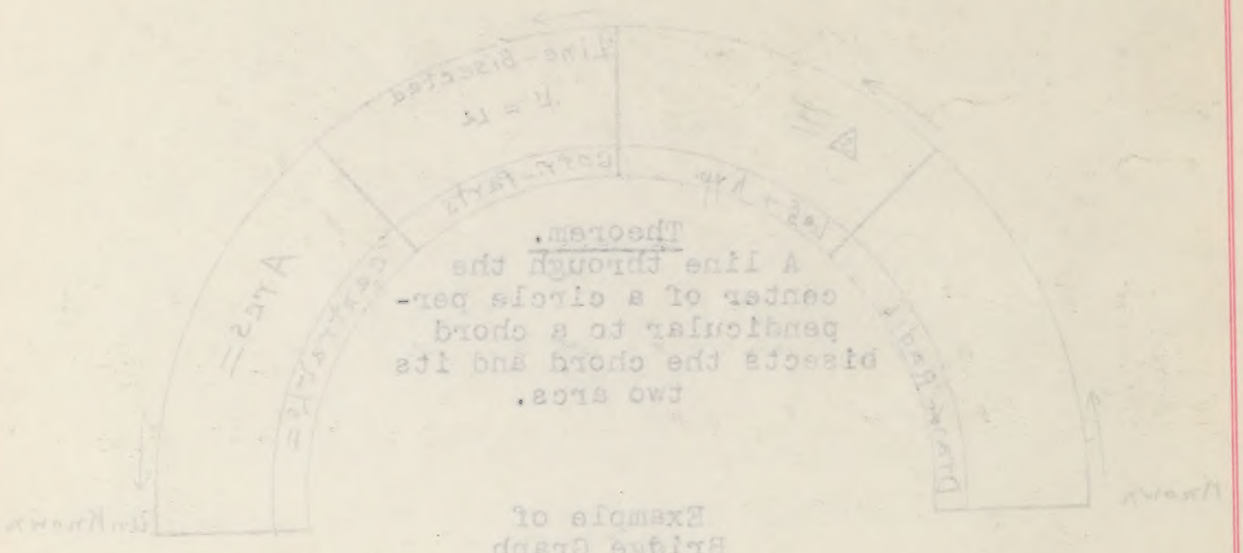
(b). The column.



Theorem: A quadrilateral with opposite sides equal is a parallegram.

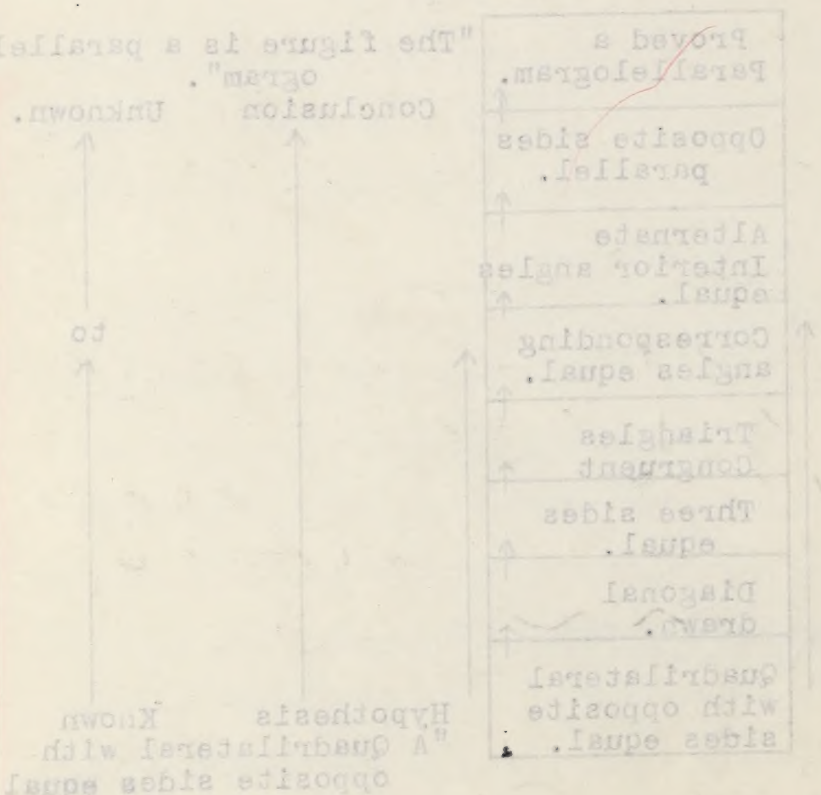
Example of block graph.

(a). The bridge.



Example of
Bridge Graph

(b). The column.

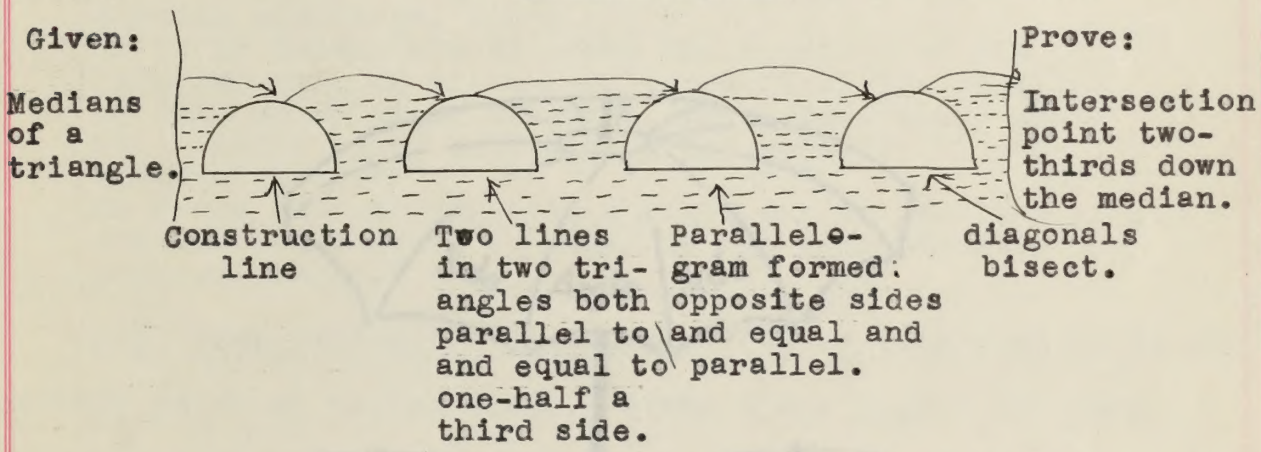


Theorem: A quadrilateral with opposite sides equal is a parallelogram.

Example of block graph.

(c). The stream. In this graph, one bank of the stream is the hypothesis; the opposite side is the conclusion. The steps necessary to cross are designated by stones labeled with the proper reading. The drawing is below. The theorem is: The medians of a triangle meet in a point two-thirds of the way from the vertex to the opposite side.

Stepping Stones.

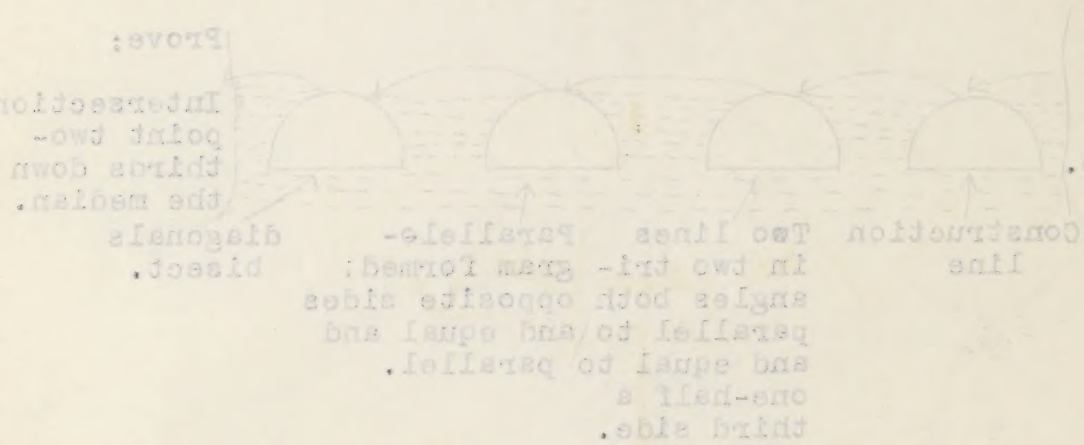


This graphic work, of course, will accompany and supplement the regular figure of the proposition.

(d). The umbrella. Another graphic method of making the interrelationships of propositions real to the pupil is by the method of umbrellas or parasols. In order to get under the umbrella the proposition must belong to the family. Thus in the congruence of triangles where the problem is to prove them congruent when they have three sides equal, if the triangles can be proved to have two sides and the included angle, they can get under that umbrella. Other instances may be taken in plane geometry, as for example getting triangles similar, having them fulfill the conditions of similarity. This graphic work brings the problem from the symbol through language to the picture, and

(c). The stream. In this graph, one bank of the stream is the hypothesis; the opposite side is the conclusion. The steps necessary to cross are designated by stones labeled with the proper reading. The drawing is below. The theorem is: The medians of a triangle meet in a point two-thirds of the way from the vertex to the opposite side.

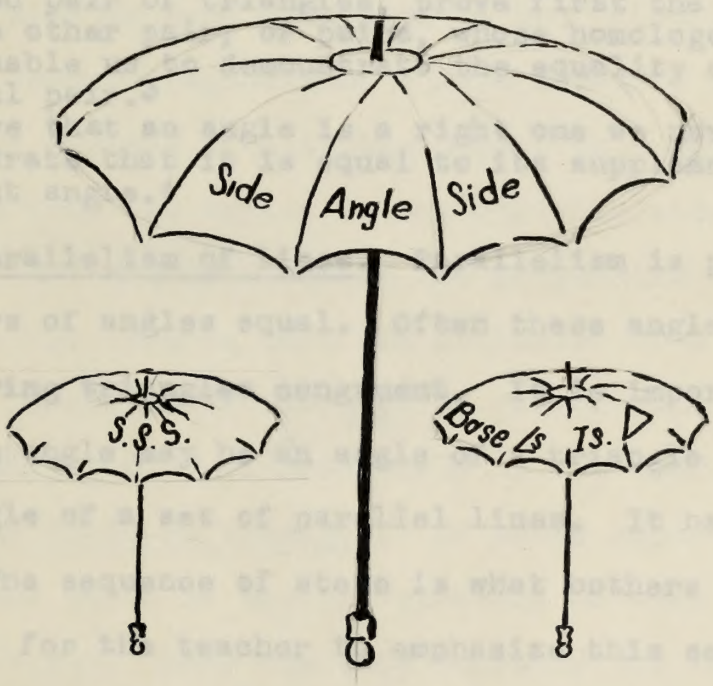
Stepping stones.



This graphic work, of course, will accompany and supplement the regular figure of the proposition.

(d). The umbrella. Another graphic method of making the interrelationships of propositions real to the pupil is by the method of umbrellas or parasols. In order to get under the umbrella the proposition must belong to the family. Thus in the congruence of triangles where the problem is to prove them congruent when they have three sides equal, if the triangles can be proved to have two sides and the included angle, they can get under that umbrella. Other instances may be taken in plane geometry, as for example getting triangles similar, having them fulfill the conditions of similarity. This graphic work brings the problem from the symbol through language to the picture, and

dramatization could bring the picture into the object if desired. Below is the illustration drawn on the board in colored chalk. It shows the two propositions on congruency by three sides, and base angles of an isosceles triangle equal, basking under the protection of the parasol of side-angle-side.



e). Methods of proof.

(1). Equality of lines, angles and triangles. Various hints on general methods of proof have been suggested by the writers. Chief among these are the methods of A. Schultze[1] who makes

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools. pp. 115-138.

dramatization could bring the picture into the object if desired. Below is the illustration drawn on the board in colored chalk. It shows the two propositions on congruency by three sides, and base angles of an isosceles triangle equal, basking under the protection of the parasol of side-angle-side.

e). Methods of proof.

(1). Equality of lines, angles and triangles. Various hints on general methods of proof have been suggested by the writers. Chief among these are the methods of A. Schultze[1] who makes

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools. pp. 115-138.

the following suggestions:

> Equality of lines and angles usually proved by means of equal triangles.[1]
> The teacher cannot emphasize this principle and its application too much....There are only a few such principles that deserve the same emphasis--method for demonstrating parallelism of lines, inequality of lines, proportionality of lines, and equivalent areas.
> If the lines or angles whose equality we wish to demonstrate are not parts of equal triangles, we have to make them parts of equal triangles by drawing additional lines.[2]
> If it is impossible to prove the equality of the required pair of triangles, prove first the equality of some other pair, or pairs, whose homologous parts will enable us to demonstrate the equality of the original pair.[3]
> To prove that an angle is a right one we usually demonstrate that it is equal to its supplementary adjacent angle.[4]

(2). Parallelism of lines. Parallelism is proved by having certain pairs of angles equal. Often these angles are made equal by having triangles congruent. It is important to emphasize that an angle may be an angle of a triangle and also an exterior angle of a set of parallel lines. It has a double identity. The sequence of steps is what bothers the pupil. It is important for the teacher to emphasize this sequence. For instance:

1. Angles are equal because they are corresponding angles of congruent triangles.
2. Their name is also alternate-interior angles, although naming them this does not prove them equal, as a drawing may illustrate.
3. They are equal by step 1, and are named in step 2. The combination of being equal and alternate-interior angles makes the lines parallel.

1. Arthur Schultze, The Teaching of Mathematics in Secondary Schools, p. 115.
2. Ibid, 127.
3. Ibid. 135.
4. Ibid. 138.

the following suggestions:

> Equality of lines and angles usually proved by means of equal triangles.[1]
>
> The teacher cannot emphasize this principle and its application too much....There are only a few such principles that deserve the same emphasis--method for demonstrating parallelism of lines, inequality of lines, proportionality of lines, and equivalent areas.
>
> If the lines or angles whose equality we wish to demonstrate are not parts of equal triangles, we have to make them parts of equal triangles by drawing additional lines.[2]
>
> If it is impossible to prove the equality of the required pair of triangles, prove first the equality of some other pair, or pairs, whose homologous parts will enable us to demonstrate the equality of the original pair.[3]
>
> To prove that an angle is a right one we usually demonstrate that it is equal to its supplementary adjacent angle.[4]

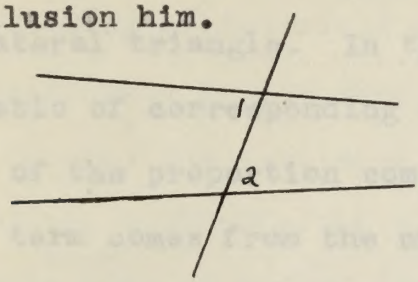
(2). <u>Parallelism of lines</u>. Parallelism is proved by having certain pairs of angles equal. Often these angles are made equal by having triangles congruent. It is important to emphasize that an angle may be an angle of a triangle and also an exterior angle of a set of parallel lines. It has a double identity. The sequence of steps is what bothers the pupil. It is important for the teacher to emphasize this sequence. For instance:

> 1. Angles are equal because they are corresponding angles of congruent triangles.
> 2. Their name is also alternate-interior angles, although naming them this does not prove them equal, as a drawing may illustrate.
> 3. They are equal by step 1, and are named in step 2. The combination of being equal and alternate-interior angles makes the lines parallel.

---

1. Arthur Schultze, <u>The Teaching of Mathematics in Secondary Schools</u>, p. 115.
2. Ibid. 127.
3. Ibid. 135.
4. Ibid. 138.

The pupil is apt to say that the angles are equal because they are alternate-interior angles. Drawings on the board will disillusion him.



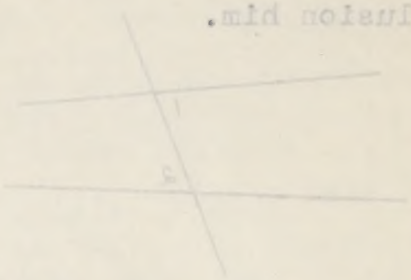
1 and 2 are alternate-interior angles, but are not equal.

Other cases where the double identity of a line occurs is where a line is a side or part of a side in two triangles, where the medians may become diagonals of a parallelogram in the proposition that the medians of a triangle meet in a point two-thirds of the distance from the vertex to the opposite side, and where the altitudes of the triangle become the perpendicular bisectors of the sides in the theorem that the altitudes of a triangle are concurrent. Graphically this may be represented by the parasol drawing as suggested just previously in the thesis.

(3). <u>The 30-60-90 triangle.</u> Of prime importance in problem solving to the pupil is the relationship between the sides of the 30-60-90 triangle. Here the side opposite the 30 degree angle is one-half of the hypotenuse. The general triangle may be represented by "a" for the side of an equilateral triangle; the altitude bisects the base and the vertex angle, making the half base "1/2 a", and the vertex angle is one-half of 60 degrees, or 30 degrees. The altitude is found by the pythagoream theorem, and is $\sqrt{3/4\ a^2}$ or $\frac{a}{2}\sqrt{3}$.

Not only in plane geometry but also in solid geometry does this fact find many usages.

The pupil is apt to say that the angles are equal because they are alternate-interior angles. Drawings on the board will distillusion him.



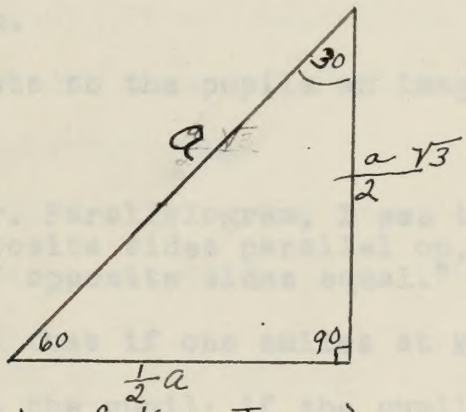
1 and 2 are alternate-interior angles, but are not equal.

Other cases where the double identity of a line occurs is where a line is a side or part of a side in two triangles, where the medians may become diagonals of a parallelogram in the proposition that the medians of a triangle meet in a point two-thirds of the distance from the vertex to the opposite side, and where the altitudes of the triangle become the perpendicular bisectors of the sides in the theorem that the altitudes of a triangle are concurrent. Graphically this may be represented by the parasol drawing as suggested just previously in the thesis.

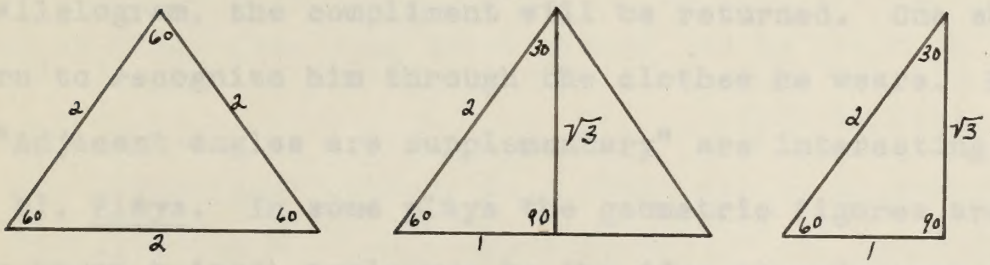
(3). <u>The 30-60-90 triangle</u>. Of prime importance in problem solving to the pupil is the relationship between the sides of the 30-60-90 triangle. Here the side opposite the 30 degree angle is one-half of the hypotenuse. The general triangle may be represented by "s" for the side of an equilateral triangle; the altitude bisects the base and the vertex angle, making the half base "1\2 s", and the vertex angle is one-half of 60 degrees, or 30 degrees. The altitude is found by the pythagorean theorem, and is $\sqrt{3\backslash 4\ s^2}$ or $\frac{s}{2}\sqrt{3}$.

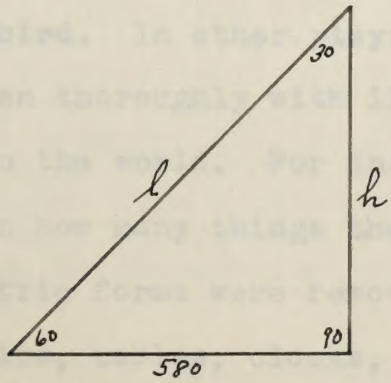
Not only in plane geometry but also in solid geometry does this fact find many usages.

For actual comparison, as in the case of similar triangles, the triangle is drawn with "2" representing a side of an equilateral triangle. In the proportion which is derived from the ratio of corresponding sides of similar triangles, two terms of the proportion come from the key triangle, while the third term comes from the unknown triangle. The unknown sides of the second triangle are found from the proportions formed. Following is the drawing and a sample problem.



Development - of - Key - Triangle

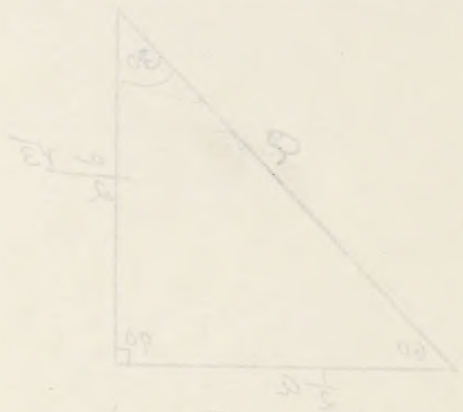




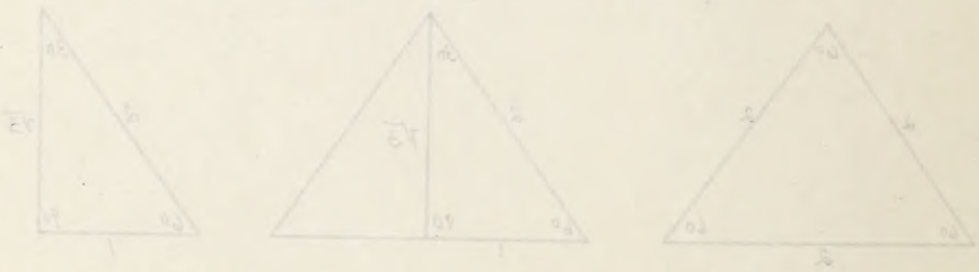
$\frac{1}{580} = \frac{\sqrt{3}}{h}$ $\frac{1}{580} = \frac{2}{l}$

$h = \sqrt{3}(580)$ $l = 2(580)$

For actual comparison, as in the case of similar triangles, the triangle is drawn with "2" representing a side of an equilateral triangle. In the proportion which is derived from the ratio of corresponding sides of similar triangles, two terms of the proportion come from the key triangle, while the third term comes from the unknown triangle. The unknown sides of the second triangle are found from the proportions formed. Following is the drawing and a sample problem.



Development - of - Key - Triangle



2. Attitude.

a). Humanizing. Turning now to the devices directed toward the attaining of favorable attitudes, the first topic is that of humanizing. As mentioned previously, the sector becomes the piece of pie; the circle, a wheel; the triangle with the exterior angles, a 4th-of-July piece of fireworks. S. S. Colvin[1] uses this idea in making the triangle of interest in his book, when he speaks of its vertices being the cities of Worcester, Providence and Boston.

The writer suggests to the pupils an imaginary conversation as follows:

> "Good morning, Mr. Parallelogram, I see that you have your coat of opposite sides parallel on, and are wearing your trousers of opposite sides equal."

The pupil is told that if one smiles at Mr. Parallelogram, he will smile back at the pupil; if the pupil recognizes Mr. Parallelogram, the compliment will be returned. One should learn to recognize him through the clothes he wears. His shoes of "Adjacent angles are supplementary" are interesting.

b). Plays. In some plays the geometric figures are made into human beings, analogous to the idea already presented in Maeterlinck's Bluebird. In other plays, the geometric situations are interwoven thoroughly with life's activities to show their importance in the world. For instance, one group of plays may center on how many things there would be left in the world if all geometric forms were removed. Such household necessities as chairs, tables, clocks, bookcases and even the

1. An Introduction to High School Teaching, p. 425.

2. Attitude.

a). Humanizing. Turning now to the devices directed toward the attaining of favorable attitudes, the first topic is that of humanizing. As mentioned previously, the sector becomes the piece of pie; the circle, a wheel; the triangle with the exterior angles, a 4th-of-July piece of fireworks. G. S. Colvin uses this idea in making the triangle of interest in his book, when he speaks of its vertices being the cities of Worcester, Providence and Boston.

The writer suggests to the pupils an imaginary conversation as follows:

"Good morning, Mr. Parallelogram, I see that you have your coat of opposite sides parallel on, and are wearing your trousers of opposite sides equal."

The pupil is told that if one smiles at Mr. Parallelogram, he will smile back at the pupil; if the pupil recognizes Mr. Parallelogram, the compliment will be returned. One should learn to recognize him through the clothes he wears. His shoes of "Adjacent angles are supplementary" are interesting.

b). Plays. In some plays the geometric figures are made into human beings, analogous to the idea already presented in Masterlinck's Bluebird. In other plays, the geometric situations are interwoven thoroughly with life's activities to show their importance in the world. For instance, one group of plays may center on how many things there would be left in the world if all geometric forms were removed. Such household necessities as chairs, tables, clocks, bookcases and even the

1. An Introduction to High School Teaching, p. 425.

walls and the houses themselves would disappear. This is a powerful device toward arousing an attitude of appreciation toward geometry of form.

c). Challenge. Some pupils may be stirred into action by a challenge as follows: "I don't think anyone can solve this one; it's pretty hard. One or two pupils did it last year." Almost always there will be an answer to this challenge from the argumentative type of pupil who has mathematical ability.

d). Games. Group competition, games with placards of geometric figures, and the drill conducted in the game spirit, all serve to produce a proper attitude.

e). Poems. The following poem brought in by one of the pupils of the writer's geometry class is illustrative.

"Oh! the dangle and the tangle of the corresponding angle,
And the equal length and straightness of the intersecting line!
How, with many a narrow finger, they tenaciously will linger
In my mind, like some sweet singer, as my brain cells they entwine."

Alma Pagliai.

f). Interrelationships. C. A Stone and J. S. Georges write:[1]

"By striving to create appreciation of all concepts and principles of mathematics the instructional materials of the unit bring into play many varied applications of these concepts and principles as means of their further realization; and if the significance of each concept and each principle in relation to the problems of life and the world is actually grasped, a love and interest for them may readily be created. The proposition is strengthened in succeeding units by creating new channels of applications and usefulness for mathematical concepts and principles."

1. Learning Product of a Unit of Instruction in Mathematics. School Science and Mathematics. (June, 1931). XXXI. pp. 695-706.

walls and the houses themselves would disappear. This is a powerful device toward arousing an attitude of appreciation toward geometry of form.

c). Challenge. Some pupils may be stirred into action by a challenge as follows: "I don't think anyone can solve this one; it's pretty hard. One or two pupils did it last year." Almost always there will be an answer to this challenge from the argumentative type of pupil who has mathematical ability.

d). Games. Group competition, games with placards of geometric figures, and the drill conducted in the game spirit, all serve to produce a proper attitude.

e). Poems. The following poem brought in by one of the pupils of the writer's geometry class is illustrative.

"Oh! the angle and the tangle of the corresponding angle,
And the equal length and straightness of the intersecting line!
How, with many a narrow finger, they tenaciously will linger
In my mind, like some sweet singer, as my brain calls they entwine."

Alma Paglial.

f). Interrelationships. G. A Stone and L. S. Georges write:[1]

"By striving to create appreciation of all concepts and principles of mathematics the instructional materials of the unit bring into play many varied applications of these concepts and principles as means of their further realization; and if the significance of each concept and each principle in relation to the problems of life and the world is actually grasped, a love and interest for them may readily be created. The proposition is strengthened in succeeding units by creating new channels of applications and usefulness for mathematical concepts and principles."

1. Learning Product of a Unit of Instruction in Mathematics. School Science and Mathematics. (June, 1931). XXXI. pp. 695-706.

E. R. Hedrick says:[1]

> "There are, it seems to me, a great many processes in mathematics which aside from any facts with which they are actually associated are very real and very important. Such processes, far more than single facts and given skills, find true application in far wider and more vital fields of human living and human thinking. Is it not possible that the true explanation of the transfer of mathematical training is now known to be more certain than was supposed a few years ago may be the transfer which occurs in what I have called the processes."

(1). General--The Analogy. First consider the more general relationship called the analogy. Illustrations follow:

Subject	Solution.
a. In bacteriology	The germ.
b. In astronomy	The star.
c. In chemistry	The element.
d. In algebra	The quantity.
e. In history	The method of consideration of a situation.
f. In geometry	The method of attacking a problem.

The general methods of analysis and synthesis may be related to the other subjects as follows:

Subject	Analysis	Synthesis.
Biology	In living matter, chemical elements.	Photo-synthesis.
Chemistry	Elements in compound	Compound built from elements.
English	Theme to topics	Topics to theme.
Algebra	Factoring	Expansion.
History	Movement to causes	Causes to movement.

1. The Reality of Mathematical Processes. The National Council of Teachers of Mathematics, The Third Yearbook, 1928, pp. 35-41

E. R. Hedrick says:[1]

> "There are, it seems to me, a great many processes in mathematics which aside from any facts with which they are actually associated are very real and very important. Such processes, far more than single facts and given skills, find true application in far wider and more vital fields of human living and human thinking. Is it not possible that the true explanation of the transfer of mathematical training is now known to be more certain than was supposed a few years ago may be the transfer which occurs in what I have called the processes."

(1). General--The Analogy. First consider the more general relationship called the analogy. Illustrations follow:

Subject	Solution.
a. In bacteriology	The germ.
b. In astronomy	The star.
c. In chemistry	The element.
d. In algebra	The quantity.
e. In history	The method of consideration of a situation.
f. In geometry	The method of attacking a problem.

The general methods of analysis and synthesis may be related to the other subjects as follows:

Subject	Analysis	Synthesis.
Biology	In living matter.	Photo-synthesis.
Chemistry	Elements in compound chemical elements.	Compound built from elements.
English	Theme to topics	Topics to theme.
Algebra	Factoring	Expansion.
History	Movement to causes	Causes to movement.

1. The Reality of Mathematical Processes. The National Council of Teachers of Mathematics, The Third Yearbook, 1928, pp. 39-41

Another general relationship may be expressed in the snow-ball being rolled to be built into a big snowman, and the propositions in geometry rolling up into the advanced propositions. Finally, general relation between success in manual training and geometry, may be shown as follows:

	Geometry	Manual Training.
Tools.	Fact well in mind---- Fact well organized-- Drill in fundamental-facts and methods. Selection of facts.--- Proper application---and assembly of steps.	Tools bright and shiny Tools in proper bins Command of proper use of tools. Selection of tools. Proper preparation of parts and assembly of parts in the whole.

(2). Specific.

(a). Physics. Here the relationships exist in the similar triangles used in problems on the inclined plane, heat expansion apparatus, and the movement of the lever in general. Triangles always enter into the study of light going through lenses or being reflected from a mirror.

Another general relationship may be expressed in the snow-ball being rolled to be built into a big snowman, and the propositions in geometry rolling up into the advanced propositions. Finally, general relation between success in manual training and geometry, may be shown as follows:

	Geometry	Manual Training.
Tools.	Fact well in mind----Tools bright and shiny Fact well organized--Tools in proper bins Drill in fundamental-Command of proper use facts and methods. of tools. Selection of facts---Selection of tools. Proper application---Proper preparation of and assembly of parts and assembly of steps. parts in the whole.	

(2). Specific.

(a). Physics. Here the relationships exist in the similar triangles used in problems on the inclined plane, heat expansion apparatus, and the movement of the lever in general. Triangles always enter into the study of light going through lenses or being reflected from a mirror.

The parallelogram is studied in connection with the resolution of forces. Examples are a wire holding a picture, a telephone wire which never can be entirely taut, and the pull on a sled. Hobson M. Zerbe[1] noted the interrelationships with physics as follows: similarity, vertical angles, pythagorean theorem, and parallelograms which involve the composition and resolution of forces.

(b). Solid Geometry. The relationships exist chiefly in the methods of proof, and the fact that solid geometry is built on plane geometry, and many of its problems are founded on it. Locus problems expand from a line to a plane in solid geometry, while plane surfaces or areas, become solid surfaces or volumes.

(c). Mechanical drawing. Instruments and constructions used in plane geometry are needed in mechanical drawing. The same geometric forms are considered. William Betz[2] notes the following relationships with geometry: shop drawings, interpretation of blueprints, making of house plans, designs, pattern making and surveying a plot of land.

1. The Elements of Plane Geometry in High School Physics, School Science and Mathematics, (June, 1930) XXX, pp. 665-667.

2. The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook. 1933, p.136.

The parallelogram is studied in connection with the resolution of forces. Examples are a wire holding a picture, a telephone wire which never can be entirely taut, and the pull on a sled. Robson M. Zerbe[1] noted the interrelationships with physics as follows: similarity, vertical angles, pythagorean theorem, and parallelograms which involve the composition and resolution of forces.

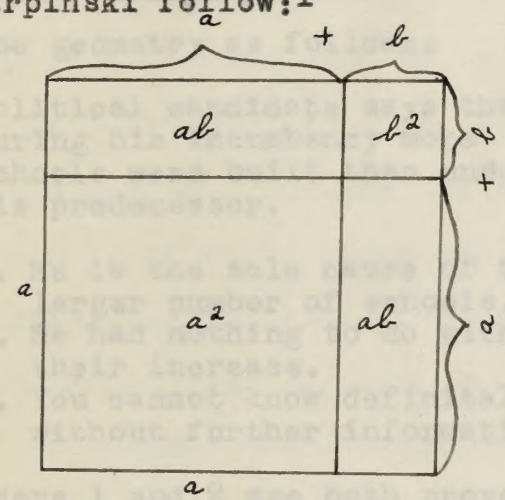
(b). Solid Geometry. The relationships exist chiefly in the methods of proof, and the fact that solid geometry is built on plane geometry, and many of its problems are founded on it. Locus problems expand from a line to a plane in solid geometry, while plane surfaces or areas, become solid surfaces or volumes.

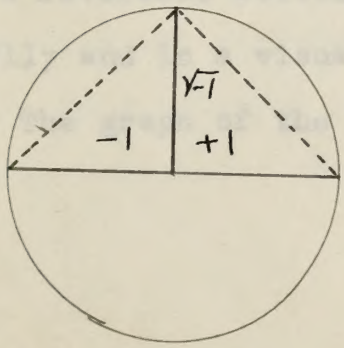
(c). Mechanical drawing. Instruments and constructions used in plane geometry are needed in mechanical drawing. The same geometric forms are considered. William Betz[2] notes the following relationships with geometry: shop drawings, interpretation of blueprints, making of house plans, designs, pattern making and surveying a plot of land.

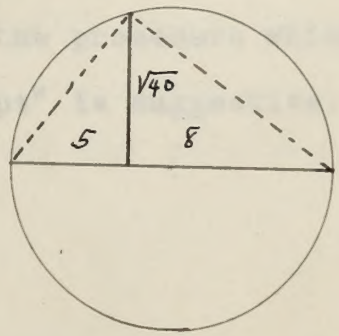
1. The Elements of Plane Geometry in High School Physics, School Science and Mathematics. (June, 1930) XXX. pp. 665-667.

2. The Teaching of Intuitive Geometry, The National Council of Teachers of Mathematics, The Eighth Yearbook. 1933, p.156.

(d). Algebra. The graphic work below shows the relationship between the multiplication of polynomials in algebra and the geometric representation by means of the area idea. The imaginary number, the square root of -1, is represented graphically by means of similar triangles, made possible by the multiplying of the extremes and means of a proportion. This makes the perpendicular from the right angle of the triangle the square root of -1. The graphic representations as suggested by Louis C. Karpinski follow:[1]

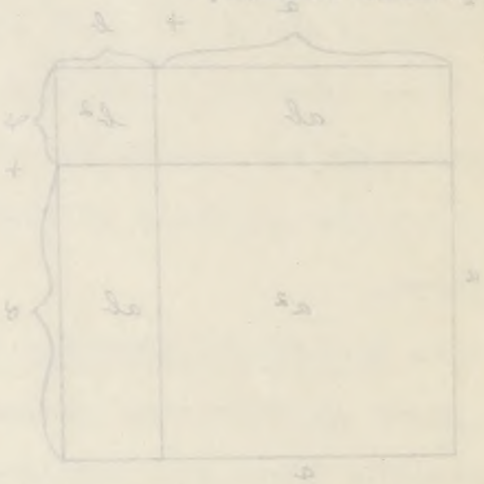






1. Unity of Algebra and Geometry, School Science and Mathematics, (May 1933) pp. 515-516.

(d). Algebra. The graphic work below shows the relationship between the multiplication of polynomials in algebra and the geometric representation by means of the area idea. The imaginary number, the square root of -1, is represented graphically by means of similar triangles, made possible by the multiplying of the extremes and means of a proportion. This makes the perpendicular from the right angle of the triangle the square root of -1. The graphic representations as suggested by Louis O. Karpinski follow:[1]



1. Unity of Algebra and Geometry, School Science and Mathematics, (May 1923) pp. 315-316.

(e). Astronomy. Getting the distance to the stars involves geometry. Here the triangle is solved when two angles and a side are known. The side is the diameter of the earth's orbit around the sun, 186,000,000 miles in length as the base line. The angles are noted at each end of the diameter as the earth comes into the positions. From these known quantities the distance to the star is calculated.

(f). Politics. Charles Salkind[1] relates politics and plane geometry as follows:

Political candidate says that during his incumbency more schools were built than under his predecessor. ------	A rectangular form may be made rigid by adding another bar.
1. He is the sole cause of the larger number of schools. -----	1. Parallel to the short sides.
2. He had nothing to do with their increase. ------	2. Parallel to the long sides.
3. You cannot know definitely without further information. ------	3. Diagonally.

Here 1 and 2 are both proved wrong; 3 will need further investigation before actually arriving at a conclusion.

(g). Room graphs of learning process. Another method of producing a favorable attitude is by explaining to the pupils both verbally and in a visual way the procedure which you are to adopt. The graph of the "concept" is suggestive. Usually

1. Functional Geometry, The Mathematics Teacher. XXV (December, 1932). pp. 485-486.

(e). Astronomy. Getting the distance to the stars involves geometry. Here the triangle is solved when two angles and a side are known. The side is the diameter of the earth's orbit around the sun, 186,000,000 miles in length as the base line. The angles are noted at each end of the diameter as the earth comes into the positions. From these known quantities the distance to the star is calculated.

(f). Politics. Charles Sarkind[1] relates politics and plane geometry as follows:

Political candidate says that------A rectangular form may during his incumbency more be made rigid by adding schools were built than under another bar. his predecessor.

1. He is the sole cause of the-----1. Parallel to the short larger number of schools. sides.
2. He had nothing to do with ------2. Parallel to the long their increase. sides.
3. You cannot know definitely------3. Diagonally. without further information.

Here 1 and 2 are both proved wrong; 3 will need further investigation before actually arriving at a conclusion.

(g). Room graphs of learning process. Another method of producing a favorable attitude is by explaining to the pupils both verbally and in a visual way the procedure which you are to adopt. The graph of the "concept" is suggestive. Usually

1. Functional Geometry, The Mathematics Teacher. XXV (December, 1932). pp. 485-486.

leading into any new difficulty may be the explanation, demonstration, experiment, drill, reading and problem approaches. Acquaint the pupils with the method being pursued and the end in view. Thus, they will see the significance of the particular task at hand.

In the graph on how to solve a problem, the solution is represented as the inner kernel, while the pupil is on the outside of the circle. Tentacles of thought are constantly sent out which wind in and out, but do not reach the center, the solution. Finally, from the array of ideas, one goes to the center. Success did not just happen; it came as the result of work. Time was spent in endeavoring to make associations. Thus, the pupil becomes aware of the necessity of continuous effort.

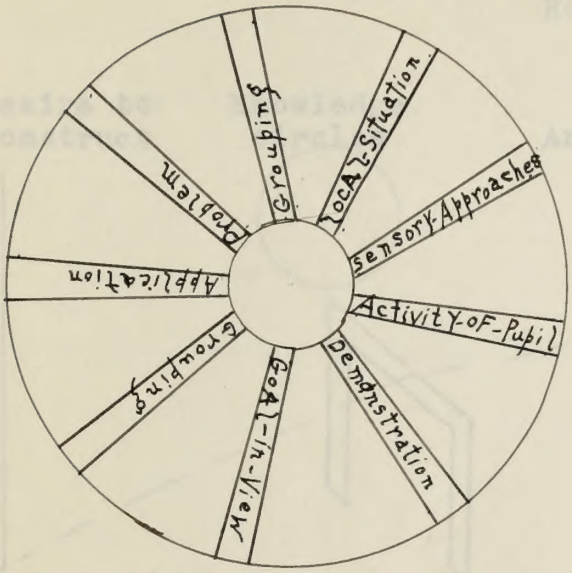
Another visual presentation involves partial learning. It is likened to the swimmer coming up from the lake bottom. Though he is almost at the surface, yet no one can see him. So with memory, to have a thing partly in mind or partly learned does not register. The next repetition may bring it. All this helps to give courage to the pupil who has not yet quite achieved. It shows him that persistence wins.

leading into any new difficulty may be the explanation, demonstration, experiment, drill, reading and problem approaches. Acquaint the pupils with the method being pursued and the end in view. Thus, they will see the significance of the particular task at hand.

In the graph on how to solve a problem, the solution is represented as the inner kernel, while the pupil is on the outside of the circle. Tentacles of thought are constantly sent out which wind in and out, but do not reach the center, the solution. Finally, from the array of ideas, one goes to the center. Success did not just happen; it came as the result of work. Time was spent in endeavoring to make associations. Thus, the pupil becomes aware of the necessity of continuous effort.

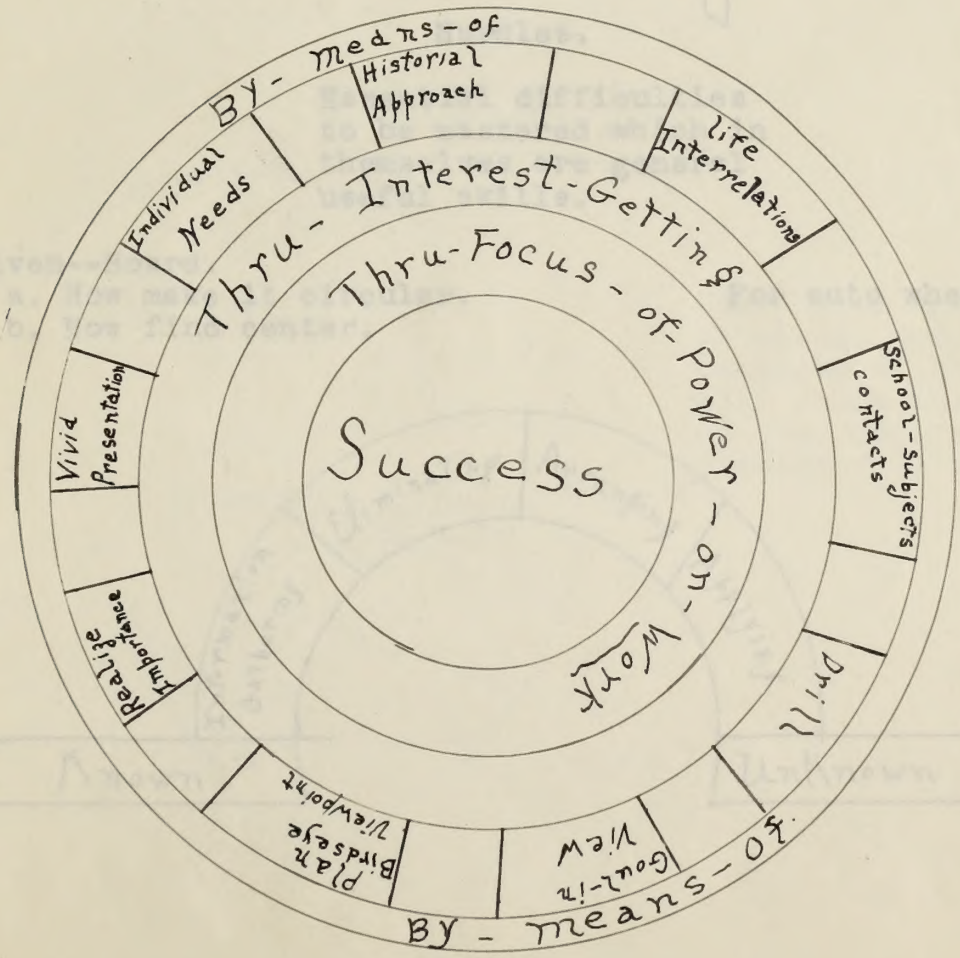
Another visual presentation involves partial learning. It is likened to the swimmer coming up from the lake bottom. Though he is almost at the surface, yet no one can see him. So with memory, to have a thing partly in mind or partly learned does not register. The next repetition may bring it. All this helps to give courage to the pupil who has not yet quite achieved. It shows him that persistence wins.

Suggested Graphs for Mathematics Room.



Concept developed through

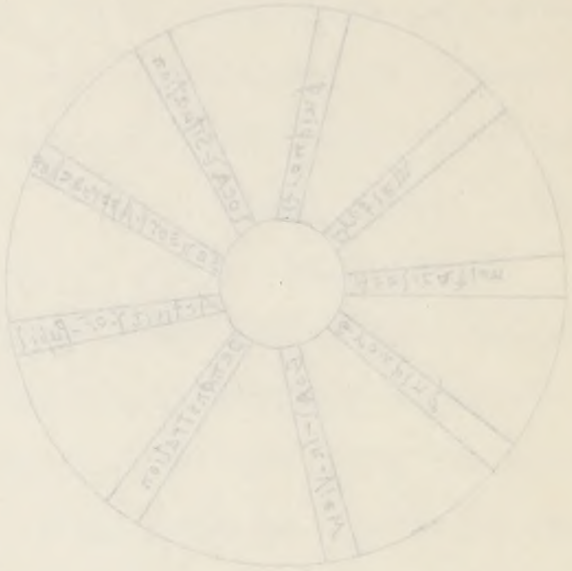
a. Local situation.
b. Pupil activity.
c. Sensory approaches.
d. Demonstration.
e. Goal in view.
f. Grouping.
g. Application.
h. Problem.
i. Test.

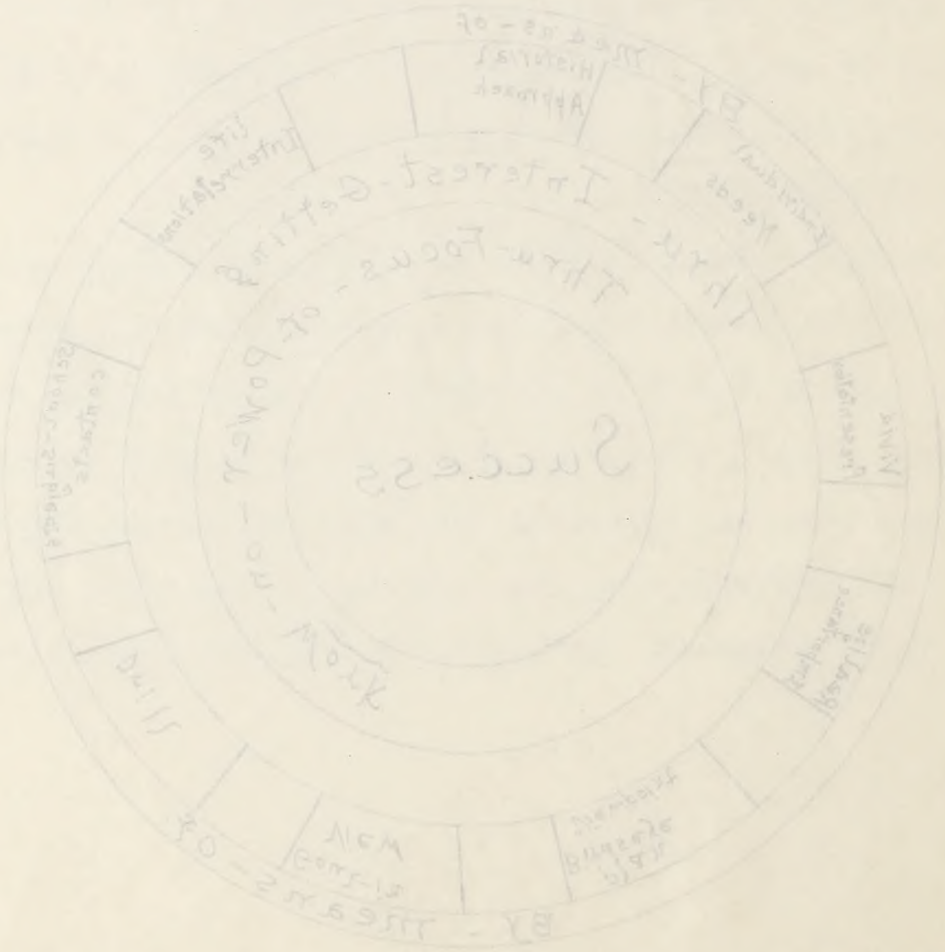


Suggested Graphs for Mathematics Room.

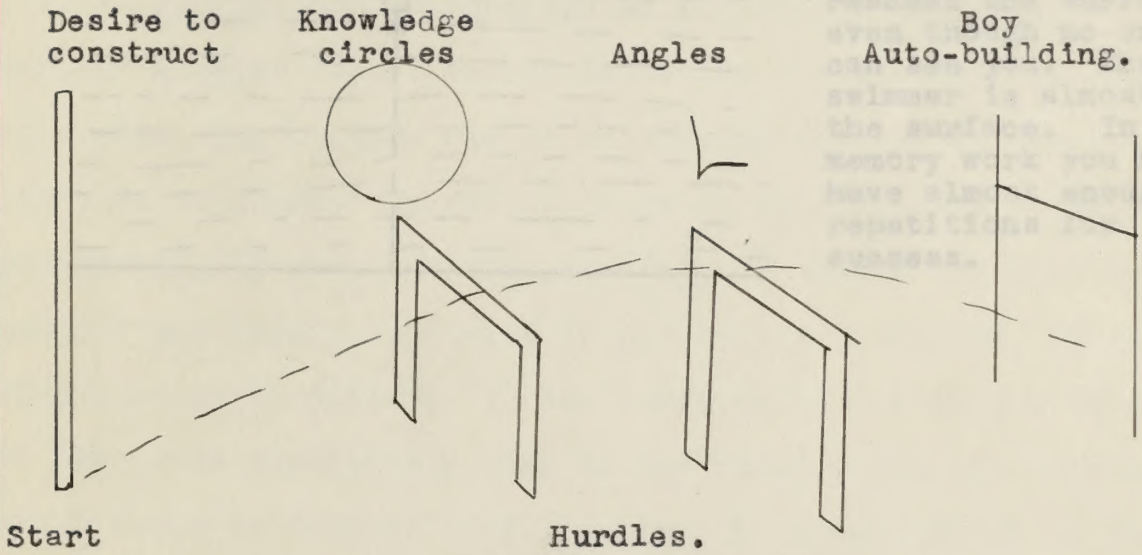
Concept developed through

a. Local situation.
b. Pupil activity.
c. Sensory approaches.
d. Demonstration.
e. Goal in view.
f. Grouping.
g. Application.
h. Problem.
i. Test.





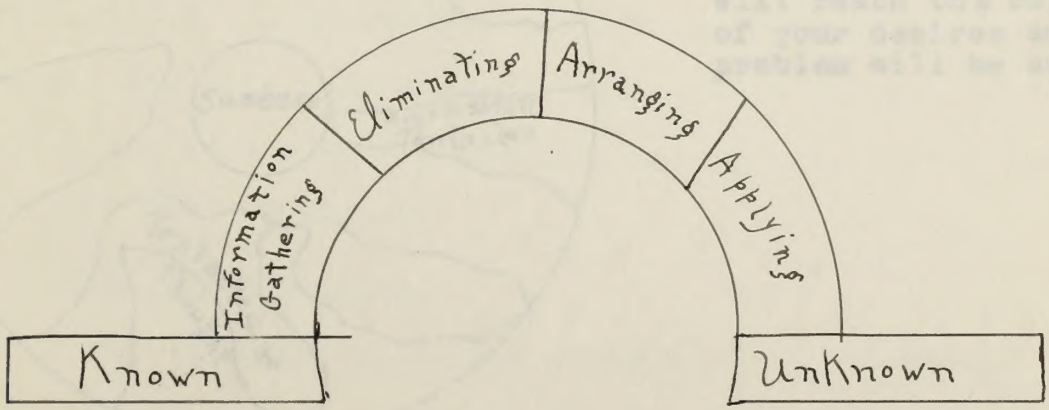
Suggested graphs for Mathematics Room.



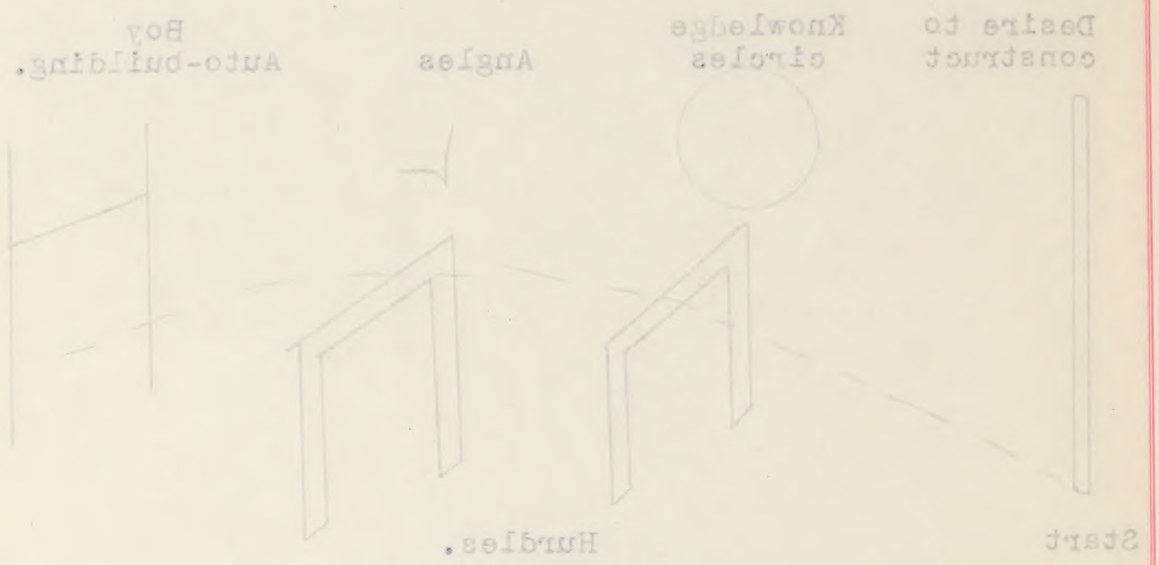
Essential difficulties to be mastered which in themselves are general useful skills.

Given--Board.
a. How make it circular.
b. How find center.

For auto wheel.



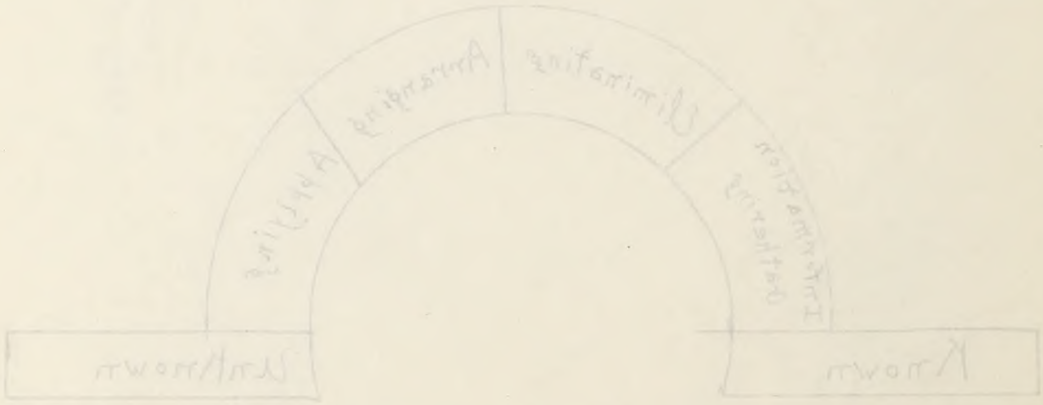
Suggested graphs for Mathematics Room.



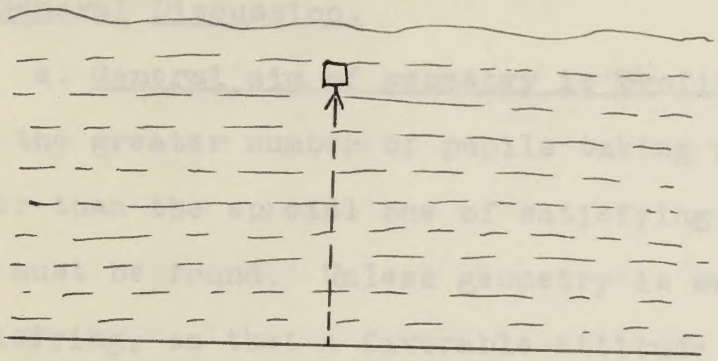
Essential difficulties to be mastered which in themselves are general useful skills.

Given--Board.
a. How make it circular.
b. How find center.

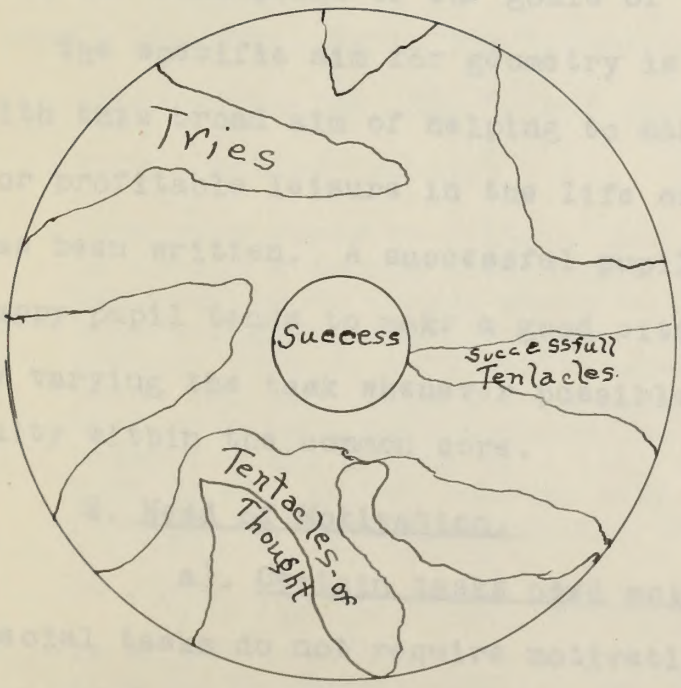
For auto wheel.



Suggested Graphs for Mathematics Room.



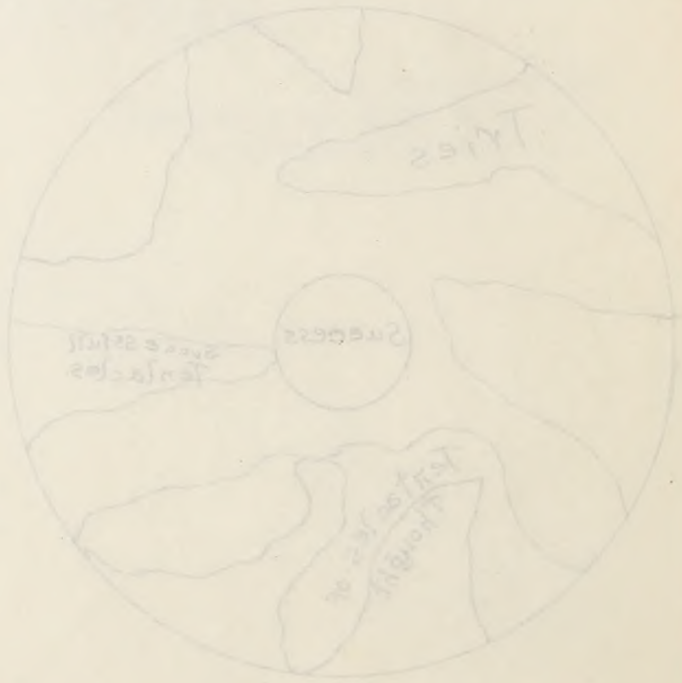
Don't give up! You may almost have reached the surface even though no one can see you. The swimmer is almost to the surface. In memory work you may have almost enough repetitions for success.



Don't give up! Keep thinking! Some little tentacle of thought will reach the object of your desires and the problem will be solved.

Suggested Graphs for Mathematics Room.

Don't give up! You may almost have reached the surface even though no one can see you. The swimmer is almost to the surface. In memory work you may have almost enough repetitions for success.



Don't give up! Keep thinking! Some little tentacle of thought will reach the object of your desires and the problem will be solved.

PART V.

SUMMARY.

A. General Discussion.

a. General aim of geometry is profitable leisure.

For the greater number of pupils taking plane geometry, values other than the special one of satisfying college entrance credits must be found. Unless geometry is sufficiently vivid and satisfying, so that a favorable attitude toward the particular subject and toward education in general is developed, no matter what success is achieved in the learning of particular theorems, or even what success may come in the added power in thinking which would be derived from practice in concentration on the mathematical task, the subject would be considered a failure from the standpoint of the goals of the modern high school.[1]

The specific aim for geometry is the leisure aim. It is with this broad aim of helping to make geometry an added vehicle for profitable leisure in the life of the pupil that this thesis has been written. A successful pupil is a happy pupil, and a happy pupil tends to make a good citizen. This success is aided by varying the task whenever possible and by varying the difficulty within the common core.

2. Need of Motivation.

a). Certain tasks need motivation. Generally, the special tasks do not require motivation for successful completion, since the original inherent interest is high and often

1. Cardinal Principles of Secondary Education, A Report of The Commission on the Reorganization of Secondary Education, Appointed by the National Education Association, No. 35 (1918), pp.5-32.

PART V.

SUMMARY.

A. General Discussion.

a. General aim of geometry is profitable leisure.

For the greater number of pupils taking plane geometry, values other than the special one of satisfying college entrance credits must be found. Unless geometry is sufficiently vivid and satisfying, so that a favorable attitude toward the particular subject and toward education in general is developed, no matter what success is achieved in the learning of particular theorems, or even what success may come in the added power in thinking which would be derived from practice in concentration on the mathematical task, the subject would be considered a failure from the standpoint of the goals of the modern high school.[1]

The specific aim for geometry is the leisure aim. It is with this broad aim of helping to make geometry an added vehicle for profitable leisure in the life of the pupil that this thesis has been written. A successful pupil is a happy pupil, and a happy pupil tends to make a good citizen. This success is aided by varying the task whenever possible and by varying the difficulty within the common core.

2. Need of Motivation.

a). Certain tasks need motivation. Generally, the special tasks do not require motivation for successful completion, since the original inherent interest is high and often

1. Cardinal Principles of Secondary Education. A Report of The Commission on the Reorganization of Secondary Education, Appointed by the National Education Association. (1918), pp.5-32.

work has already been done in the particular field. Of such are the special tasks of manual training, or design work, as they touch plane geometry.

The real work of applying techniques in the motivation of plane geometry rests in that common part in which all pupils are to participate. Here it is that vividness of the task to be done and clear ways of accomplishing the work become important. Giving courage to the pupil and keeping that courage level is necessary for the reaching of the goal. One way to accomplish this is by the elimination of fear, as noted in the introduction.

b). The "slow pupil" needs motivation. It is to be assumed that that rare individual, who has within himself the dynamic power of going ahead by himself, will achieve regardless of how much or how little he may be motivated from without. But such pupils are rare; it is the teacher's lot to deal more often with the other extremity, the "slow" pupil.

Virgil Mallory notes:[1]

> "One soon comes to believe that much of the dullness is merely a rather intelligent decision on the pupil's part not to do things which he has decided have little importance or value for him. Pupil activity and laboratory work in mathematics come to have more significance and one becomes more fully convinced of the fact that working a book problem has little in common with measuring the height of a real flagpole. Most important of all is the success that comes to the pupil in finding that there are things that he can do well."

He asks:

> "Is it not possible that seeming inability to do mathematics is frequently nothing but a failure on the teacher's part to convince the pupil of the worth-whileness of the material of instruction?"

1. Mathematics for the Slow Moving Pupil, The Mathematics Teacher November, 1933, (XXVI), p. 396.

work has already been done in the particular field. Of such are the special tasks of manual training, or design work, as they touch plane geometry.

The real work of applying techniques in the motivation of plane geometry rests in that common part in which all pupils are to participate. Here it is that vividness of the task to be done and clear ways of accomplishing the work become important. Giving courage to the pupil and keeping that courage level is necessary for the reaching of the goal. One way to accomplish this is by the elimination of fear, as noted in the introduction.

b). The "slow pupil" needs motivation. It is to be assumed that that rare individual, who has within himself the dynamic power of going ahead by himself, will achieve regardless of how much or how little he may be motivated from without. But such pupils are rare; it is the teacher's lot to deal more often with the other extremity, the "slow" pupil.

Virgil Mallory notes:[1]

> "One soon comes to believe that much of the dullness is merely a rather intelligent decision on the pupil's part not to do things which he has decided have little importance or value for him. Pupil activity and laboratory work in mathematics come to have more significance and one becomes more fully convinced of the fact that working a book problem has little in common with measuring the height of a real flagpole. Most important of all is the success that comes to the pupil in finding that there are things that he can do well."

He asks:

> "Is it not possible that seeming inability to do mathematics is frequently nothing but a failure on the teacher's part to convince the pupil of the worth-whileness of the material of instruction?"

1. Mathematics for the Slow Moving Pupil. The Mathematics Teacher November, 1933, (XXVI), p. 326.

In view of the foregoing, Mallory suggests that the sole reason for giving slow moving groups mathematical training is to interest them in the quantitative aspects of the world in which they live, and to improve their ability to see and understand relations. Their success, he notes, should be determined by improvement along these lines.

3. Plane Geometry offers opportunities for motivation similar to those in grade subjects. As one studies the various devices by which plane geometry may be motivated, one becomes more and more conscious of the truth that this subject is as fertile a field in many respects as any other school subject in any other place in the school ladder.

a). Illustrations. Take, for instance, the vitally important principle of learning through the isolation of the difficulty, no matter whether it is in the first grade learning the phonetic table, or in the high school learning plane geometry, the more complete the segregation of the new difficulty, and the more gradual the introduction of the difficulty into the more complex situations, the greater the chance of success.

Whether one turns to the ideas of humanizing the grade reading lesson by relating it to the chickens, dogs, and cats of the home, or whether one thinks of the angles made by the hands of the clock, or the range of the dog hitched to a slide wire as in the illustration of the locus in the high school, there is the same principle of relating to the familiar, or making interesting by making the lesson meaningful.

Or again, whether it be pictures of lakes and rivers in the

In view of the foregoing, Mallory suggests that the sole reason for giving slow moving groups mathematical training is to interest them in the quantitative aspects of the world in which they live, and to improve their ability to see and understand relations. Their success, he notes, should be determined by improvement along these lines.

3. Plane Geometry offers opportunities for motivation similar to those in grade subjects. As one studies the various devices by which plane geometry may be motivated, one becomes more and more conscious of the truth that this subject is as fertile a field in many respects as any other school subject in any other place in the school ladder.

a). Illustrations. Take, for instance, the vitally important principle of learning through the isolation of the difficulty, no matter whether it is in the first grade learning the phonetic table, or in the high school learning plane geometry, the more complete the segregation of the new difficulty, and the more gradual the introduction of the difficulty into the more complex situations, the greater the chance of success.

Whether one turns to the ideas of humanizing the grade reading lesson by relating it to the chickens, dogs, and cats of the home, or whether one thinks of the angles made by the hands of the clock, or the range of the dog hitched to a slide wire as in the illustration of the locus in the high school, there is the same principle of relating to the familiar, or making interesting by making the lesson meaningful.

Or again, whether it be pictures of lakes and rivers in the

geography book or whether it be the story of the geometry proposition told in picture form, there is the translation of the symbol of english and geometric language to the picture language, as an aid to understanding.

One can touch on when and where to drill, or on the dressing up in an attractive way by means of colors and dramatics, or by means of the teacher's own enthusiastic personality, but whether one takes a junior high school civics class or a high school geometry class, the opportunity awaits in each field.

The interrelation with outside life and with school life is but a form of relating to the familiar, and here again, whether the child is sent home to study the heating system in his own house, or to bring in a list of geometric forms in his own room, the same idea is at work in either junior high general sciences, or in senior high plane geometry.

In the matter of organizing procedure or knowledge to make meaningful, whether the material is organized around high school debating and composition, or whether the principle of grouping and continuity in plane geometry is used, each field is favorable.

Thus it is seen that geometry is a fertile field for motivation; it has opportunity for the operation of the same laws of learning that play a part in grade learning, and it has also similar opportunities for individual tasks, as in other school subjects of either grade or high school level.

4. Plan of the summary. The plan of this summary is to present devices suggested in the thesis under such representative

geography book or whether it be the story of the geometry proposition told in picture form, there is the translation of the symbol of English and geometric language to the picture language as an aid to understanding.

One can touch on when and where to drill, or on the dressing up in an attractive way by means of colors and dramatics, or by means of the teacher's own enthusiastic personality, but whether one takes a junior high school civics class or a high school geometry class, the opportunity awaits in each field.

The interrelation with outside life and with school life is but a form of relating to the familiar, and here again, whether the child is sent home to study the heating system in his own house, or to bring in a list of geometric forms in his own room, the same idea is at work in either junior high general science, or in senior high plane geometry.

In the matter of organizing procedure or knowledge to make meaningful, whether the material is organized around high school debating and composition, or whether the principle of grouping and continuity in plane geometry is used, each field is favorable.

Thus it is seen that geometry is a fertile field for motivation; it has opportunity for the operation of the same laws of learning that play a part in grade learning, and it has also similar opportunities for individual tasks, as in other school subjects of either grade or high school level.

4. <u>Plan of the summary</u>. The plan of this summary is to present devices suggested in the thesis under such representative

general principles as are outlined by Thomas.[1] These principles are suggested in relation to grade subjects, such as, reading, writing, arithmetic and civics. The plan here is to consider them in relation to the high school subject of plane geometry, and to list representative devices which operate under these fundamental ideas. Thomas' outline includes "Preparation and Mental Set", "The Lesson for Habits and Skill", "The Lesson for Appreciation", and "The Lesson for Mastering Knowledge". Of necessity the emphasis will vary with the types of lessons, and it will be seen geometry is a subject which has the greatest emphasis on the lesson for appreciation.

a). Preparation and the mental set. In considering the preparation and the mental set, it is noted that each response is conditioned by previous experiences and responses. "Events, words and instructions have meaning to the learner only as interpreted through such of his previous experiences as are actively associated in consciousness with the present situation".[2] It is further noted that "the situation and material must at all times be connected in some way with one or more of the fundamental interests of the pupil and thus give significance and value to him."[3] This might all be summed up in that there needs to be a relation to the familiar, and that this relationship will bring the mind into condition for favorable attention and

---

1. Frank W. Thomas, Principles and Technique of Teaching. pp. i--xii—1-410.

2. Ibid, p. 135.

3. Ibid, p. 140.

general principles as are outlined by Thomas.[1] These principles are suggested in relation to grade subjects, such as, reading, writing, arithmetic and civics. The plan here is to consider them in relation to the high school subject of plane geometry, and to list representative devices which operate under these fundamental ideas. Thomas' outline includes "Preparation and Mental Set", "The Lesson for Habits and Skill", "The Lesson for Appreciation", and "The Lesson for Mastering Knowledge". Of necessity the emphasis will vary with the types of lessons, and it will be seen geometry is a subject which has the greatest emphasis on the lesson for appreciation.

a). Preparation and the mental set. In considering the preparation and the mental set, it is noted that each response is conditioned by previous experiences and responses. "Events, words and instructions have meaning to the learner only as interpreted through such of his previous experiences as are actively associated in consciousness with the present situation".[2] It is further noted that "the situation and material must at all times be connected in some way with one or more of the fundamental interests of the pupil and thus give significance and value to him."[3] This might all be summed up in that there needs to be a relation to the familiar, and that this relationship will bring the mind into condition for favorable attention and

---

1. Frank W. Thomas, Principles and Technique of Teaching. pp. 1--xii--1-410.

2. Ibid, p. 135.

3. Ibid, p. 140.

response. Again, instruction must be on the level of the learner and the constant use of illustrative material is of value.[1] This phase of mind-set is called the "ideational". Its success depends upon vividness of the impression.

(1). Illustrations. Illustrations in the thesis under the relating to the familiar are the definition of geometry, and its relation to natural and man-created work. In the unit on orientation examples are given where the attention of the pupils is called to the geometry in the world. The geometric fact that two straight lines cross in a point is made meaningful by referring to the locating of a forest fire, a fence corner, a city house, and a ship at sea. The angle is related to the compass, hands of a watch, the open book, and the radio dial. Hypothesis and conclusion take on meaning in that unit, where the familiar grammatical construction of making the hypothesis the subject of the sentence, and the conclusion the predicate, is noted. In sports, a fishing trip and a picnic party are suggested as topics for making the terms "hypothesis" and "conclusion" real in the minds of the pupils. The applied problem as related to the familiar is found in the brace used in connection with the unit on the isosceles triangle.

In the section on proportion, relation to the familiar is made by means of photography, pictures, drawings, dressmaking, algebra, trigonometry, chemistry, physics, medicine, music and the playground. The photograph out of proportion, the enlargement of drawings either by squares or by the pantograph, the

1. Frank W. Thomas, op. cit. pp. 142-143.

response. Again, instruction must be on the level of the learner and the constant use of illustrative material is of value.[1] This phase of mind-set is called the "idealional". Its success depends upon vividness of the impression.

(1). Illustrations. Illustrations in the thesis under the relating to the familiar are the definition of geometry, and its relation to natural and man-created work. In the unit on orientation examples are given where the attention of the pupils is called to the geometry in the world. The geometric fact that two straight lines cross in a point is made meaningful by referring to the locating of a forest fire, a fence corner, a city house, and a ship at sea. The angle is related to the compass, hands of a watch, the open book, and the radio dial. Hypothesis and conclusion take on meaning in that unit, where the familiar grammatical construction of making the hypothesis the subject of the sentence, and the conclusion the predicate, is noted. In sports, a fishing trip and a picnic party are suggested as topics for making the terms "hypothesis" and "conclusion" real in the minds of the pupils. The applied problem as related to the familiar is found in the brace used in connection with the unit on the isosceles triangle.

In the section on proportion, relation to the familiar is made by means of photography, pictures, drawings, dressmaking, algebra, trigonometry, chemistry, physics, medicine, music and the playground. The photograph out of proportion, the enlargement of drawings either by squares or by the pantograph, the

1. Frank W. Thomas, op. cit. pp. 142-143.

proper widths of tucks, the imaginary number, the trigonometric tables, the inclined plane, and the expansion apparatus, the chemical problems, the method of mixtures, the graduating instrument for the violin, the teeter board, are all examples of the applied problem, and of interrelationship with other studies and with the familiar. Not all these contacts may be real to all pupils, but each pupil will have knowledge of some of the above fields.

In the section on loci is found also many contacts. The movies, the wheel in motion either revolving around an axis which is either stationary or going forward, the dog on the slide wire, the opening door, the motion of the hands of the clock, are all examples of getting instruction on the level of the learner and within his interests.

Still another phase of the preparation and mental set is the "feeling phase of the mind-set". It is noted "The pupil's temporary 'mood' plays a part in his reaction to any kind of situation and material, and the consequent need for the teacher to plan constantly to provide for the most favorable mind-set which can be induced in preparation for a contemplated learning-activity."[1] Here the teacher should know when and where to press questions. The skill is intangible. It is one of those mystic qualities of the good teacher which cannot be slotted because the success of its application depends upon complete understanding of the particular pupil at the particular moment.

1. Frank W. Thomas, op. cit. p. 141.

proper widths of tracks, the imaginary number, the trigonometric tables, the inclined plane, and the expansion apparatus, the chemical problems, the method of mixtures, the graduating instrument for the violin, the tester board, are all examples of the applied problem, and of interrelationship with other studies and with the familiar. Not all these contacts may be real to all pupils, but each pupil will have knowledge of some of the above fields.

In the section on load is found also many contacts. The movies, the wheel in motion either revolving around an axis which is either stationary or going forward, the dog on the slide wire, the opening door, the motion of the hands of the clock, are all examples of getting instruction on the level of the learner and within his interests.

Still another phase of the preparation and mental set is the "feeling phase of the mind-set". It is noted "The pupil's temporary 'mood' plays a part in his reaction to any kind of situation and material, and the consequent need for the teacher to plan constantly to provide for the most favorable mind-set which can be induced in preparation for a contemplated learning-activity."[1] Here the teacher should know when and where to press questions. The skill is intangible. It is one of those mystic qualities of the good teacher which cannot be slotted because the success of its application depends upon complete understanding of the particular pupil at the particular moment.

1. Frank W. Thomas, op. cit. p. 141.

To the degree that the teacher does understand the pupil, to just this degree will success come in handling these moods. The writer feels its importance, and strives within the class period to fathom when to call for extreme activity on the part of the particular pupil, and when to let the pupil remain less active. The mood itself is often bound up with out-of-school life. Discipline cases come from a misunderstanding of moods.

b). The Lesson for habits and skills.

(1). The correct start. First consider the correct start, the making sure that the learner possesses a clear mental image of the thing to be practised. One example of this in the thesis is found in the unit on the drill in picking out the hypothesis and conclusion. It was previously found that the learner is given a clear mental image by first arousing his interest through attachment to the familiar subject and predicate in english grammar, and through a relation to his own out-of-school activities such as the fishing trip or the picnic.

Humaneness and humor are introduced into the start of the drill procedure by endowing geometric figures with life, and clothing them with human characteristics. Thus, in the drill procedure, it is suggested that one must be friends with Mr. Parallelogram, if he is to be a friend. One recognizes him by his trousers of "opposite sides equal", and his shoes of "adjacent angles supplementary".

The clear mental image of the definition of angle is achieved through reference to objective material such as the globe, the pointer, the compass, the watch, the book, and the

To the degree that the teacher does understand the pupil, to just this degree will success come in handling these moods. The writer feels its importance, and strives within the class period to fathom when to call for extreme activity on the part of the particular pupil, and when to let the pupil remain less active. The mood itself is often bound up with out-of-school life. Discipline cases come from a misunderstanding of moods.

b). The Lesson for habits and skills.

(1). The correct start. First consider the correct start, the making sure that the learner possesses a clear mental image of the thing to be practised. One example of this in the thesis is found in the unit on the drill in picking out the hypothesis and conclusion. It was previously found that the learner is given a clear mental image by first arousing his interest through attachment to the familiar subject and predicate in english grammar, and through a relation to his own out-of-school activities such as the fishing trip or the picnic.

Humanness and humor are introduced into the start of the drill procedure by endowing geometric figures with life, and clothing them with human characteristics. Thus, in the drill procedure, it is suggested that one must be friends with Mr. Parallelogram, if he is to be a friend. One recognizes him by his trousers of "opposite sides equal", and his shoes of "adjacent angles supplementary".

The clear mental image of the definition of angle is achieved through reference to objective material such as the globe, the pointer, the compass, the watch, the book, and the

radio dial. The idea of angle as rotation of a line about a fixed line is illustrated by the vertices of a triangle, and the exterior angle of a polygon cut from the figure and placed together, so that they will make a half and complete revolution respectively.

Attention to the correct reading of an angle when more than one angle meets at a vertex also makes a clear mental image. In fact many of the difficulties in advanced parts of any work have their seed in confused or erroneous early mental images.

The drill on the included angle is first preceded by having the pupil make cardboard models of triangles which may be superimposed if they have two sides and the included angle equal. Flat wooden triangles may be shown with finger tracings to demonstrate the word "included". What happens when an angle is equal but not an included angle may also be drawn on the board. In the thesis there is mention of the use of colored chalk, verbal description, board drawings, humorous situations, and the use of placards for the securing of proper attitude and a correct mental image of the difficulty of included angle for drill work.

The same care in getting a correct start must be used in obtaining a clear idea of the meaning of the word "corresponding" angle or side. Here the proportion is written in words at first.

Another aid in getting a correct mental image is to use the see-it-grow, or class-built method, whereby the proposition is developed under the eye of the pupil. In this way, the

radio dial. The idea of angle as rotation of a line about a fixed line is illustrated by the vertices of a triangle, and the exterior angle of a polygon cut from the figure and placed together, so that they will make a half and complete revolution respectively.

Attention to the correct reading of an angle when more than one angle meets at a vertex also makes a clear mental image. In fact many of the difficulties in advanced parts of any work have their seed in confused or erroneous early mental images.

The drill on the included angle is first preceded by having the pupil make cardboard models of triangles which may be superimposed if they have two sides and the included angle equal. Flat wooden triangles may be shown with finger tracings to demonstrate the word "included". What happens when an angle is equal but not an included angle may also be drawn on the board. In the thesis there is mention of the use of colored chalk, verbal description, board drawings, humorous situations, and the use of placards for the securing of proper attitude and a correct mental image of the difficulty of included angle for drill work.

The same care in getting a correct start must be used in obtaining a clear idea of the meaning of the word "corresponding" angle or side. Here the proportion is written in words at first.

Another aid in getting a correct mental image is to use the see-it-grow, or class-built method, whereby the proposition is developed under the eye of the pupil. In this way, the

presentation becomes meaningful to him.

In the testing program itself the writer has changed from testing for complete propositions to testing during the first lessons for short steps of the proposition.

As noted previously, the places where drill may be applied to plane geometry, though important, are few.

(2). Varieties of guiding images. The second principle considered is that of varieties of guiding images. It is noted that not only visual, but auditory and kinaesthetic or motor images are important, and warning is given that no amount of mere telling, lecturing, or criticising clumsy work will compensate for an inadequate mental image. Thomas[1] believes certain kinds of work demands emphasis on a certain kind of image.

(a). Illustrations. In the drill on the definition of the word "angle", opportunity for individual interests and capacities may be provided for by varied approaches and varied tasks. Some pupils will learn through hand work such as drawing triangles with included angles, or making wooden models to illustrate them. Other pupils will have access to the definition through the visual path by seeing the figures on the board drawn with colors. Still others will get the concept best from the auditory image by having the teacher explain, or by having definitions read to them.

The important thing to observe is that the preferred type of guiding image varies with different individuals.[2] In the

1. Frank W. Thomas, op. cit. p. 152.

2. Frank W. Thomas, op. cit. p. 155.

presentation becomes meaningful to him.

In the testing program itself the writer has changed from testing for complete propositions to testing during the first lessons for short steps of the proposition.

As noted previously, the places where drill may be applied to plane geometry, though important, are few.

(2). Varieties of guiding images. The second principle considered is that of varieties of guiding images. It is noted that not only visual, but auditory and kinaesthetic or motor images are important, and warning is given that no amount of mere telling, lecturing, or criticizing clumsy work will compensate for an inadequate mental image. Thomas[1] believes certain kinds of work demands emphasis on a certain kind of image.

(a). Illustrations. In the drill on the definition of the word "angle", opportunity for individual interests and capacities may be provided for by varied approaches and varied tasks. Some pupils will learn through hand work such as drawing triangles with included angles, or making wooden models to illustrate them. Other pupils will have access to the definition through the visual path by seeing the figures on the board drawn with colors. Still others will get the concept best from the auditory image by having the teacher explain, or by having definitions read to them.

The important thing to observe is that the preferred type of guiding image varies with different individuals.[2] In the

1. Frank W. Thomas, op. cit. p. 152.

2. Frank W. Thomas, op. cit. p. 155.

determination of the hypothesis and conclusion, for instance, one pupil may learn best by a graphic method, another by reference to english grammar, and a third, whose mind wanders out-of-doors, will find his entry by way of the fishing trip. Or again, hand practice for one, through setting off in the proposition the hypothesis and conclusion, may be all important. In fact, it is the experience of the writer that many a time a pupil understands the difficulty, but when that understanding is put to a test through writing, without previous practice, the test is a failure.

(3). The pupil must discover the essential value of the drill for him. Another principle suggested is that the pupil must discover the essential value of the drill to him as the means of attainment of some desirable end, and then must realize that its mastery is necessary to the accomplishment of his own purpose.[1] This principle may be attached to geometry in high school through graphic representations of the steps in propositions. The mountain graph, the stream, the arch, all show the necessity of each step. Immediate connection at times with propositions in advance may be made, as in the algebraic approach to proportion. Here the algebra to be learned or reviewed may be immediately attached to the plane geometry proposition. Thus the pupil sees the need of the algebraic processes to be undertaken.

Another instance may be noted where a pupil can be told in advance that if he gets the triangle well in mind, that in

1. Frank W. Thomas, op. cit. p. 161.

determination of the hypothesis and conclusion, for instance, one pupil may learn best by a graphic method, another by reference to English grammar, and a third, whose mind wanders out-of-doors, will find his entry by way of the fishing trip. Or again, hand practice for one, through setting off in the proposition the hypothesis and conclusion, may be all important. In fact, it is the experience of the writer that many a time a pupil understands the difficulty, but when that understanding is put to a test through writing, without previous practice, the test is a failure.

(3). The pupil must discover the essential value of the drill for him. Another principle suggested is that the pupil must discover the essential value of the drill to him as the means of attainment of some desirable end, and then must realize that its mastery is necessary to the accomplishment of his own purpose.1 This principle may be attached to geometry in high school through graphic representations of the steps in propositions. The mountain,graph, the stream, the arch, all show the necessity of each step. Immediate connection at times with propositions in advance may be made, as in the algebraic approach to proportion. Here the algebra to be learned or review may be immediately attached to the plane geometry proposition. Thus the pupil sees the need of the algebraic processes to be undertaken.

Another instance may be noted where a pupil can be told in advance that if he gets the triangle well in mind, that in

1. Frank W. Thomas, op. cit. p. 161.

the part of the text on circles he will be in a familiar field, because triangles appear in circles. The same is true in the study of polygons.

In brief, to show a pupil where, in the journey, the particular step fits, is important. He wishes to know where he is going and how far he has to go. And it is this latter part of how far he is to go which is so vital to the pupil in keeping a record of his own progress. This has been noted in the thesis in the section on classroom procedure where it is suggested that pupils keep notebooks of their own errors.

c). The Lesson for Appreciation.

(1). Emotional Identification with Task. When one turns to the lesson for appreciation, it is found that the pupil must identify himself emotionally with the task either by active participation in a physical way, or by mental participation through the imagination. Thus the boy scout achieves this activity in a physical act of a good turn daily, while the reader of the adventure story lives mentally the part of the hero.

Thomas[1] cites this technique as providing for three steps in the achieving of appreciative attitudes. The first is the gaining of the appropriate mental set, the recalling of experiences in harmony with the feeling desired. The second is the provision of class activities so that the pupil will participate emotionally in the situation, identifying himself with the cause or its champions. The third step is an extension of the second;

1. Frank W. Thomas, op. cit. p. 199.

the part of the text on circles he will be in a familiar field, because triangles appear in circles. The same is true in the study of polygons.

In brief, to show a pupil where, in the journey, the particular step fits, is important. He wishes to know where he is going and how far he has to go. And it is this latter part of how far he is to go which is so vital to the pupil in keeping a record of his own progress. This has been noted in the thesis in the section on classroom procedure where it is suggested that pupils keep notebooks of their own errors.

c). The Lesson for Appreciation.

(1). Emotional Identification with Task. When one turns to the lesson for appreciation, it is found that the pupil must identify himself emotionally with the task either by active participation in a physical way, or by mental participation through the imagination. Thus the boy scout achieves this activity in a physical act of a good turn daily, while the reader of the adventure story lives mentally the part of the hero.

Thomas[1] cites this technique as providing for three steps in the achieving of appreciative attitudes. The first is the gaining of the appropriate mental set, the recalling of experiences in harmony with the feeling desired. The second is the provision of class activities so that the pupil will participate emotionally in the situation, identifying himself with the cause or its champions. The third step is an extension of the second;

1. Frank W. Thomas, op. cit. p. 199.

it is the inauguration of some further line of activity through which the new feeling may find expression and satisfaction.

(a). Illustrations. The full significance which the appreciation lesson plays in geometry comes to light when one examines the thesis for material on the mental set toward the subject. To prove that geometry is not just a book subject but a real part of life, its relation to nature in the animal and mineral world and to man in his various activities is noted. Whether one looks at the flower or the honey-comb, the crystal of salt or the leaf on a tree, one finds geometric form. Whether it be the lining up of trees being planted in an apple orchard, or the location of a ship at sea, geometric laws are there. One looks at quilts, wall paper, or oil cloth at home, there one observes geometric designs. The furniture is ever repeating some geometric rule. The carpenter builds his brace with the geometric form of the triangle.

In the chapter on proportion and similarity, it is noted that the "force times distance" law is entertwined with geometry in the inclined plane where triangles are similar, and that in the heat expansion apparatus, the magnification is based on similar triangles. In chemistry, the graphic representations of equation problems brings geometric forms. Turning to the study of trigonometry, its very essence comes from similar triangles. All the trigonometric tables are built on the principle of proportion. In music, the "graduating" of a violin is done through magnification, which again involves proportion. If one goes to the playground, the moving teeter board forms

it is the inauguration of some further line of activity through which the new feeling may find expression and satisfaction.

(a). Illustrations. The full significance which the appreciation lesson plays in geometry comes to light when one examines the thesis for material on the mental set toward the subject. To prove that geometry is not just a book subject but a real part of life, its relation to nature in the animal and mineral world and to man in his various activities is noted. Whether one looks at the flower or the honey-comb, the crystal of salt or the leaf on a tree, one finds geometric form. Whether it be the lining up of trees being planted in an apple orchard, or the location of a ship at sea, geometric laws are there. One looks at quilts, wall paper, or oil cloth at home, there one observes geometric designs. The furniture is ever repeating some geometric rule. The carpenter builds his brace with the geometric form of the triangle.

In the chapter on proportion and similarity, it is noted that the "force times distance" law is entertwined with geometry in the inclined plane where triangles are similar, and that in the best expansion apparatus, the magnification is based on similar triangles. In chemistry, the graphic representations of equation problems brings geometric forms. Turning to the study of trigonometry, its very essence comes from similar triangles. All the trigonometric tables are built on the principle of proportion. In music, the "graduating" of a violin is done through magnification, which again involves proportion. If one goes to the playground, the moving teeter board forms

similar triangles, and the swing describes the arc of a circle as it soars aloft to the gleeful shouts of the children.

At school, manual training and mechanical drawing could not exist without geometric forms. Even in algebra the conception of imaginary numbers is based on proportion, and the expansion of a binomial shown graphically borrows from geometry.

All these are taken up in detail in the thesis chiefly in the units on orientation, definitions, proportion and similarity, and in the miscellaneous devices. They are for the purpose of producing the proper mental set. They are aimed to make the pupil enthusiastic, and eager to begin and to continue the study of geometry. Through them the pupil is to emotionally identify himself with the task both in a physical and in an imaginative way.

The inherent difficulties peculiar to geometry have to be met and overcome if the pupil is to identify himself emotionally with the actual development of geometric procedure. The difficulty of flatland, as noted in the introduction, may be overcome by relation to objects such as pieces of pie, ice cream, and candy. For instance, instead of simply a triangle, the geometric form becomes attached to the above sweets.

The abstract nature of geometry is provided for by translation to objects. An example of this is found in the introduction section where the symbol, verbal language, picture and object table is given.

The difficulty of the new procedure of proof is overcome by a gradual introduction through orientation and through drill in

similar triangles, and the swing describes the arc of a circle as it soars aloft to the gleeful shouts of the children.

At school, manual training and mechanical drawing could not exist without geometric forms. Even in algebra the conception of imaginary numbers is based on proportion, and the expansion of a binomial shown graphically borrows from geometry.

All these are taken up in detail in the thesis chiefly in the units on orientation, definitions, proportion and similarity, and in the miscellaneous devices. They are for the purpose of producing the proper mental set. They are aimed to make the pupil enthusiastic, and eager to begin and to continue the study of geometry. Through them the pupil is to emotionally identify himself with the task both in a physical and in an imaginative way.

The inherent difficulties peculiar to geometry have to be met and overcome if the pupil is to identify himself emotionally with the actual development of geometric procedure. The difficulty of flatland, as noted in the introduction, may be overcome by relation to objects such as pieces of pie, ice cream, and candy. For instance, instead of simply a triangle, the geometric form becomes attached to the above sweets.

The abstract nature of geometry is provided for by translation to objects. An example of this is found in the introduction section where the symbol, verbal language, picture and object table is given.

The difficulty of the new procedure of proof is overcome by a gradual introduction through orientation and through drill in

picking out the known and unknown quantities, the hypothesis and conclusion.

The section on illusions is for the special purpose of creating a felt need for the demonstration, and an emotionalized attitude toward participation in the demonstration which will augur well for the success of the pupil.

All this is for the purpose of getting the pupil to emotionally identify himself with the task that it may be real to him.

(2). Provision for class activities must be made. Next consider the second principle for the lesson on appreciation which is providing class activities so that the pupil will participate emotionally in the situation. This is the actual learning of the lesson. It involves the class period chiefly. Its success rests in the resourcefulness and sympathy of the teacher. In the thesis, it involves all those techniques which go to make the lesson interesting and vivid; these appear to a large extent in the unit on miscellaneous devices for clarity and attitude.

Its core is essentially the "we" feeling which is induced through the see-it-grow and class-built methods, as outlined in the section on classroom procedure. It provides primarily for the physical activity of the pupil in the construction work, and in the keeping of his own score and the noting of his own errors and their correction. It is to be here noted, however, that a pupil may be active though not physically active. That is, he may be reacting in a very real way mentally, and his

picking out the known and unknown quantities, the hypothesis and conclusion.

The section on illusions is for the special purpose of creating a felt need for the demonstration, and an emotionalized attitude toward participation in the demonstration which will augur well for the success of the pupil.

All this is for the purpose of getting the pupil to emotionally identify himself with the task that it may be real to him.

(2). Provision for class activities must be made. Next consider the second principle for the lesson on appreciation which is providing class activities so that the pupil will participate emotionally in the situation. This is the actual learning of the lesson. It involves the class period chiefly. Its success rests in the resourcefulness and sympathy of the teacher. In the thesis, it involves all those techniques which go to make the lesson interesting and vivid; these appear to a large extent in the unit on miscellaneous devices for clarity and attitude.

Its core is essentially the "we" feeling which is induced through the see-it-grow and class-built methods, as outlined in the section on classroom procedure. It provides primarily for the physical activity of the pupil in the construction work, and in the keeping of his own score and the noting of his own errors and their correction. It is to be here noted, however, that a pupil may be active though not physically active. That is, he may be reacting in a very real way mentally, and his

mind be keenly alert to all that is going on. In short, he may be identifying himself emotionally with the situation with great zest, even though he may not be physically active even to the point of verbal expression. This condition varies with the personality.

However, with pupils of high school age, physical activity is of great help in learning. As noted previously, perfect understanding may dwindle to failure when putting the material in writing, unless previous practice has taken place. The author, therefore, urges again and again throughout the year, that the pupils write their proofs on paper, thus giving themselves the examination ahead of time.

The class activity of the individual pupils will vary as their interests and capacities. Emotional identification will come only when work is suited to the level of the learner. Thus all the skill in portioning out questions and tasks according to these interests and abilities is skill directed to the successful identifying of each pupil with the work involved. Emotional identification with the task may involve the solution of difficult problems with one pupil, design work with another, or easier propositions with a third. As soon as the task is out of focus with the pupil, emotional identification stops.

The idea of translating the symbol to the object also is with the aim of securing emotional identification. The taking back to familiar things, to attractive things such as household designs, and the use of colored chalk is all toward this same end. All interrelations with other school subjects is

mind be keenly alert to all that is going on. In short, he may be identifying himself emotionally with the situation with great zest, even though he may not be physically active even to the point of verbal expression. This condition varies with the personality.

However, with pupils of high school age, physical activity is of great help in learning. As noted previously, perfect understanding may dwindle to failure when putting the material in writing, unless previous practice has taken place. The author, therefore, urges again and again throughout the year, that the pupils write their proofs on paper, thus giving themselves the examination ahead of time.

The class activity of the individual pupils will vary as their interests and capacities. Emotional identification will come only when work is suited to the level of the learner. Thus all the skill in portioning out questions and tasks according to these interests and abilities is skill directed to the successful identifying of each pupil with the work involved. Emotional identification with the task may involve the solution of difficult problems with one pupil, design work with another, or easier propositions with a third. As soon as the task is out of focus with the pupil, emotional identification stops.

The idea of translating the symbol to the object also is with the aim of securing emotional identification. The taking back to familiar things, to attractive things such as house-hold designs, and the use of colored chalk is all toward this same end. All interrelations with other school subjects is

but an effort to find a warm spot in the particular pupil's heart which will cause him to identify himself emotionally with the geometric task. All devices to isolate the difficulty which build toward vivid teaching are but to make the path smooth for this identification.

The erasing of parts of the figure to restore to original simplicity and to show the difficulty in isolation is another example. Still another way is the focussing on the new difficulty by means of colored chalk or broad lines. Even the drills themselves with their satisfying success reactions build toward this active participation. The graphic method of showing the process of the proposition is another element working toward the same end to continue a favorable attitude toward the task.

Humorous situations such as the photograph out of focus, the fish, the stilts, masks, and the fly, as cited in the section on proportion and similarity, all aim at making the pupil an interested participant, rather than an artificially forced pupil of the "there-in-body-only" type. It has been said that if one can but harness fifteen per cent of the pupil's mental life to the school task that one need not worry about accomplishment of the pupil. It is suspected that the reason much high school learning has left the pupil, like rain washing from a duck's back, is because it never was really there in the first place. It was superimposed, artificial, and involved only the surface activity of the pupil; it did not seep in and become really his; he did not enter whole heartedly into it.

but an effort to find a warm spot in the particular pupil's heart which will cause him to identify himself emotionally with the geometric task. All devices to isolate the difficulty which build toward vivid teaching are but to make the path smooth for this identification.

The erasing of parts of the figure to restore to original simplicity and to show the difficulty in isolation is another example. Still another way is the focussing on the new difficulty by means of colored chalk or broad lines. Even the drills themselves with their satisfying success reactions build toward this active participation. The graphic method of showing the process of the proposition is another element working toward the same end to continue a favorable attitude toward the task.

Humorous situations such as the photograph out of focus, the fish, the stilts, masks, and the fly, as cited in the section on proportion and similarity, all aim at making the pupil an interested participant, rather than an artificially forced pupil of the "there-in-body-only" type. It has been said that if one can but harness fifteen per cent of the pupil's mental life to the school task that one need not worry about accomplishment of the pupil. It is suspected that the reason much high school learning has left the pupil, like rain washing from a duck's back, is because it never was really there in the first place. It was superimposed, artificial, and involved only the surface activity of the pupil; it did not seep in and become really his; he did not enter whole heartedly into it.

(3). Provision for further activity outside the classroom. The third step in the technique of teaching the lesson on appreciation is "the inauguration of some further line of activity through which the new feeling may find expression and added satisfaction".[1]

It is here where the section on provision for individual differences plays a part. The writer has noted that the interest may vary in being hand activity such as drawing or designing, or mechanical drawing, or manual training, or the interest may be in verbal presentation such as plays or talks, or the problem-minded pupil may present an original, or work out a puzzle. Those interested in science may present the interrelations in that direction.

The keynote of this technique is that the activity of the pupil is his own inherent choice, and that he has a variety of activities from which to choose.

The writer was much interested in offering to the class the opportunity to copy pictures by means of squares as described in the section on proportion and similarity. Many of the poorer students in problem work gained new life and enthusiasm through this opportunity.

Several boys have made wooden models with much gusto, and others, pantographs.

The vehicles by which these individual activities may be realized is noted in the unit on provision for individual differences. They include the mathematics club for the socially-

1. Frank W. Thomas, op. cit. p. 199.

(3). Provision for further activity outside the classroom.

The third step in the technique of teaching the lesson on appreciation is "the inauguration of some further line of activity through which the new feeling may find expression and added satisfaction".[1]

It is here where the section on provision for individual differences plays a part. The writer has noted that the interest may vary in being hand activity such as drawing or designing, or mechanical drawing, or manual training, or the interest may be in verbal presentation such as plays or talks, or the problem-minded pupil may present an original, or work out a puzzle. Those interested in science may present the interrelations in that direction.

The keynote of this technique is that the activity of the pupil is his own inherent choice, and that he has a variety of activities from which to choose.

The writer was much interested in offering to the class the opportunity to copy pictures by means of squares as described in the section on proportion and similarity. Many of the poorer students in problem work gained new life and enthusiasm through this opportunity.

Several boys have made wooden models with much gusto, and others, pantographs.

The vehicles by which these individual activities may be realized is noted in the unit on provision for individual differences. They include the mathematics club for the socially-

1. Frank N. Thomas, op. cit. p. 129.

minded geometry pupil. The mathematics room affords opportunity for the art-minded. For the verbal-minded, bulletin boards and reference tables give activity, while for the hand-minded, geometrical instruments and models are satisfying.

Everyone is about his own task, which he himself has selected and in which he has a special interest. This is the third step in the technique of teaching the lesson for appreciation, so that a continuance of the emotional identification of the pupil with the task may be achieved.

d). The Lesson for Mastering Knowledge.

(1). Associations must involve thoughtful recognition of relationships. Turning to the lesson for mastering knowledge, Thomas observes:[1]

> "In short, the most valuable kind of associations are those resulting from a thoughtful recognition of relationships, and from a discovery, through application, of the logical uses to which the information may be put."
>
> "It should be obvious, also, that the probability of recall is greatly increased by having a number and variety of associations established. .....
> .....
> "When facts are given application and put to use in something resembling the normal setting in which they function, they immediately take on from that relationship a rich variety of association peculiarly adapted to facilitate recall...."
>
> "The type of learning procedure which develops the strongest and most desirable associations for recalling information is precisely the procedure which employs and tends to develop the pupil's facility in using and applying information. Accordingly, in our search for the fundamental principles upon which to base a sound technique of teaching this type of lesson, we come directly to the psychology of thinking and problem solving."

1. Frank W. Thomas, op. cit. pp. 217-218.

minded geometry pupil. The mathematics room affords opportunity for the art-minded. For the verbal-minded, bulletin boards and reference tables give activity, while for the hand-minded, geometrical instruments and models are satisfying.

Everyone is about his own task, which he himself has selected and in which he has a special interest. This is the third step in the technique of teaching the lesson for appreciation, so that a continuance of the emotional identification of the pupil with the task may be achieved.

d). The Lesson for Mastering Knowledge.

(1). Associations must involve thoughtful recognition of relationships. Turning to the lesson for mastering knowledge, Thomas observes:[1]

> "In short, the most valuable kind of associations are those resulting from a thoughtful recognition of relationships, and from a discovery, through application, of the logical uses to which the information may be put."
>
> "It should be obvious, also, that the probability of recall is greatly increased by having a number and variety of associations established. .....
> .....
> "When facts are given application and put to use in something resembling the normal setting in which they function, they immediately take on from that relationship a rich variety of association peculiarly adapted to facilitate recall...."
>
> "The type of learning procedure which develops the strongest and most desirable associations for recalling information is precisely the procedure which employs and tends to develop the pupil's facility in using and applying information. Accordingly, in our search for the fundamental principles upon which to base a sound technique of teaching this type of lesson, we come directly to the psychology of thinking and problem solving."

1. Frank W. Thomas, op. cit. pp. 217-218.

To summarize the above principles, one might say that teaching must have meaning in order to be remembered, and that this meaning is given through rich associations and rich applications. The studying of the given situation with its associations serves to make the mind dwell long enough and with enough intensity, so that the procedure becomes a part of the student, instead of the pupil being artificially whitewashed with isolated facts which drip off with the first rainstorm of other interests.

(a). The principle of organization and continuity is important. First examine the thesis to see wherein devices are suggested to bring about the above situation. First note the associations which result from thoughtful recognition of relationships. The kernel of this idea rests, perhaps, in such material as the unit on Organization and Continuity.

i). Illustrations. Graphic examples are given of the relationship between theorems concerning areas, and the measurements of angles in connection with the circle. Another illustration is that of the product and mean proportion theorems.

In the introductory unit are to be found examples of relationship of various geometric symbols working through the english language and the picture language, to the actual object.

Parallel steps of procedure in learning which involve first the general learning situation, and then learning with specific subjects such as athletics, music and geometry are noted. The aim is to have the pupil perceive the significant relationships and to appreciate the work which must be done in conquering any

To summarize the above principles, one might say that teaching must have meaning in order to be remembered, and that this meaning is given through rich associations and rich applications. The studying of the given situation with its associations serves to make the mind dwell long enough and with enough intensity, so that the procedure becomes a part of the student, instead of the pupil being artificially whitewashed with isolated facts which drip off with the first rainstorm of other interests.

(a). <u>The principle of organization and continuity is important</u>. First examine the thesis to see wherein devices are suggested to bring about the above situation. First note the associations which result from thoughtful recognition of relationships. The kernel of this idea rests, perhaps, in such material as the unit on Organization and Continuity.

1). <u>Illustrations</u>. Graphic examples are given of the relationship between theorems concerning arcs, and the measurements of angles in connection with the circle. Another illustration is that of the product and mean proportion theorems.

In the introductory unit are to be found examples of relationship of various geometric symbols working through the english language and the picture language, to the actual object.

Parallel steps of procedure in learning which involve first the general learning situation, and then learning with specific subjects such as athletics, music and geometry are noted. The aim is to have the pupil perceive the significant relationship and to appreciate the work which must be done in conquering any

situation.

The various graphs throughout the thesis also tend to produce thoughtful recognition of relationships, because they enable the pupil to see clearly the entire journey and the goal.

In addition to setting a favorable situation for thoughtful recognition of relationships through proper organization and grouping, the thesis makes use of the applications of the geometric principles. In the unit on individual differences, for instance, the interrelationships with the pupil's own interests in other activities both within the school curriculum and outside the school suggests further rich associations which again give real meaning to the task, thus setting the situation favorable for recall.

The various graphs for the mathematics room, illustrating the thought procedure necessary is both interesting and useful to the pupil. Here he has a chance to observe how his own mind works. Among these graphs are those on achieving success. They show a variety of approaches for success, the method of problem solving, and the necessity of repetitions. Still other graphs as illustrated by the parasol shows how the method of obtaining a proof is derived from bringing the unknown theorem under a known theorem. The facts about the unknown are true because similar facts about the known are true. Thus the unknown has been related to the known.

The unit on constructions and measurements serves as an activity for the hand-minded pupil which furthers thoughtful recognition of relationships and offers excellent opportunities

situation.

The various graphs throughout the thesis also tend to produce thoughtful recognition of relationships, because they enable the pupil to see clearly the entire journey and the goal.

In addition to setting a favorable situation for thoughtful recognition of relationships through proper organization and grouping, the thesis makes use of the applications of the geometric principles. In the unit on individual differences, for instance, the interrelationships with the pupil's own interests in other activities both within the school curriculum and outside the school suggests further rich associations which again give real meaning to the task, thus setting the situation favorable for recall.

The various graphs for the mathematics room, illustrating the thought procedure necessary is both interesting and useful to the pupil. Here he has a chance to observe how his own mind works. Among these graphs are those on achieving success. They show a variety of approaches for success, the method of problem solving, and the necessity of repetitions. Still other graphs as illustrated by the parasol shows how the method of obtaining a proof is derived from bringing the unknown theorem under a known theorem. The facts about the unknown are true because similar facts about the known are true. Thus the unknown has been related to the known.

The unit on constructions and measurements serves as an activity for the hand-minded pupil which furthers thoughtful recognition of relationships and offers excellent opportunities

for self discovery of geometric laws. Many of the errors of thinking in geometry may be removed by making a drawing of the situation. Among these instances might be noted the tendency of placing too many conditions on a line. This is considered under the unit on the isosceles triangle.

The hand work, such as opening doors, and finger tracings of the geometric laws involved, as found in the section on the locus, again furnishes opportunities for thoughtful recognition of relationships, and thus rich associations.

The very mathematics room itself, as outlined in the unit on provision for individual differences, affords an attractive setting. The variety of sensory approaches also makes for added power.

Thomas[1] notes that occasional summarizing of progress avoids confusion and wandering, that this summarizing is especially needed in the mathematical problem. Especially significant for this idea is the method of graphic analysis as illustrated throughout the thesis in the mountain, bridge, or stream form.

It is in this way that memory work is made powerful by becoming a real part of the pupil. When he absorbs willingly and understandingly, the knowledge stays with him because the various facts are attached to vital principles.

> "The lad could not realize that the mass of facts and formulas had been so interrelated by thought and use that one could be reproduced from another, and that the retention of the whole actually required less effort than he was expending in his struggle, to maintain his grip upon a few elusive facts....."[1]

1. Frank W. Thomas, p. 220.

for self discovery of geometric laws. Many of the errors of thinking in geometry may be removed by making a drawing of the situation. Among these instances might be noted the tendency of placing too many conditions on a line. This is considered under the unit on the isosceles triangle.

The hand work, such as opening doors, and finger tracings of the geometric laws involved, as found in the section on the locus, again furnishes opportunities for thoughtful recognition of relationships, and thus rich associations.

The very mathematics room itself, as outlined in the unit on provision for individual differences, affords an attractive setting. The variety of sensory approaches also makes for added power.

Thomas[1] notes that occasional summarizing of progress avoids confusion and wandering, that this summarizing is especially needed in the mathematical problem. Especially significant for this idea is the method of graphic analysis as illustrated throughout the thesis in the mountain, bridge, or stream form.

It is in this way that memory work is made powerful by becoming a real part of the pupil. When he absorbs willingly and understandingly, the knowledge stays with him because the various facts are attached to vital principles.

> "The lad could not realize that the mass of facts and formulas had been so interrelated by thought and use that one could be reproduced from another, and that the retention of the whole actually required less effort than he was expending in his struggle, to maintain his grip upon a few elusive facts...."[1]

1. Frank W. Thomas, p. 220.

The youthful mind finds it hard to grasp the idea that memory work can be so related that effort and efficiency are greatly increased. Small boys marvel at the gigantic "memories" of adults in talking, visualizing that the adult must have "remembered" each isolated word, whereas in reality the adult did no such thing, but had in mind ideas, and grouped around those ideas associations illustrating them.

In conclusion, it is to be emphasized that Thomas is not a standard, but merely typical, and that this summary is designed to show the ease with which requirements of good teaching are observed.

The youthful mind finds it hard to grasp the idea that memory work can be so related that effort and efficiency are greatly increased. Small boys marvel at the gigantic "memories" of adults in talking, visualizing that the adult must have "remembered" each isolated word, whereas in reality the adult did no such thing, but had in mind ideas, and grouped around those ideas associations illustrating them.

In conclusion, it is to be emphasized that Thomas is not a standard, but merely typical, and that this summary is designed to show the ease with which requirements of good teaching are observed.

BIBLIOGRAPHY

(Books)

Angell, James Rowland. Psychology, pp.163,164,187. Fourth Edition, Revised, New York: Henry Holt and Company, 1904, 1908.

Breslich, Ernest R. Problems in Teaching Secondary School Mathematics. pp. 247-249, 279, 289, 290, 296-299, 302. Chicago, Illinois: The University of Chicago Press.

Burnham, William H. The Normal Mind. New York: D. Appleton and Company, 1924. vii * 702 pp.

Burnham, William H. The Wholesome Personality. New York: D.Appleton and Company, 1932. vii * 713 pp.

Calkins, Mary Whiton, A First Book in Psychology. p. 72. Fourth Edition, Revised, New York: The MacMillan Company, 1914.

Colvin, Stephen Sheldon. An Introduction to High School Teaching. p. 425. New York: The MacMillan Company, 1922, 1917.

Cowley, Elizabeth Buchanan. Plane Geometry. pp. 30, 45, 61, 79, 283-284. New York: Siliver, Burdett and Company, 1932.

Dewey, John and Evelyn. Schools of To-morrow. New York: Dutton and Company, 1915. 316 pp.

Dewey, John. The School and Society. Chicago: Revised Edition. Chicago: University of Chicago Press, 1930. 164 pp.

Hassler, Jasper C., and Smith, Rolland R. The Teaching of Secondary Mathematics. pp. 191, 317, 323, 324, 331, 332, 347, 348 356, 360-365. Edited by Earle Raymond Hedrick, New York: The MacMillan Company, 1930.

Montessori, Maria, The Montessori Method. p. 276. Fifth Edition, Translated from the Italian by Anne E.George. New York: Frederick A.Stokes Company, 1912.

Morrison, Henry C. The Practice of Teaching in the Secondary School. pp. 79-99. Chicago, Illinois: University of Chicago Press, 1926.

Myers, Charles S. A Text Book of Experimental Psychology. pp. 287-288, 290. Second Edition, Cambridge: at the University Press. Longmans, Green and Company, Fourth Avenue and 30th. St. New York. 1911.

BIBLIOGRAPHY

(Books)

Angell, James Rowland. Psychology, pp. 163, 164, 167. Fourth Edition, Revised. New York: Henry Holt and Company, 1904, 1908.

Breslich, Ernest R. Problems in Teaching Secondary School Mathematics. pp. 247-249, 279, 289, 290, 296-299, 302. Chicago, Illinois: The University of Chicago Press.

Burnham, William H. The Normal Mind. New York: D. Appleton and Company, 1924. vii + 702 pp.

Burnham, William H. The Wholesome Personality. New York: D.Appleton and Company, 1932. vii + 413 pp.

Calkins, Mary Whiton. A First Book in Psychology. p. 72. Fourth Edition, Revised. New York: The MacMillan Company, 1914.

Colvin, Stephen Sheldon. An Introduction to High School Teaching. p. 425. New York: The MacMillan Company, 1922, 1917.

Cowley, Elizabeth Buchanan. Plane Geometry. pp. 30, 45, 61, 79, 283-284. New York: Silver, Burdett and Company, 1932.

Dewey, John and Evelyn. Schools of To-morrow. New York: Dutton and Company, 1915. 316 pp.

Dewey, John. The School and Society. Chicago: Revised Edition. Chicago: University of Chicago Press, 1920. 164 pp.

Hassler, Jasper C., and Smith, Roland R. The Teaching of Secondary Mathematics. pp. 191, 317, 323, 324, 331, 332, 347, 355, 360-365. Edited by Earle Raymond Hedrick. New York: The MacMillan Company, 1930.

Montessori, Maria. The Montessori Method. p. 276. Fifth Edition. Translated from the Italian by Anne E.George. New York: Frederick A.Stokes Company, 1912.

Morrison, Henry C. The Practice of Teaching in the Secondary School. pp. 79-99. Chicago, Illinois: University of Chicago Press, 1926.

Myers, Charles S. A Text Book of Experimental Psychology. pp. 287-288, 290. Second Edition. Cambridge: at the University Press. Longmans, Green and Company, Fourth Avenue and 30th St. New York. 1911.

(Books) (cont.)

Parker, Samuel Chester. Methods of Teaching in High Schools. pp. 169-229. Revised Edition, Boston: Ginn and Company, 1915-1920.

Pillsbury, W. B. The Essentials of Psychology. pp. 176, 196, 198, Revised Edition, New York: The MacMillan Company, Boston, 1920, 1911.

Shibli, J. Recent Developments in the Teaching of Geometry. pp. 103, 104, 223, 227, 228, Pennsylvania: J. Shibli, Publisher. State College. 1932.

Schultze, Arthur, The Teaching of Mathematics in Secondary Schools. pp. 30, 32-36, 45-46, 48-49, 77, 110-112, 113, 115-118, 127, 135, 138, 152, 160-161, 184, 197-206, 218-225. New York: The MacMillan Company, 1924.

Stone, John C., and Mallory, Vergil S. Stone-Mallory Modern Solid Geometry. p. 520. Chicago: Benjamin H. Sanborn & Company, 1933.

Strader, William W. and Rhoads, Lawrence D. Plane Geometry pp. 42, 45, 52. Philadelphia, Pa: The John C. Winston Company, 1927.

Thomas, Frank W. Principles and Technique of Teaching. pp. 227-240. The Riverside Press, Cambridge, Houghton Mifflin Company, 1927.

Wilson, Harry B., Kyte, George C., and Lull, Herbert G., Modern Methods in Teaching. pp. 92-110. New York: Silver, Burdett and Company, 1924.

Young, J. W. A. The Teaching of Mathematics in the Elementary and Secondary School. pp. 263, 266, 267, 284. New York: Longmans, Green and Company, 1931.

(Books) (cont.)

Parker, Samuel Chester. Methods of Teaching in High Schools. pp. 169-229. Revised Edition, Boston: Ginn and Company, 1915-1920.

Pillsbury, W. B. The Essentials of Psychology. pp. 179, 190, 198. Revised Edition, New York: The MacMillan Company, Boston, 1920, 1911.

Shibli, J. Recent Developments in the Teaching of Geometry. pp. 103, 104, 225, 227, 228. Pennsylvania: J. Shibli, Pottsville. State College. 1932.

Schultze, Arthur. The Teaching of Mathematics in Secondary Schools. pp. 30, 32-36, 45-46, 48-49, 77, 110-112, 113, 115-118, 127, 135, 138, 152, 160-161, 184, 197-206, 218-225. New York: The MacMillan Company, 1924.

Stone, John C., and Mallory, Virgil S. Stone-Mallory Modern Solid Geometry. p. 320. Chicago: Benjamin H. Sanborn & Company, 1933.

Strader, William W. and Rhoads, Lawrence D. Plane Geometry pp. 42, 46, 52. Philadelphia, Pa: The John C. Winston Company, 1927.

Thomas, Frank W. Principles and Technique of Teaching. pp. 227-240. The Riverside Press, Cambridge, Houghton Mifflin Company, 1927.

Wilson, Harry B., Kyte, George C., and Lull, Herbert G., Modern Methods in Teaching. pp. 92-110. New York: Silver, Burdett and Company, 1924.

Young, J. W. A. The Teaching of Mathematics in the Elementary and Secondary School. pp. 223, 226, 227, 284. New York: Longmans, Green and Company, 1931.

(Magazines)

Anning, Norman, "High School Mathematics Club". The Mathematics Teacher, XXVI (February, 1933) pp. 70-76.

Beatley, Ralph. "Second Report of the Committee on Geometry." The Mathematics Teacher. XXVI (October, 1933) pp. 266-271. Presented to the National Council of Teachers of Mathematics at its Annual Meeting in Minneapolis on February 24, 1933.

Beatley, Ralph. "Preliminary Report of the Committee on Geometry". The Mathematics Teacher, XXV (November, 1932). pp. 427-428.

Blaisdell, Walter H. "Daily Testing." The Mathematics Teacher, XXVI (April, 1933). pp. 227-234.

Christofferson, H. C. "A Fallacy in Geometric Reasoning". The Mathematics Teacher, XXIII (January, 1930), pp. 19-22.

Cook, Mary R. "Stimulate The Interest in Mathematics by Creating a Mathematical Atmosphere." The Mathematics Teacher. XXIV (April, 1931), pp. 248-254.

Engle, B. T. "Proportion". School Science and Mathematics. XXXII (March, 1932), pp. 268-271.

Engle, B. T. "Constructing a Transit as a Project in Geometry". The Mathematics Teacher. XXIV (November, 1931) pp.444-447.

Georges, J.S. "Learning Product of a Unit of Instruction in Mathematics." School Science and Mathematics. XXXI (June, 1931), pp. 695-706.

Hoag, Ruth. "Source of Program Material and Some Types of Program Work by High School Mathematics Club". The Mathematics Teacher. XXIV (December, 1931). pp. 492-502

Horn, Aaron. "Book Propositions in Teaching Geometry". The Mathematics Teacher. XXV (February, 1932), pp. 76-78.

Hatton, Mary C. "The Mathematics Club". The Mathematics Teacher, XX (January, 1927). pp. 39-45.

Hennessey, F. P. "The Principles of Continuity". The Mathematics Teacher. XXIV (January, 1931). p. 32-40.

Johnson, J. T. "Adapting Instructional Material to Individual Differences in Learning." The Mathematics Teacher, XXVI (April, 1933), pp. 193-199.

(Magazines)

Anning, Norman. "High School Mathematics Club". The Mathematics Teacher, XXVI (February, 1933) pp. 70-76.

Beatley, Ralph. "Second Report of the Committee on Geometry." The Mathematics Teacher. XXVI (October, 1933) pp. 266-271. Presented to the National Council of Teachers of Mathematics at its annual Meeting in Minneapolis on February 24, 1933.

Beatley, Ralph. "Preliminary Report of the Committee on Geometry". The Mathematics Teacher, XXV (November, 1932). pp. 427-428.

Blackwell, Walter H. "Daily Testing." The Mathematics Teacher, XXVI (April, 1933). pp. 227-234.

Christofferson, H. O. "A Fallacy in Geometric Reasoning". The Mathematics Teacher, XXIII (January, 1930), pp. 19-22.

Cook, Mary R. "Stimulate The Interest in Mathematics by Creating a Mathematical Atmosphere." The Mathematics Teacher. XXIV (April, 1931), pp. 248-254.

Engle, B. T. "Proportion". School Science and Mathematics. XXXII (March, 1932), pp. 268-271.

Engle, B. T. "Constructing a Transit as a Project in Geometry". The Mathematics Teacher. XXIV (November, 1931) pp.444-447.

Georges, J.S. "Learning Product of a Unit of Instruction in Mathematics." School Science and Mathematics. XXXI (June, 1931), pp. 695-705.

Hoag, Ruth. "Source of Program Material and Some Types of Program Work by High School Mathematics Club". The Mathematics Teacher. XXIV (December, 1931). pp. 492-502

Horn, Aaron. "Book Propositions in Teaching Geometry". The Mathematics Teacher. XXV (February, 1932), pp. 75-78.

Hutton, Mary O. "The Mathematics Club". The Mathematics Teacher, XX (January, 1927). pp. 38-45.

Hennessey, F. P. "The Principles of Continuity". The Mathematics Teacher. XXIV (January, 1931). p. 32-40.

Johnson, J. T. "Adapting Instructional Material to Individual Differences in Learning." The Mathematics Teacher, XXVI (April, 1933), pp. 193-199.

(Magazines) (cont.)

Jacobs, J. M. "Reaching the Individual in Mathematics Instruction." School Science and Mathematics, XXXI (May, 1931), pp. 575-585.

Karpinski, Louis C. "Unity of Algebra and Geometry". School Science and Mathematics, XXXI (May, 1933), pp. 515-516.

McDill, R. M.
The Mathematics Teacher, XXI[V] (January, 1931), p. 16.

Mossman, Edith L. "A Mathematics Room That Speaks for Itself". School Science and Mathematics, XXXIII (April, 1933), pp. 423-430.

Murray, Ken "How Eyes Tell Lies". Quoted from "Popular Mechanics." (April 26, 1930). p. 28.

Nyeberg, Joseph A.
The Mathematics Teacher, XXVI (October, 1933) pp. 395-399.

Nyeberg, Joseph A. "Proving the Equality of the Base Angles of the Isosceles Triangle." The Mathematics Teacher. XXII (October, 1929), p. 318.

Nyeberg, Joseph A. "Verbatim Report of A Recitation in Geometry". School Science and Mathematics, XXXIII (February, 1933). p. 68.

Parks, Hapgood G. "A Mathematics Teacher Seeks Guidance in Guidance". The Mathematics Teacher, XXVI (February, 1933) pp. 89-98.

Pratt, Gertrude V. "Popularizing Plane and Solid Geometry". The Mathematics Teacher, XXI (November, 1928), p. 413.

Reeve, William D.
The Mathematics Teacher, XXVI (March, 1933). p. 161.

Ramon, W. R. "Geometry Measures Land." The Mathematics Teacher, XXIII (April, 1930) p. 246.

(Magazines) (cont.)

Jacobs, J. M. "Reaching the Individual in Mathematics Instruction." School Science and Mathematics, XXXI (May, 1931), pp. 575-585.

Karpinski, Louis C. "Unity of Algebra and Geometry". School Science and Mathematics, XXXI (May, 1933), pp. 515-516.

McDill, R. M.
The Mathematics Teacher, XXIV (January, 1931), p. 16.

Moseman, Edith L. "A Mathematics Room That Speaks for Itself". School Science and Mathematics, XXXIII (April, 1933), pp. 425-430.

Murray, Ken "How Eyes Tell Lies". Quoted from "Popular Mechanics." (April 26, 1930). p. 28.

Nyberg, Joseph A.
The Mathematics Teacher, XXVI (October, 1933) pp. 395-399.

Nyberg, Joseph A. "Proving the Equality of the Base Angles of the Isosceles Triangle." The Mathematics Teacher. XXII (October, 1929), p. 318.

Nyberg, Joseph A. "Verbatim Report of A Recitation in Geometry". School Science and Mathematics, XXXIII (February, 1933). p. 88.

Parke, Hapgood G. "A Mathematics Teacher Seeks Guidance in Guidance". The Mathematics Teacher, XXVI (February, 1933) pp. 89-98.

Pratt, Gertrude V. "Popularizing Plane and Solid Geometry". The Mathematics Teacher, XXI (November, 1928), p. 413.

Reeve, William D.
The Mathematics Teacher, XXVI (March, 1933). p. 161.

Ramon, W. R. "Geometry Measures Land." The Mathematics Teacher, XXIII (April, 1930) p. 246.

(Magazines) (cont.)

Richmond, Vesta A. "A Number of Things for Beginners in Geometry". The Mathematics Teacher, XX (March, 1927), pp. 142-149.

Rabourn, S. B. F. "A Few Classroom Devices to Stimulate Interest in Mathematics". The Mathematics Teacher, XX (October, 1927), pp. 328-333.

Salkind, Charles. "Functional Geometry". The Mathematics Teacher, XXI (December, 1932), pp. 485-486.

Schnell, Leroy H. "An Attempt to Apply the Principles A Progressive Education to the Teaching of Geometry". The Mathematics Teacher, XXVI (March, 1933), pp. 163-175.

Selleck, George H. "Loco-phobia--It's Causes and Cure." The Mathematics Teacher, XXII (November, 1929). pp. 382-384.

Smith-Reeve-Morse. "Text and Tests in Plane Geometry". The Mathematics Teacher, XXVI (February, 1933), pp. 119-121.

Woodford, E. B. "Teaching Locus Concept in Plane Geometry" The Mathematics Teacher. XXVI (February, 1933), pp. 105-106.

Zerbe, Hobson, M. "The Elements of Plane Geometry in High School Physics". School Science and Mathematics, XXX (June, 1930), pp. 665-667.

(Magazines) (cont.)

Richmond, Vesta A. "A Number of Things for Beginners in Geometry". The Mathematics Teacher, XX (March, 1927), pp. 142-149.

Rabourn, S. B. F. "A Few Classroom Devices to Stimulate Interest in Mathematics". The Mathematics Teacher, XX (October, 1927), pp. 328-333.

Salkind, Charles. "Functional Geometry". The Mathematics Teacher, XXI (December, 1928), pp. 483-488.

Schnell, Leroy E. "An Attempt to Apply the Principles A Progressive Education to the Teaching of Geometry". The Mathematics Teacher, XXVI (March, 1933), pp. 165-175.

Sallack, George H. "Loco-phobia--It's Causes and Cure." The Mathematics Teacher, XXII (November, 1929), pp. 382-384.

Smith-Reeve-Morse. "Text and Tests in Plane Geometry". The Mathematics Teacher, XXVI (February, 1933), pp. 119-121.

Woodford, E. B. "Teaching Locus Concept in Plane Geometry" The Mathematics Teacher, XXVI (February, 1933), pp. 108-106.

Zerbe, Hobson, M. "The Elements of Plane Geometry in High School Physics". School Science and Mathematics, XXX (June, 1930), pp. 655-657.

(Bulletins)

Allen, Gertrude E. An Experiment in Redistribution of Material for High School Geometry, 1930, pp. 73, 81-82. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Allen, Gertrude E. Reorganization of Course of Study for Senior High School Mathematics, 1927, pp. 256, 267-268, 298. The National Council of Teachers of Mathematics. The Second Yearbook.

Beatley, Ralph. Coherence and Diversity in Secondary Mathematics, 1933, p. 198. The National Council of Teachers of Mathematics, The Eighth Yearbook.

Benz, H. E. A Summary of Some Scientific Investigations of the Teaching of High School Mathematics, 1933, pp. 29-30. The National Council of Teachers of Mathematics, The Eighth Yearbook.

Betz, William. The Teaching of Intuitive Geometry, New York Syllabus in Junior High School Mathematics, 1928-1933, pp. 84-95 116, 133, 135, 136, 144, 151, 155. The National Council of Teachers of Mathematics. The Eighth Yearbook.

Betz, William. Quotation from Olive A. Kee, A Mathematical Atmosphere, 1928, pp. 268. The Third Yearbook. The National Council of Teachers of Mathematics. The Fifth Yearbook. pp. 138-140.

Birkhoff, George D., and Beatley, Ralph. A New Approach to Elementary Geometry, 1930, p. 87. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Breslich, Ernest R. Demonstrative Geometry and Functional Thinking, 1928, p. 51. The National Council of Teachers of Mathematics. The Third Yearbook.

Gugle, Marie, and others. Recreational Values Obtained Through Mathematics Club, 1926. pp. 194-200. The National Council of Teachers of Mathematics. The First Yearbook.

(Bulletins)

Allen, Gertrude E. An Experiment in Redistribution of Material for High School Geometry, 1930, pp. 73, 81-82. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Allen, Gertrude E. Reorganization of Courses of Study for Senior High School Mathematics, 1927, pp. 256, 267-268, 298. The National Council of Teachers of Mathematics. The Second Yearbook.

Beatley, Ralph. Coherence and Diversity in Secondary Mathematics, 1933, p. 198. The National Council of Teachers of Mathematics, The Eighth Yearbook.

Bene, H. E. A Summary of Some Scientific Investigations of the Teaching of High School Mathematics, 1933, pp. 29-30. The National Council of Teachers of Mathematics, The Eighth Yearbook.

Betz, William. The Teaching of Intuitive Geometry, New York Syllabus in Junior High School Mathematics, 1928-1933, pp. 84-95 116, 133, 135, 136, 144, 151, 155. The National Council of Teachers of Mathematics. The Eighth Yearbook.

Betz, William. Quotation from Olive A. Lee, A Mathematical Atmosphere, 1928, pp. 268. The Third Yearbook. The National Council of Teachers of Mathematics. The Fifth Yearbook. pp. 138-140.

Birkhoff, George D., and Beatley, Ralph. A New Approach to Elementary Geometry, 1930, p. 87. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Breslich, Ernest R. Demonstrative Geometry and Functional Thinking, 1928, p. 81. The National Council of Teachers of Mathematics. The Third Yearbook.

Gugle, Marie, and others. Recreational Values Obtained Through Mathematics Club, 1926. pp. 194-200. The National Council of Teachers of Mathematics. The First Yearbook.

(Bulletins) (cont.)

Hedrick, E. R. The Reality of Mathematical Processes, 1928. pp. 35-41. The National Council of Teachers of Mathematics. The Third Yearbook.

Lide, Edwin S. Instructions in Mathematics. Bulletin, 1930, p. 51, Number 17. Monograph 23. United States Department of the Interior. Office of Education.

Orleans, Joseph B. A Unit of Demonstrative Geometry for the Ninth Year. 1930, pp. 45, 51-52. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Reeve, William D. United States. 1929, pp. 135-150, 162. The National Council of Teachers of Mathematics. The Fourth Yearbook.

The Reorganization of Mathematics in Secondary Education. A Report by The National Committee on Mathematical Requirements 1923, p. 34. under the Auspices of The Mathematical Association of America, Incorporated, The Mathematical Association of America, Incorporated.

Swenson, John A. Graphic Methods of Teaching Congruence in Geometry. 1930. pp. 96-97, 99-100. The National Council of Teachers of Mathematics, The Fifth Yearbook.

Schlauch, W. S. Mathematics as an Interpreter of Life, 1928. pp. 28. The National Council of Teachers of Mathematics. The Third Yearbook.

Schlauch, W. S. The Analytical Method in the Teaching of Geometry, 1930. pp. 134-144. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Schloring, Raleigh. Suggestions for the Solution of the Important Problems That Have Arisen in the Last Quarter of the Century, 1930. p. 105. The National Council of Teachers of Mathematics. The Fifth Yearbook.

The Problem of Mathematics in Secondary Education. Department of the Interior, Bureau of Education, Bulletin, 1920, Number 1. pp. 12-13.

The Teaching of Geometry, 1930. The National Council of Teachers of Mathematics, The Fifth Yearbook. Bureau of Publication. Teachers College, Columbia University, New York City.

(Bulletins) (cont.)

Hedrick, E. R. The Reality of Mathematical Processes. 1928. pp. 36-41. The National Council of Teachers of Mathematics. The Third Yearbook.

Lide, Edwin S. Instructions in Mathematics. Bulletin, 1920, p. 31, Number IV. Monograph 23. United States Department of the Interior. Office of Education.

Orleans, Joseph B. A Unit of Demonstrative Geometry for the Ninth Year. 1930. pp. 45, 31-32. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Reeve, William D. United States. 1929. pp. 159-160, 162. The National Council of Teachers of Mathematics. The Fourth Yearbook.

The Reorganization of Mathematics in Secondary Education. A Report by The National Committee on Mathematical Requirements 1923. p. 34. under the Auspices of The Mathematical Association of America, Incorporated. The Mathematical Association of America, Incorporated.

Swenson, John A. Graphic Methods of Teaching Congruence in Geometry. 1930. pp. 96-97, 99-100. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Schlauch, W. S. Mathematics as an Interpreter of Life. 1928. pp. 28. The National Council of Teachers of Mathematics. The Third Yearbook.

Schlauch, W. S. The Analytical Method in the Teaching of Geometry, 1930. pp. 134-144. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Schloring, Ralph. Suggestions for the Solution of the Important Problems That Have Arisen in the Last Quarter of the Century, 1930. p. 105. The National Council of Teachers of Mathematics. The Fifth Yearbook.

The Problem of Mathematics in Secondary Education. Department of the Interior, Bureau of Education, Bulletin, 1920, Number 1. pp. 12-13.

The Teaching of Geometry, 1930. The National Council of Teachers of Mathematics, The Fifth Yearbook. Bureau of Publication. Teachers College, Columbia University, New York City.

(Bulletins) (cont.)

The Teaching of Mathematics in the Secondary School, 1933, The National Council of Teachers of Mathematics. The Eighth Yearbook. Bureau of Publications. Teachers College. Columbia University, New York City.

Upton, Clifford B. The Use of Indirect Proof in Geometry and in Life. 1930, pp. 102-133. The National Council of Teachers of Mathematics, The Fifth Yearbook.

Wilt, Mary C. Teaching Plane and Solid Geometry Simultaneously. 1932. p. 66. The National Council of Teachers of Mathematics, The Fifth Yearbook.

Wolff, George. The Mathematical Collection, 1933. pp. 216-263. The National Council of Teachers of Mathematics. The Eighth Yearbook.

Young, John W. Symmetry. 1930. pp. 145-148. The National Council of Teachers of Mathematics, The Fifth Yearbook.

(Bulletins) (cont.)

The Teaching of Mathematics in the Secondary School. 1933. The National Council of Teachers of Mathematics. The Eighth Yearbook. Bureau of Publications. Teachers College. Columbia University, New York City.

Upton, Clifford B. The Use of Indirect Proof in Geometry and in Life. 1930. pp. 102-133. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Wilt, Mary C. Teaching Plane and Solid Geometry Simultaneously. 1932. p. 66. The National Council of Teachers of Mathematics. The Fifth Yearbook.

Wolff, George. The Mathematical Collection. 1933. pp. 216-263. The National Council of Teachers of Mathematics. The Eighth Yearbook.

Young, John W. Symmetry. 1930. pp. 145-148. The National Council of Teachers of Mathematics. The Fifth Yearbook.

(Unpublished Material)

Eddy, Louis Barbour. "Motivating the Study of Geometry." Unpublished Master's Thesis, Chicago University, 1930.

Templeton, Agnes Wallace. "A Survey of the Magazine, 'The Mathematics Teacher'." Unpublished Master of Arts'Thesis of Boston University, 1932.

Wolf, Alma. "The Teaching of Secondary Mathematics." Unpublished Master of Arts' Thesis of Boston University, 1932.

(Monograph)

Bradley, A. Day. The Geometry of Repeating Design and Geometry of Design for High Schools. 1933. v-vi--3-131.

(Unpublished Material)

Eddy, Louis Barnour. "Motivating the Study of Geometry." Unpublished Master's Thesis, Chicago University, 1930.

Templeton, Agnes Wallace. "A Survey of the Magazine 'The Mathematics Teacher'." Unpublished Master of Arts' Thesis of Boston University, 1932.

Wolf, Alma. "The Teaching of Secondary Mathematics." Unpublished Master of Arts' Thesis of Boston University, 1932.

(Monograph)

Bradley, A. Day. The Geometry of Repeating Design and Geometry of Design for High Schools. 1933. V-vi--3-131.

INDEX

Algebra, 120, 129, 139, 145, 224, 240, 243.

Analogies, 17, 18, 221, 222.

Angle, Applications,
- Book, 235, 237.
- Compass, 235, 237.
- Pointer, 237.
- Radio Dial, 47, 235, 238.

Angle, Reading, 49.
- Rotation, 46, 48, 238.
- Size, 47.
- Sum, 48.

Angle, Included, 52.
- Definition, 50.
- Drill, 99, 238.
- Test, 100.
- Wooden Triangles, 238.

Applied Problems
- Area polygon, 158-159.
- Brace, 106-107.
- Building, 67, 106.
- City House, 42, 44, 235.
- Expansion, 135-136, 236, 242.
- Fence corner, 42, 44, 235.
- Flag pole, 67.
- Forest fire, 41, 43, 235.
- Inclined plane, 236, 242.
- River, 97-99.
- Ship, 42, 45, 235, 242.
- Tank, 176.

Approach, gradual,
- Demonstrations, 22.
- Exercises, 22.
- Testing, 23.

Assignments, 33, 48.

Cartoon,
- Language necessity, 39.

Challenge, 220.

Chemistry, 138-139.

Class built, 184, 238.

Colored chalk, 212, 245-246.
- Angle, 207, 218.
- Congruency, 100.
- Hypothesis, 94.
- Included angle, 50.
- Lines, 95, 100, 104, 208.
- Mountain, 108.
- Three roads, 114.

Conference groups, 187.

Congruency,
- Application,
  - Wage chart, 103
- Applied problem,
  - River, 97-98.
- Assignments,
  - Bringing milk bottles, etc. 102.
  - Differing, 171-172.
  - Dissection, 104.
  - Drawing, 104.
  - Individual, 101.
  - Literary articles, 102.
  - Superposition, 104.
  - Tracing, 104.
- Construction,
  - Cardboard triangles, 99.
  - Wooden triangles, 99.
  - Wooden models, 102, 104.
- Examples,
  - Castings, Lasts, Reprints 105.
- Proof, 115-119.
- Use, Finding unknown side of triangle, 102.

INDEX

Airshore, 120, 129, 139, 145, 224, 240, 243.

Analogies, 17, 18, 221, 222.

Angle, Applications,
Book, 235, 237.
Compass, 235, 237.
Pointer, 237.
Radio Dial, 47, 235, 238.

Angle, Reading, 49.
Rotation, 46, 48, 238.
Size, 47.
Sum, 48.

Angle, Included, 52.
Definition, 50.
Drill, 99, 238.
Tool, 100.
Wooden Triangles, 238.

Applied Problems

Area polygon, 158-159.
Brace, 106-107.
Building, 67, 106.
City House, 42, 44, 235.
Expansion, 135-136, 236, 242.
Frame corner, 42, 44, 235.
Flag pole, 67.
Forest fire, 41, 43, 235.
Inclined plane, 236, 242.
River, 97-99.
Ship, 42, 45, 235, 242.
Tank, 176.

Approach, gradual,
Demonstrations, 22.
Exercises, 22.
Testing, 23.

Assignments, 33, 48.

Cartoon,
Language necessity, 39.

Chalkboard, 220.

Chemistry, 138-139.

Class built, 184, 238.

Colored chalk, 212, 245-246.
Angle, 207, 216.
Congruency, 100.
Hypothesis, 94.
Included angle, 50.
Lines, 95, 100, 104, 208.
Mountain, 108.
Three roads, 114.

Conference group, 187.

Congruency,
Application,
Base chart, 103
Applied problem,
River, 97-98.
Assignments,
Bringing milk bottles, etc. 102.
Differing, 171-172.
Dissection, 104.
Drawing, 104.
Individual, 101.
Literary articles, 102.
Superposition, 104.
Tracing, 104.
Construction,
Cardboard triangles, 99.
Wooden triangles, 99.
Wooden models, 102, 104.
Examples,
Cabtings, Lists, Reprints, 105.
Proof, 115-119.
Use, Finding unknown side of triangle, 102.

INDEX- Cont.

Constructions,
Advantages of, 62-63.
Disadvantages, 63-64.
Difficulties, 65.
Gothic windows, 59.
Kite, 59.
List, 58.
Pantograph, 67, 124-125.
Protractor, 67.
Quatrefoils, 59.
Star polygon, 59.
Trefoils, 59.
Transit, 67.
Wooden models, 239.

Continuity,
Graphic, 162.
Measurement of angles by arcs, 160, 162.
Product of segments, 160-161.
Pythagorean, 160-161.
Trapezoid and triangle, 161.

Definitions,
Corresponding sides, 128.
Geometry, 30, 235.
Need of, 38-39.
Teaching, 37.

Demonstration,
Approach gradual, 22.
Evaluation of, 186.

Difficulties, General,
Abstract nature, 10,243.
Cumulative nature, 8,15.
Fear, 25-26.
Flatland, 7-9.
New Procedure, 21, 243.
Reflective nature, 8, 21.

Difficulty isolated, 15,231,246.
Approach gradual, 15.
Definitions, 55-56.
Enlargement, 16.
Included angle, 51.
Line in relief, 16,246.
Parallel steps, 17
Restoration to simple situation, 19-20,246.
See-it-grow, 18, 66,67, 183.
Slow motion, 19.
Slow question, 19.
Slow answer, 19.

Distorted Figure, 209-211.

Direction sheets, 187.

Divine Proportion, 128.

Double Identities, 159, 216

Drawing, 212.

Drill,
Characteristics, 202.
Correct start, 201.
Directions, 203-205.
General, 233.
Hypothesis, 93.
Included angle, 52,99.
Necessity, 198-199.
Note books, 202.
Place of, 199-202.
Reason for, 197-198.
Sheet, 187.

Equality,
Lines, angles, triangles, 215-216.

Errors, Common, 14, 251.
Congruency, 51-55.
Construction lines, 65-66.
List, 190. 109-112.
Recording, 189.
Similar triangles, 133.
Tests on, 189.

Exercises,
Gradual complexity, 22.

Familiar, 13, 245.
Confections, 5, 14, 37, 51-52, 219, 243.

INDEX- Cont.

Constructions,
Advantages of, 62-63.
Disadvantages, 63-64.
Difficulties, 65.
Gothic windows, 59.
Kite, 59.
Lint, 58.
Pantograph, 67, 124-125.
Protractor, 67.
Quatrefoils, 59.
Star polygon, 59.
Trefoils, 59.
Transit, 67.
Wooden models, 219.

Continuity,
Graphic, 162.
Measurement of angles
by area, 160, 162.
Product of segments,
160-161.
Pythagorean, 160-161.
Trapezoid and triangle, 161.

Definitions,
Corresponding sides, 128.
Congruity, 30, 235.
Need of, 38-39.
Teaching, 37.

Demonstration,
Approach gradual, 22.
Evaluation of, 185.

Difficulties, General,
Abstract nature, 10, 243.
Cumulative nature, 8, 15.
Fear, 25-26.
Fatigue, 7-9.
New procedure, 21, 243.
Reflective nature, 6, 21.

Difficulty Isolated, 15, 231, 246.
Approach gradual, 15.
Definitions, 55-56.
Enlargement, 16.
Included angle, 51.
Line in relief, 16, 246.
Parallel steps, 17
Reaction to simple
situation, 19-20, 246.
Eye-hand-grow, 18, 66, 67,
183.
Slow motion, 19.
Slow question, 19.
Slow answer, 19.

Distorted Figure, 209-211.

Division Areas, 187.

Divine Proportion, 128.

Double Identities, 189, 216

Drawing, 212.

Drill,
Characteristics, 202.
Correct start, 201.
Directions, 203-205.
General, 233.
Hypothesis, 93.
Included angle, 52, 99.
Necessity, 198-199.
Note books, 202.
Place of, 199-202.
Reason for, 197-198.
Sheet, 187.

Equality,
Lines, angles, triangles,
215-216.

Error, Common, 14, 251.
Congruency, 51-55.
Construction lines, 65-66.
List, 190. 109-112.
Reporting, 189.
Similar triangles, 133.
Tests on, 189.

Exercises,
Gradual complexity, 22.

Familiar, 13, 248.
Conversations, 5, 14, 37,
51-52, 219, 243.

INDEX-- Cont.

Familiar,
Games, 14, 220.
Checkberboard, 31.
Cribbage board, 31.
Puzzle board, 30.
Teeter board, 235, 243.
Homes, 13.
Embroidery, 32
Furniture, 242.
Quilts,242.
Oil Cloth, 32,40,242.
Wall paper, 40, 242.
Miscellaneous,
Building, 67
Fence, 42,44,235
Flag pole, 67.
Ship, 42,235.
Water tank, 67.
Nature,
Flowers, 242.
Honeycomb, 30,242.
Leaf, 242.
Minerals, 30.
Parks, 31.
Pictures, 235.
Public gardens, 31.
Salt crystals, 242.
Snow flakes, 31.
Occupations,
Dressmaking, 235
Lookout station,41-43.
Medicine, 235.
Photography, 235.
School,
Bell,clock, cylinder, 31.
School Subjects,
Algebra, 235.
Chemistry, 235.
Drawing, 235.
Physics, 235.
Trigonometry, 235
Sports,
Baseball diamond, 14,18,30.
Football, 31.
Hockey, 31.
Picnic, 90,235.

Fear, 25,26

Folding, 211

Goal, 230.

Golden ratio, 128.

Graphs,
Algebraic, 234.
Area of quadrilaterals, 157.
Bridge, 213,228,240.
Column, 213.
Concept, 227.
Isosceles triangls, 108.
Arc measurement, 162.
Memory, 229.
Mountain, 108, 165,240.
Parasol, 214-215.
Polygons, 54, 157.
Success, 228-229.
Stream, 132, 214, 240.
Three Roads, 114.
Thinking, 229.

Grouping,
Polygons, 53-54, 156-157.

Humanizing,
Angle, 198.
Circle, 30,37,219, 243.
Locus, 232.
Polygons, 53.
Parallelogram, 198-199, 219, 237.
Transversal, 90.
Triangle, 37, 219.
Sector, 219.

Humor,
Big feet, 126.
Bugs, 100.
Fish, 126, 246.
Fly, 126,246.
Masks, 126,246.
Stilts, 126,246.

Polygons, 53-54.

Hypothesis and conclusion, 235.

Declarative sentence difficulty, 89.

INDEX-- Cont.

Familiar,
Games, 14, 220.
Checkerboard, 31.
Cribbage board, 31.
Puzzle board, 30.
Tester board, 235, 243.
Homes, 15.
Embroidery, 32
Furniture, 242.
Quilts, 242.
Oil Cloth, 32, 40, 242.
Wall paper, 40, 242.
Miscellaneous,
Building, 67
Fence, 42, 44, 235
Flag-pole, 67.
Ship, 42, 235.
Water tank, 67.
Nature,
Flowers, 242.
Honeycomb, 30, 242.
Leaf, 242.
Minerals, 30.
Parks, 31.
Pictures, 235.
Public gardens, 31.
Salt crystals, 242.
Snow flakes, 31.
Occupations,
Dressmaking, 235
Lookout station, 41-43.
Medicines, 235.
Photography, 235.
School,
Ball, clock, cylinder, 31.
School Subjects,
Algebra, 235.
Chemistry, 235.
Drawing, 235.
Physics, 235.
Trigonometry, 235
Sports,
Baseball diamond, 14, 18, 30.
Football, 31.
Hockey, 31.
Tennis, 90, 235.

Fear, 25, 26

Folding, 211

Goal, 230.

Golden ratio, 128.

Graphs,
Algebraic, 234.
Area of quadrilaterals, 157.
Bridge, 213, 228, 240.
Column, 213.
Concept, 227.
Isosceles triangle, 108.
Arc measurement, 162.
Memory, 229.
Mountain, 108, 165, 240.
Parabol, 214-215.
Polygons, 54, 157.
Success, 228-229.
Stream, 132, 214, 240.
Three Roads, 114.
Thinking, 229.

Grouping,
Polygons, 53-54, 156-157.

Humanizing,
Angle, 198.
Circle, 30, 37, 219, 243.
Locus, 232.
Polygons, 53.
Parallelogram, 198-199, 219, 237.
Transversal, 90.
Triangle, 37, 219.
Sector, 219.

Humor,
Big feet, 126.
Bugs, 103.
Fish, 126, 246.
Fly, 126, 246.
Masks, 126, 246.
Stilts, 126, 246.

Polygons, 53-54.

Hypothesis and conclusion, 235.

Declarative sentence difficulty, 89.

Hypothesis and Conclusion (Cont.)
Drill, 93.
English grammar, 237.
Hypo-meaning, 93.
Objective material, 90,235.
Pictures, 92.
Test, 94.
Notation,
Arrows, 94.
Braces, 95.
Colored chalk, 94.
Square for right angle, 94.

Illusions, General, 69,71,244.
Angles, 79.
Continuation of line, 82.
Circles and parallel lines, 77.
Dots, 83.
Electric light, 81.
Lindbergh, 73.
Ladder, 83.
Lines, 76, 78.
Oar, 80.
Parallel lines, 82.
Parallel lines with transversals, 82.
Parallel lines with cross lines, 82-83.
Parallel lines with blocks, 83.
Rabbite-duck, 84.
Squares, 79.
Stairs, 79.
Trapezoids, 82.

Illusions, After images,
Lady in chair, 85.
Rider and horse, 85.
Cross, 85.

Illusions, Complementary Colors, 85-86.

Illusions, Other Sensory,
Dry ice, 86.
Ice, 86.
Boat and bridge, 87.
Auto riding, 87.
Airplane position, 86.
Train motion, 86.

Illusions,
Instruments correcting,
Compass, 87.
Speedometer, 87.
Thermometer, 87.
Watch, 87.

Imaginary numbers, 236,243.

Individual interests,
Hand, 247.
Coloring, 34, 173.
Designing, 34, 173, 245.
Drawing, 34, 173, 247.
Lettering, 34, 173.
Manual training, 34, 172, 247.
Mapping, 34, 173.
Painting, 34, 173.

Problem, 173, 245.
Puzzles, 34.

Scientific,
Astronomy, 34, 173.
Photography, 34, 173.
Physics, 173.

Social,
Bulletin board, 173.
Exhibits, 173.
Games, 34.

Verbal,
Anecdotes, 172.
Biographies, 173.
Dictionary of terms, 172.
History, 172.
Plays, 34, 172, 247.
Magazine articles, 173.
Newspaper articles, 173.
Reports, 173.
Talks, 34, 247.
Writing, 34.

Hypothesis and Conclusion (Cont.)
Drill, 93.
English grammar, 237.
Hypo-meaning, 93.
Objective material, 90,235.
Pictures, 92
Text, 94.
Notation,
Arrows, 94.
Braces, 95.
Colored chalk, 94.
Square for
right angle, 94.

Illustrations, General, 69,71,244.
Angles, 75.
Continuation of line, 82.
Circles and parallel lines, 77.
Dots, 83.
Electric light, 81.
Lindbergh, 73.
Ladder, 83.
Lines, 76, 78.
Oar, 80.
Parallel lines, 82.
Parallel lines
with transversals, 82.
Parallel lines
with cross lines, 82-83.
Parallel lines
with blocks, 83.
Rabbit-duck, 84.
Squares, 75.
Stairs, 75.
Trapezoids, 82.

Illustrations, After images,
Lady inkbelt, 85.
Rider and horse, 85.
Cross, 85.

Illustrations, Complementary Colors,
85-86.

Illustrations, Other Sensory,
Dry ice, 86.
Ice, 86.
Boat and bridge, 87.
Auto riding, 87.
Airplane position, 86.
Train motion, 86.

Illustrations,
Instruments of reading,
Compass, 87.
Speedometer, 87.
Thermometer, 87.
Watch, 87.

Imaginary numbers,
236,243.

Individual interests,
Hand, 247.
Coloring, 34, 173.
Designing, 34, 173,
245.
Drawing, 34, 173,
247.
Lettering, 34, 173.
Manual training,
34, 172,247.
Mapping, 34, 173.
Painting, 34, 173.

Problem, 173, 245.
Puzzles, 34.

Scientific,
Astronomy, 34, 173.
Photography, 34, 173.
Physics, 173.

Social,
Bulletin board, 173.
Exhibits, 173.
Games, 34.

Verbal,
Anecdotes, 172.
Biographies, 173.
Dictionary of
terms, 172.
History, 172.
Plays, 34, 172, 247.
Magazine articles,
173.
Newspaper articles,
173.
Reports, 173.
Talks, 34,247.
Writing, 34.

Inhibitions, 26

Instruments,
List, 61-62.

Interrelationships, 233.
Algebra, 224.
Astronomy, 225.
Manual Training, 222.
Mechanical drawing, 223.
Physics, 222-223.
Politics, 225.
Solid Geometry, 223.

Isosceles triangle,
Brace, 106-107.
Construction, 116.
Methods of proof, 118.
Mountain graph, 108.
Perpendiculars, 118-119.
Superposition, 116-117.

Lines, 208-209.

Loci,
Airplane propeller, 146.
Algebraic,graph, 145.
Automobile gears, 147.
Chemistry, 147-148.
Chalk, 144.
Clock, 236.
Cow, 143.
Design, 147.
Dog, 143, 236.
Door, 144-145, 236.
Engine, 147.
Eraser, 144-145, 236.
Fly wheel, 143, 236.
Levers, 143.
Machinery, 143.
Manual training, 147.
Milk separator, 147.
Movies, 145.
Order of proof, 149-150.
Pen, 144.
Pencil, 144.
Pointer, 144.
Proof, 150.
Pumping station, 143.
Questions, 148.
Solid geometry, 145.
Suggestions, 151-152.
Tracing, 144.
Wooden models, 144.

Manual Training, 147, 222, 243.

Mathematics Club, 175-178. 247.

Mathematics Room, 178-180, 248.

Meaning of Geometry, 30.

Measurement,
Opportunities for, 60-61.

Mechanical Drawing, 243.

Medicine, 139.

Methods of proof, 163-166.
Analytic, 169, 189.
Heuristic, 166.
Indirect, 167-168.
Inductive, 167.
Synthetic, 166, 169.

Notation, 208.

Note books, 187.

Objective material,
Book, 46, 237.
Cardboard, 209, 237.
Compass, 46, 237.
Globe, 46, 237.
Pointer, 46.
Locus material,classroom, 145.
Locus material, physics laboratory, 147.
Radio disk, 47, 248.
Watch, 46.
Wooden triangles, 100,237.

Organization,
Advantages of, 155-156.
Illustrations, quadrilateral areas, 156-157.

Pantograph, 124-125, 247.

Parallelism, 216-217.

Photography, 124.

Inhibitions, 26

Instruments,
List, 61-62.

International Exhibitions, 253.
Algebra, 224.
Astronomy, 225.
Manual Training, 222.
Mechanical drawing, 223.
Physics, 222-223.
Politics, 225.
Solid Geometry, 223.

Isosceles triangle,
Brace, 106-107.
Construction, 116.
Methods of proof, 118.
Mountain grade, 108.
Perpendicular, 118-119.
Superposition, 116-117.

Lines, 208-209.

Lost,
Airplane propeller, 146.
Algebraic graph, 145.
Automobile gears, 147.
Chemistry, 147-148.
Chalk, 144.
Clock, 236.
Cow, 143.
Design, 147.
Dog, 143, 236.
Door, 144-145, 236.
Engine, 147.
Eraser, 144-145, 236.
Fly wheel, 143, 236.
Levers, 143.
Machinery, 143.
Manual training, 147.
Milk separator, 147.
Movies, 145.
Order of proof, 149-150.
Pen, 144.
Pencil, 144.
Pointer, 144.
Proof, 150.
Pumping station, 143.
Quadrations, 148.
Solid geometry, 145.
Suggestions, 151-152.
Tracing, 144.
Wooden models, 144.

Manual Training, 147, 222, 243.

Mathematics Club, 175-176. 247.

Mathematics Room, 176-180. 248.

Meaning of Geometry, 30.

Measurement,
Opportunities for, 60-61.

Mechanical Drawing, 243.

Meditine, 139.

Methods of proof, 165-166.
Analytic, 169, 189.
Heuristic, 166.
Indirect, 167-168.
Inductive, 167.
Synthetic, 166, 169.

Notation, 208.

Note books, 187.

Objective material,
Book, 46, 237.
Cardboard, 209, 237.
Compass, 46, 237.
Globe, 46, 237.
Pointer, 46.
Loose material, classroom, 145.
Loose material, physics laboratory, 147.
Radio dial, 47, 248.
Watch, 46.
Wooden triangles, 100, 237.

Organization,
Advantages of, 155-156.
Illustrations, quadrilateral areas, 156-157.

Pantograph, 124-125, 247.

Parallelism, 216-217.

Photography, 124.

Physics, 135-136, 222-223, 235.

Pictures, enlargement, 124,247.

Plays, 219.

Poems, 220.

Polygons, 53-54.

Proportion and Similarity,
Algebra, 120,129, 235.
Applied problems,
Chemical calculations, 138-139, 122, 234, 242.
Expansion rod, 135-136, 242.
Graduating violin, 140.
Inclined plan, 136-137, 242.
Medicine mixing, 139.
Teeter board, 140-141.
Trade school, 140

Calculations, 122.

Congruence a special case, 129.
Construction, pantograph, 124-125.
Dressmaking, 122, 127, 235.
Enlarging pictures, 124,235.
Error, common, 133.
General method, 132.
Golden ratio, 128.
Graphic representati on, 132.
Human body, 127, 134.
Humor,
Big feet, fish, fly, mask, stilts, 126.
Indentation of steps, 133.
Literature, outline, 123.
Pantograph, 124-125.
Photography, 123,235.
Proof, 132.
Test, 134.
Value,
Line equality, 130,
Trigonometry tables, 130. 236.
Time saver, 131,242.

Protractor, 67.

Pythagorean, 160-161.

Pupils, 188

Reasons for taking, 195.

Safety margin, 23.

Sensory approaches, 11-12.

Similarity,
See proportion.

Symmetry,
Definition, 35.
Illustrations,- berries, flowers, grapes, leaves sand dollars, shells, spheres, starfish, 34.

eys, ears, hands, clothing, gloves, shoes, furniture, picture, butterlies, vase, shapely grees, balanced stage, kite, isosceles triangle, fruit, orange sections, games, teeter board, basketball floor, checker board, parchesi board, 35.

Slow pupil, 231.

Solid Geometry, 7,9,145,223.

Teacher,
Personality of, 24-25.
Place of, 185.

Teeter board, 140-141,242.

Testing, 188-194, 23, 94, 134.

Trade school, 140.

Transit, 67.

Triangle, 30-60-90, 217-218.

Trigonometry 130, 236,242.

Universality, 24.

Violin, 140, 236, 242.

Wooden models, 239, 247.

Work books, 187.

Pyramids, 135-136, 222-223, 235.

Pictures, enlargement, 124,247.

Plays, 219.

Poems, 220.

Polygons, 53-54.

Proportion and Similarity.
Algebra, 120, 129, 235.
Applied problems.
Chemical calculations, 138-139, 122, 234, 242.
Expansion rod, 135-136, 242.
Graduating violin, 140.
Inclined plan, 136-137, 242.
Mixing mixture, 139.
Tester board, 140-141.
Trade school, 140

Calculations, 122.

Congruence a special case, 129.
Construction, pantograph, 124-125.
Dressmaking, 122, 127, 235.
Enlarging pictures, 124,235.
Error, common, 133.
General method, 132.
Golden ratio, 128.
Graphic representation, 132.
Human body, 127, 134.
Humor.
Big feet, flat, fly, mask, stilts, 126.
Indentation of steps, 133.
Literature, outline, 123.
Pantograph, 124-125.
Photography, 125,235.
Proof, 132.
Test, 134.
Value.
Line equality, 130.
Trigonometry tables, 130, 236.
Time saver, 131,242.

Protractor, 67.

Pythagoreans, 166-161.

Pupils, 158

Reasons for taking, 195.

Safety margin, 23.

Senary approaches, 11-12.

Similarity.
See proportion.

Symmetry.
Definition, 35.
Illustrations, - berries, flowers, grapes, leaves, sand dollars, sea lile, spheres, starfish, 34.

eye, ears, hands, clothing, gloves, shoes, furniture, picture, butterflies, vase, chapely cross, balanced stage, kite, isosceles triangle, fruit, orange sections, games, tester board, basket ball floor, checker board, parchesi board, 35.

Slow mudl, 231.

Solid Geometry, 7,9, 145,223.

Teacher.
Personality of, 24-25.
Place of, 185.

Tester board, 140-141,242.

Testing, 188-194, 23, 94, 194.

Trade school, 140.

Transit, 67.

Triangle, 30-66-90, 217-218.

Trigonometry 130, 236,242.

Universality, 24.

Violin, 140, 236, 242.

Wooden models, 239, 247.

Work books, 187.

BOSTON UNIVERSITY
1 1719 02556 3976